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Modelling and migration with orthogonal isochron rays

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ABSTRACT

For increasing time values, isochrons can be regarded as expanding wavefronts and their perpendicular lines as the associated orthogonal isochron rays. The speed of the isochron movement depends on the medium velocity and the source-receiver position. We introduce the term equivalent-velocity to refer to the speed of isochron movement. In the particular case of zero-offset data, the equivalent velocity is half of the medium velocity. We use the concepts of orthogonal isochron-rays and equivalent velocity to extend the application of the exploding reflector model to non-zero offset imaging problems. In particular, we employ these concepts to extend the use of zero-offset wave-equation algorithms for modelling and imaging common-offset sections. In our imaging approach, the common-offset migration is implemented as a trace-by-trace algorithm in three steps: equivalent velocity computation, data conditioning for zero-offset migration and zero-offset wave-equation migration. We apply this methodology for modelling and imaging synthetic common-offset sections using two kinds of algorithms: finite-difference and split-step wavefield extrapolation. We also illustrate the isochron-ray imaging methodology with a field-data example and compare the results with conventional common-offset Kirchhoff migration. This methodology is attractive because it permits depth migration of common-offset sections or just pieces of that by using wave-equation algorithms, it extends the use of robust zero-offset algorithms, it presents favourable features for parallel processing, it permits the creation of hybrid migration algorithms and it is appropriate for migration velocity analysis.

INTRODUCTION

Isochron rays are curves associated with propagating isochrons, which is with surfaces that are related to seismic reflections with the same two-way traveltime. Isochron surfaces play an important role in seismic imaging because they are closely related to the impulse response of depth migration. They are fundamental to the general theory of 'data mapping', first presented geometrically by Hubral, Schleicher and Tygel (1996) and theoretically in its companion paper, Tygel, Schleicher and Hubral (1996), then referred to as 'unified approach to seismic imaging'. Bleistein, Cohen and Stockwell (2001) emphasized the importance of isochrons in the establishment of integral formulas for inversion.

Iversen (2004) introduced the term isochron ray for trajectories associated with surfaces of equal two-way time, i.e., isochron surfaces and suggested the potential use of isochron rays in future implementations of prestack depth migration. Here we exploit the idea and present a methodology that makes use of the isochron ray concept to perform prestack depth migration. We consider as isochron rays the curves that are perpendicular to the isochrons associated with the image produced by the migration of a single finite-offset seismic trace. That is, isochron rays are the orthogonal trajectories to isochrons. We use the term orthogonal isochron rays to distinguish this concept from the one introduced by

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Iversen (2004), which involves non-orthogonal trajectories. Although they are different, both methods are used to propagate isochrons. The reason to keep the term isochron ray arises from the analogy between the wavefront propagation and the isochron 'movement'. For simplicity, we sometimes use the term 'isochron ray' with the meaning 'orthogonal isochron ray'.

Our imaging approach consists in a trace-by-trace algorithm, wherein each finite-offset input trace is first conditioned for zero-offset extrapolation and then migrated using an equivalent velocity model. The conditioned data is a gather composed by the repetition of a time-shifted version of the input trace and the equivalent-velocity model corresponds to the isochron speed distribution.

We present two imaging strategies. In the first approach the prestack depth migration is achieved by performing the downward continuation of the conditioned data along the orthogonal isochron rays, followed by the application of the zero-offset imaging condition. The second imaging approach consists in a reverse-time migration algorithm, where the conditioned data is reversed and injected into the equivalent velocity model along an isochron defined by the time-shift previously applied on the input trace.

The main purpose of this research is the development of zero-offset wave-equation modelling and migration algorithms for finite offset problems. In particular, this methodology can be useful in the implementation of migration velocity analysis methods based on offset continuation (Silva 2005). Also, the presented methodology is attractive because (1) it permits depth migration of common-offset gathers using wave-equation algorithms, (2) it extends the use of robust zero-offset algorithms to the common-offset case, (3) it is based on algorithms that are appropriate for parallel processing and (4) it permits to combine different imaging algorithms.

THE ORTHOGONAL ISOCHRON RAY CONCEPT

Given a source-receiver pair and a fixed reflection traveltime, the related isochron is the surface that answers the question: where are the possible reflection points located? In other words, an isochron surface is the image of points with the same reflection time. Kinematically, an isochron surface can be viewed as a hypothetical reflector whose reflections from the source *S* are recorded simultaneously at the receiver *R*. For any point *M* belonging to an isochron surface, the traveltime measured along the path *SMR* does not vary.

The isochron surfaces play an important role in seismic imaging, especially in prestack depth migration. For a single trace composed of a sequence of impulses, the image produced by depth migration is represented by a set of isochrons. The longer the reflection time, the greater the distance between the source-receiver pair and the isochron. The shape of the isochrons depends on the velocity field, the reflection time and the spatial location of the source and receiver. In a homogeneous medium, every isochron has an ellipsoidal shape whose focus points are located at the source and receiver position, while the eccentricity is defined by the seismic wave velocity and the reflection time. The shallowest isochron surface tends to collapse into a straight line connecting the source-receiver pair.

Consider a sequence of depth migrated images where the input data consist in a sequence of seismic impulses with varying reflection time. Figure 1(a–c) illustrates the isochron 'movement' for the constant velocity case. Notice that the impulse time increases by the same amount with the first being just a little longer than the direct arrival traveltime. The initial surface (first isochron) observed in the first image moves in depth, changing its shape and acting as a propagating wavefront. If a point of the initial surface is selected and followed during the sequence, its orthogonal trajectory will define a curve. We refer to this curve as an isochron ray because it acts as a ray, while the moving isochron plays the role of a propagating wavefront. According to this abstraction, isochron rays may be defined as orthogonal lines to isochron surfaces. This concept is a special case of the one introduced by Iversen (2004), which comprises non-orthogonal trajectories. Keeping the analogy between the isochron 'movement' and the wave propagation, we can say we use isotropic isochron rays while Iversen (2004) used anisotropic.

Equivalent velocity media

The propagating isochron 'moves' through the model with a speed that is different from the wave propagation velocity. The isochron propagation velocity depends on the source-receiver location and the medium velocity and it varies even in isotropic-homogeneous media.

For a given source-receiver pair, we can imagine a hypothetical medium with the velocity distribution identical to that of the isochron velocity propagation. We refer to this as the equivalent velocity medium. There are two features that characterize the equivalent velocity field. First, isochron propagation velocity depends on the isochron ray direction, which means it can be multivalued in the presence

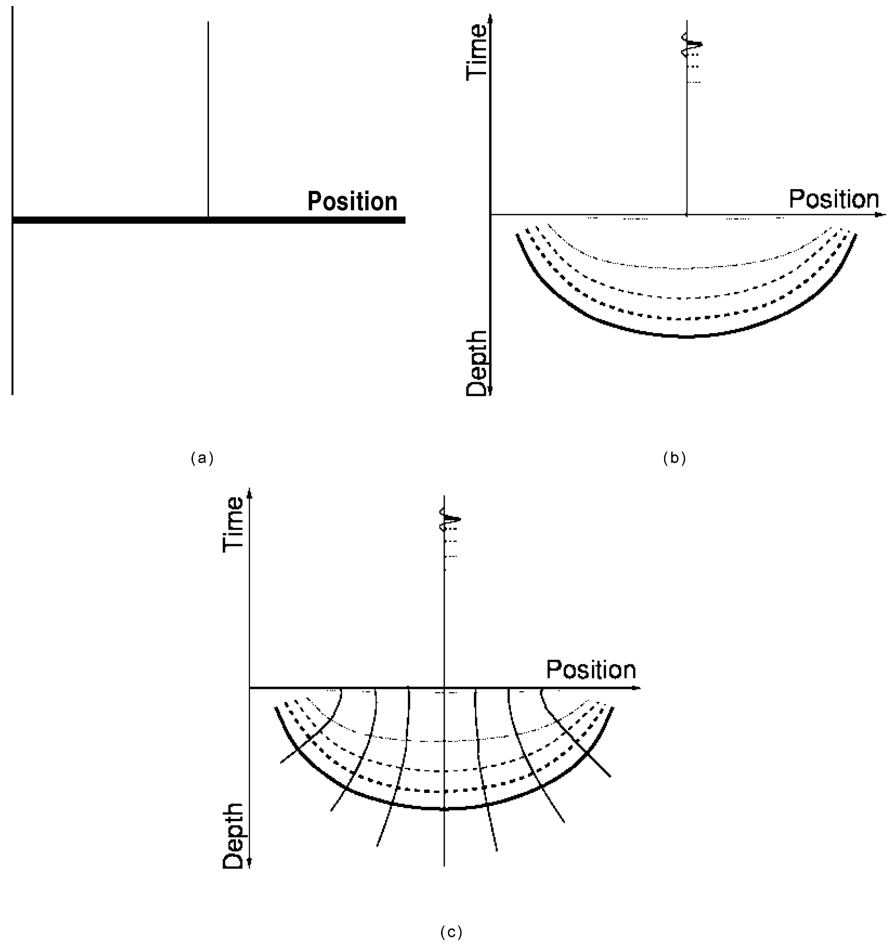

Figure 1 Propagating isochron: (a) isochron for the minimum two-way traveltime (solid blue line), (b) isochrons for increasing two-way traveltime (the smaller the traveltime, the tinier the dashed line) (c) isochron rays (solid red lines).

of caustics. Second, equivalent velocity fields present a singularity in the line connecting the source-receiver pair, which corresponds to the hypothetical starting isochron.

Depending on the complexity of the original velocity model, we distinguish two cases of determining equivalent velocity and isochron ray tracing. One is based on the assumption of the absence of caustics and the other is the general case where no restrictions are imposed on the velocity model.

Let us assume a smooth seismic velocity model with no caustics. Consider a source-receiver pair located in a horizontal plane where the velocity does not vary in a small slab between the source and the receiver. In this situation, the isochron-field can be reproduced by a hypothetical experiment where the seismic source is a horizontal segment connecting the source-receiver pair and the seismic velocity is the equivalent velocity medium. All the orthogonal isochron rays are perpendicular to the line source. In the vertical plane that contains the source-receiver pair, all the orthogonal isochron rays are vertical at their starting point. As presented in appendix A, orthogonal isochron rays obey Snell's law and the wavefronts are the surfaces of constant traveltime in space satisfying the eikonal equation:

$$\left(\frac{\partial_\tau}{\partial x}\right)^2 + \left(\frac{\partial_\tau}{\partial y}\right)^2 + \left(\frac{\partial_\tau}{\partial z}\right)^2 = \frac{1}{V_{eq}^2(x, y, z)}, \tag{1}$$

where τ is the two-way traveltime and V_{eq} is the equivalent velocity.

In the absence of triplications, the equivalent velocity medium can be directly determined by applying the eikonal equation on the precomputed two-way traveltime map. This velocity field can be used to migrate the conditioned data using conventional zero-offset migration algorithms. In the presence of triplications, equivalent velocity media are multivalued and cannot be derived from traveltime maps.

In this case, the isochron propagation velocity should be represented in isochron ray coordinates and has to be computed using a proper isochron ray-tracing algorithm. Also, the migration should use wavefield extrapolation in isochron ray coordinates. This case falls outside the scope of this paper and remains subject to future research.

Orthogonal isochron ray-tracing without media restriction

The isochron propagation can be performed by applying basic principles of wave propagation. For simplicity, consider an isochron in a 2D medium. In Fig. 2(a), the point M is the intersection of three curves:

- $z = \zeta_s(x, S, t_s)$ is the wavefront that comes from the source position, $S = (x_s, z_s)$, at the time t_s.
- $z = \zeta_r(x, R, t_r)$ is the wavefront that comes from the receiver position, $R = (x_r, z_r)$, at the time t_r.

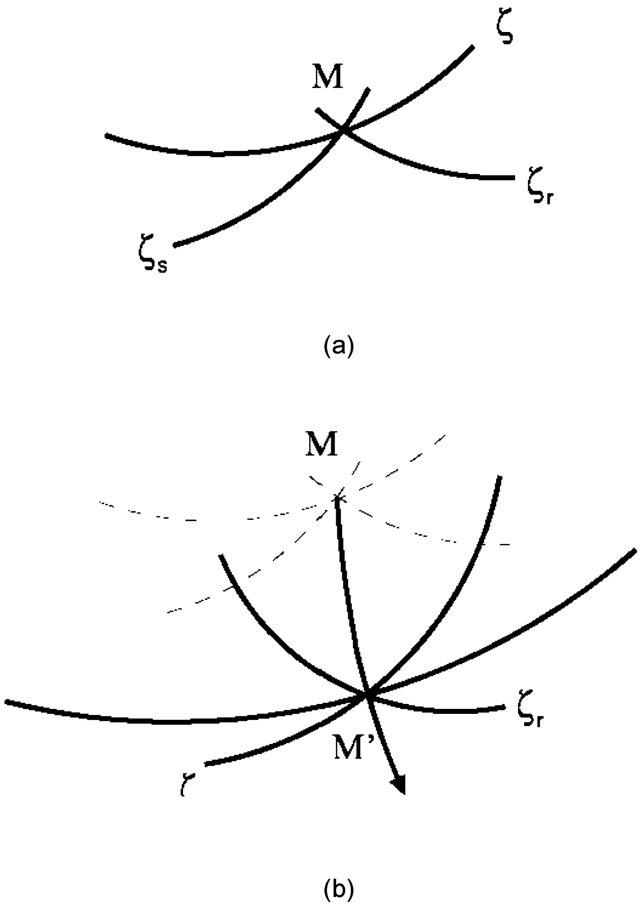

Figure 2 Isochron ray-tracing scheme: (a) wavefronts at t_s (blue line) and t_r (red line), isochron at $t_r + t_s$ (black line), (b) wavefronts at $t_s + \delta t/2$ and $t_r + \delta t/2$, isochron at $t_r + t_s + \delta t$. The isochron ray is the curve determined by the trajectory from M to M’ (green line).

- $z = \zeta(x, S, R, t_s + t_r)$ is the isochron corresponding to the source-receiver pair SR at the two-way traveltime $\tau = t_s + t_r$.

Figure 2(b) shows the isochrons after a propagation time of $\delta t/2$, the wavefront $z = \zeta_s(x, S, t_s)$ moves to $z = \zeta_s(x, S, t_s + \delta_t/2)$, the wavefront $z = \zeta_r(x, R, t_r)$ moves to $z = \zeta_r(x, R, t_r + \delta_t/2)$, the isochron $z = \zeta(x, S, R, t_s + t_r)$ moves to $z = \zeta(x, S, R, t_s + t_r + \delta t)$ and the intersection point moves to M'. In 2D models, the triple intersection point can be found by merely locating the intersection between the source and receiver wavefronts. In this case, an orthogonal isochron ray can be traced just by mapping the intersection points step by step.

In the case of 3D, the intersection between the source and receiver wavefronts is a curve instead of a point. Consequently the use of the unknown isochron is needed for determining the intersection point. The isochron is unknown but it can be defined as the envelope of intersection lines between source and receiver wavefronts whose total traveltime is constant.

The exploding reflector model

The exploding reflector model (Loewenthal *et al.* 1985) is widely applied in both seismic modelling and imaging algorithms. Although it is an approximation that cannot be reproduced by a physical experiment, it leads to simple, robust and efficient algorithms. Zero-offset seismic data can be modelled and migrated by a large number of approaches, such as Kirchhoff, finite-differences and Gaussian beams. For zero-offset data, the two-way traveltime of primary reflections with normal-incidence angle can be computed by tracing normal rays in a half-velocity medium.

The orthogonal isochron rays play a role analogous to normal rays, i.e., they are perpendicular to the reflectors and the traveltime measured along them is the time on the two-way path: source-reflector-receiver. While the normal rays can be traced using the half-velocity medium, the isochron-rays need an equivalent velocity medium that depends on the source and receiver location. Therefore we need to define an equivalent velocity medium for each source-receiver pair. Another important difference between normal rays and orthogonal isochron rays is the take-off (or emergence) angle. While normal rays can assume any direction at the recording surface, the isochron rays are perpendicular to it when the first layer is homogeneous, see Figs 3(a) and 3(b). Because of the analogy described above, we suggest the expression ‘equivalent exploding reflector model’ to refer to the association of isochron rays and equivalent velocity model.

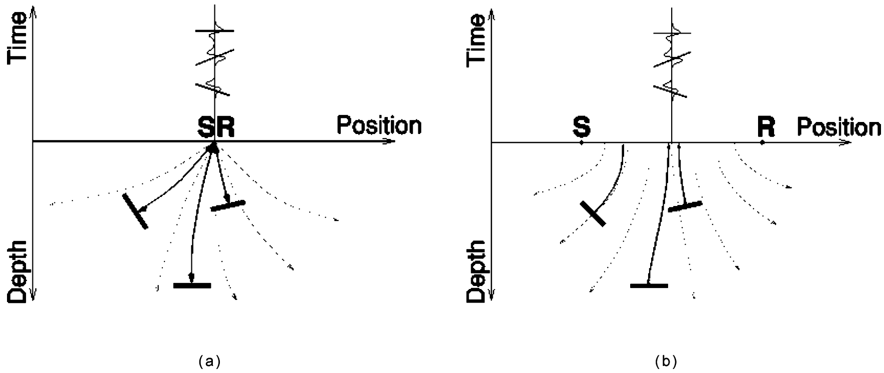

Figure 3 Illustration of data mapping with: a) normal rays and half-velocity for zero-offset data, b) isochron rays and equivalent velocity for common-offset data.

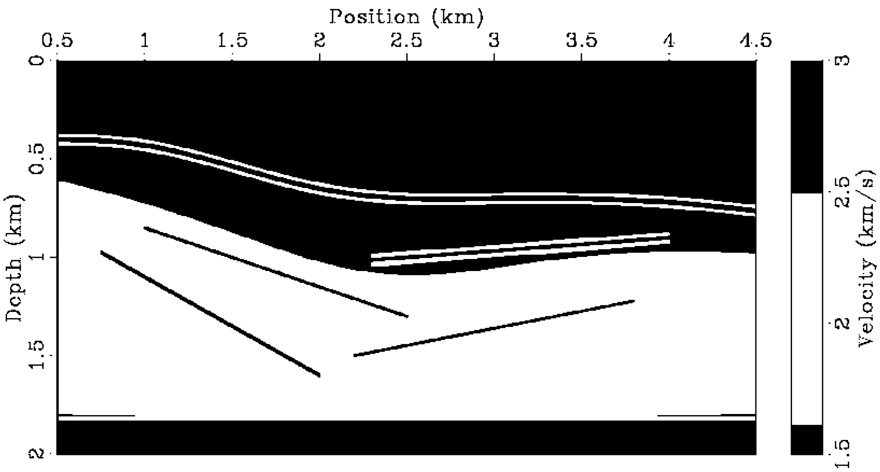

Figure 4 2D seismic model showing the superposition of interfaces and the velocity field.

A distinctive characteristic between normal rays and isochron rays is the relationship between the ray parameter and the time slope. In general, normal rays are labelled by the take-off angle, which is related to time slope. Orthogonal isochron rays are labelled by the take-off position, which is not directly related with the time slope.

IMAGING USING ORTHOGONAL ISOCHRON RAYS

The objective here is to demonstrate that the concepts of orthogonal isochron rays and equivalent velocity can be applied to extend the use of zero-offset wave-equation algorithms for modelling and migration of non zero-offset data. We do not advocate the use of these techniques in substitution of standard procedures but we believe they are useful to address imaging problems in general. In particular, we believe they have applications in redatuming, offset continuation, data regularization and migration velocity analysis. Also, they permit modelling and migration of individual traces using 3D wave-equation algorithms, which is a useful property to match seismic data with and well-log information.

Modelling

In this section, we present two examples in which we extend the use of zero-offset algorithms to finite offset data by applying the concepts of orthogonal isochron rays, equivalent velocity and exploding reflector model. We generate three common-offset sections with the methods: finite-difference, split-step wavefield extrapolation and Kirchhoff. The third dataset was generated by conventional Kirchhoff modelling to be used as a benchmark.

The 2D seismic model consists in six interfaces immersed in a smooth velocity field, where the wave velocity propagation varies from 1500 m/s at the shallow part, to 3000 m/s at the bottom. Figure 4 shows the velocity model and the

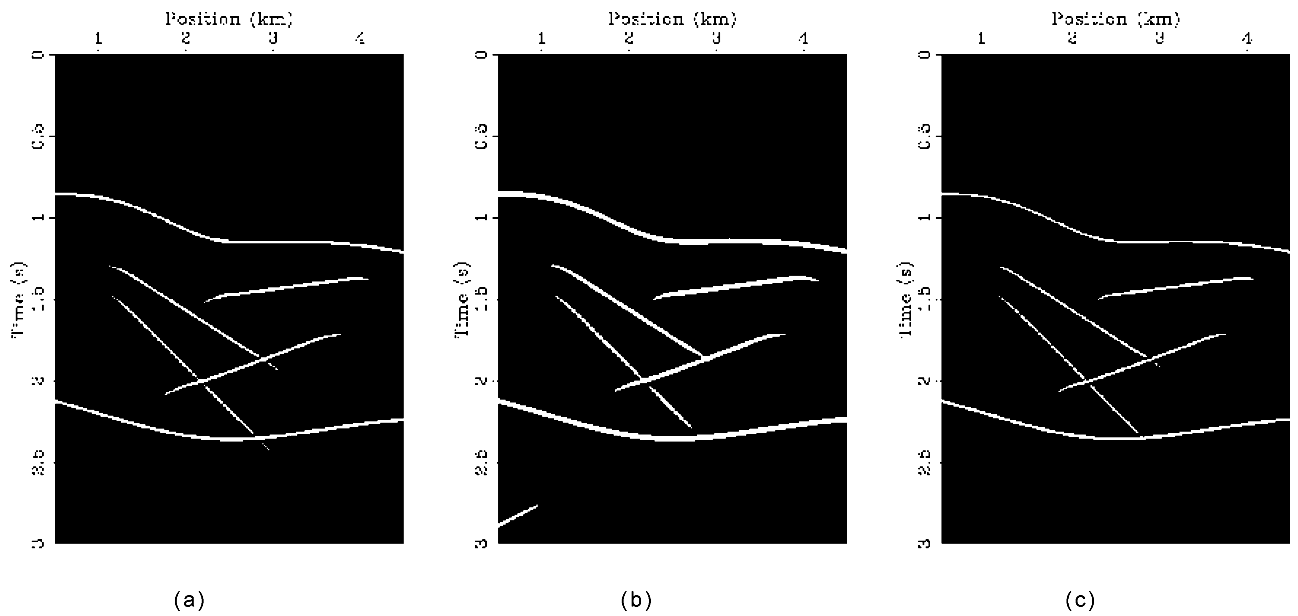

Figure 5 Common-offset gathers: (a) conventional Kirchhoff modelling, (b) isochron-ray finite-difference modelling and (c) isochron-ray wavefield extrapolation modelling.

interfaces. For all cases, the source-receiver pair is located on the horizontal plane at $z = 0$.

For Kirchhoff modelling, a Ricker wavelet with dominant frequency of 20 Hz was used. The integration was performed using a spatial interval of 1 m without any special care about the dynamic aspects. Figure 5(a) shows the synthetic Kirchhoff common-offset section for the half-offset $h = 500$ m.

For both finite-difference and wavefield extrapolation modelling, the common-offset section was constructed trace-by-trace using an equivalent velocity medium for each CMP position. The equivalent velocity media were defined following the steps:

- Build the traveltime map $t_s(x, z)$ from the source location to all points of the model using an eikonal solver algorithm.
- Repeat the previous step from the receiver location, obtaining the traveltime map $t_r(x, z)$.
- Add the two maps to obtain the two-way traveltime map $\tau = t_s + t_r$.
- Find the spatial partial derivatives of the two-way traveltime (τ).
- Apply the eikonal equation to determine the equivalent velocity for every grid point.

The equivalent velocity media has a singularity between the source-receiver pair, where the velocity goes to infinity. Thus we need to adopt special procedures to avoid numerical instability in this region. The applied procedures are different for each case. However, both are based on an analytical solution for the isochron propagation in the vicinity of the source-receiver pair. In Fig. 6 we present the equivalent-velocity field for the central position of the seismic model presented in Fig. 4.

For finite difference modelling, we avoid numerical instability by clipping or limiting the equivalent velocity field and placing the receivers along an isochron located away from the singularity. A good choice for locating this recording isochron is a region where the wave propagation is constant because it is an ellipse in this case.

The recorded wavefield contains information from all directions but only information that travels along orthogonal isochron rays should be considered. In other words, we have to sum the amplitudes along isochrons, which is equivalent to a stack of the information collected in the recording isochron along the time. Figure 5(b) is the common-offset gather modelled by the finite-difference approach, in which the recording isochron is an ellipse with minor axis of 0.35 km and the equivalent velocity field is limited at 0.14 km. Each trace of this gather is the result of the stacking of all traces recorded along an isochron whose midpoint between the source and receiver corresponds to the trace location. Figure 7(a) corresponds to the seismogram recorded at the central position of the seismic model.

For modelling by wavefield extrapolation, Fig. 5(c), the adopted procedure consists in recording the wavefield at a horizontal plane below the singularity. Since the

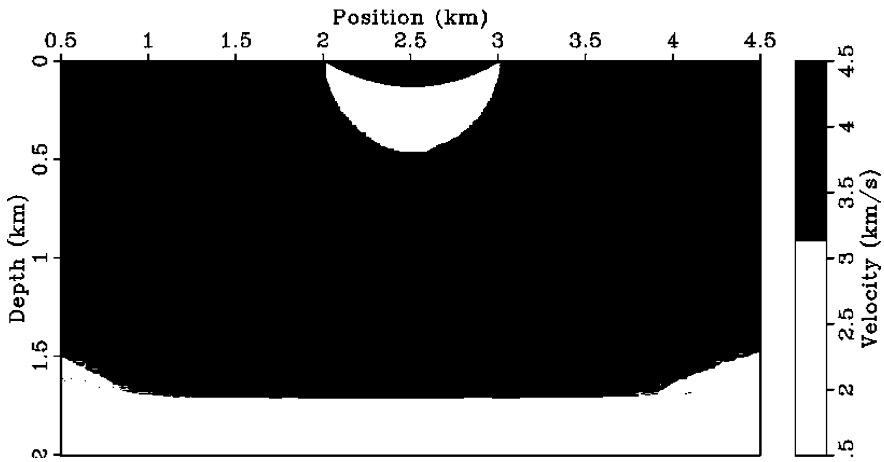

Figure 6 Equivalent velocity for the central position of the seismic model clipped at 4.5 km/s.

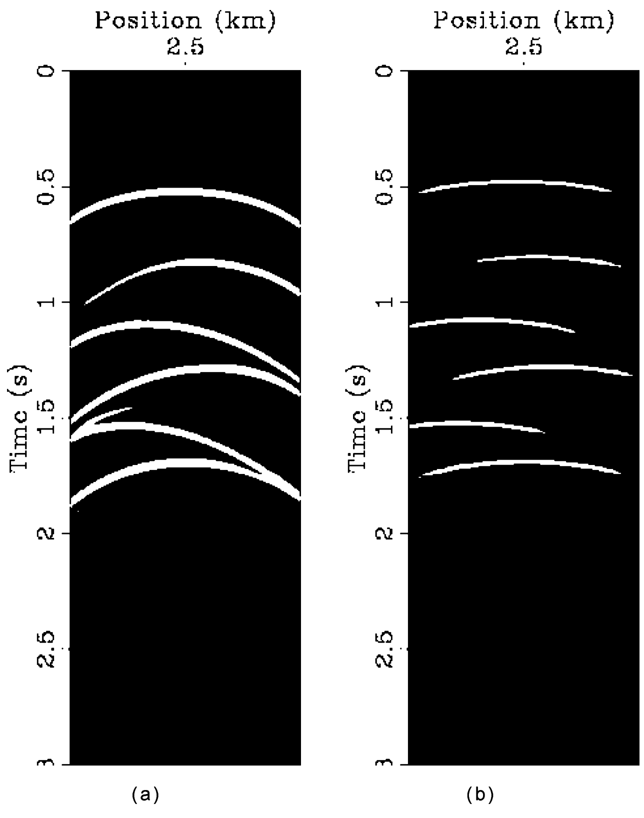

Figure 7 (a) Finite-difference recorded wavefield and (b) extrapolated wavefield recorded at $z = 5$ m.

source-receiver pair is located at the surface $z = 0$, the singularity is taken out of the equivalent velocity model just by excluding a tiny slab with a thickness corresponding to the vertical sample interval, which is 5 m in this example. The wavefield is extrapolated from the bottom to the one-sample deep surface where it is recorded, see Fig. 7b). After applying a variable time-shift to compensate for the slab removing, the data are stacked to generate the modelled trace. The initial time of the modelled trace corresponds to the direct arrival traveltime. The procedure described above is independently applied for all desired output positions of the modelled common-offset gather. Alternatively, the problem of avoiding the singularity can be addressed by redatuming the data from the surface to a deeper plane.

The common-offset isochron-ray modelled sections present artefacts due to truncation effects and numerical dispersion. Truncation creates shadows for all reflection events, which can be attenuated by using an appropriated taper function in the stacking procedures. Numerical dispersion in the finite-difference modelling also contributes to the presence of artefacts and to the distortion of the modelled seismic pulse, altering its shape and affecting its frequency content. Avoiding numerical dispersion in finite-difference isochron-ray modelling is a hard task because of the high-velocity zone present in the equivalent velocity field.

Migration

In principle, all of the zero-offset migration methods based on the exploding reflector model can have their use extended to finite-offset gathers by making use of orthogonal isochron rays and equivalent velocity media. In this section, we discuss two cases: migration by wavefield extrapolation (WEM) and reverse time migration (RTM). In both cases, the isochron ray migration can be implemented in a trace-by-trace algorithm. For each trace, the following steps are carried out: 1) computation of the equivalent velocity model, 2) creation of the conditioned data for wavefield reconstruction, 3) migration of the conditioned data by a zero-offset algorithm using the equivalent velocity model and 4) addition of the migration result to the image.

The conditioning data procedure is not the same for WEM and RTM but in both cases a half-derivative followed by a time shift is applied to the input trace. For the WEM case, the input trace is time-shifted by a negative amount that corresponds to the traveltime measured along the raypath connecting the source and receiver via a fictitious reflection point. The conditioned data gather for WEM is obtained by repeating the shifted trace for each trace position located between the source-receiver, while the remaining positions are filled with zeros. In the RTM case the time-shift is also negative and it corresponds to the time of an isochron where the reverse-data is injected. This isochron should be as far as possible from the singularity. A good choice would be an ellipse when the source-receiver pair is located in a homogeneous region. In addition to the required steps described above, linear spatial tapering is applied to the conditioned data to avoid the presence of artefacts at the final image.

Figure 8(a) corresponds to the zero-offset image obtained by conventional wavefield extrapolation migration, having as input a zero-offset Kirchhoff section. Figures 8(b) and 8(c) are the common-offset WEM and RTM images, respectively. In both cases, the synthetic Kirchhoff common-offset section was used as input. As expected, the stretching factor is greater in common-offset images. As in the modelling, the observed artefacts in isochron-ray images are due to truncation effects and numerical dispersion.

Comments

Amplitude information is not preserved in the described modelling and migration approaches but the development of true-amplitude algorithms is feasible by including appropriated weight function in the stacking procedure.

The equivalent velocity field can be expressed as a function of two factors, which are related to the effective velocity medium and to the direction of specular rays, respectively. For smooth velocity models, the directional factor is mostly affected by the overburden medium and has its sensitivity to model changes related to residual moveouts.

In the presence of strong contrasts in the velocity model, a special care is needed in the computation of traveltime tables in order to avoid the presence of head waves, which can produce anomalous values in the equivalent velocity field.

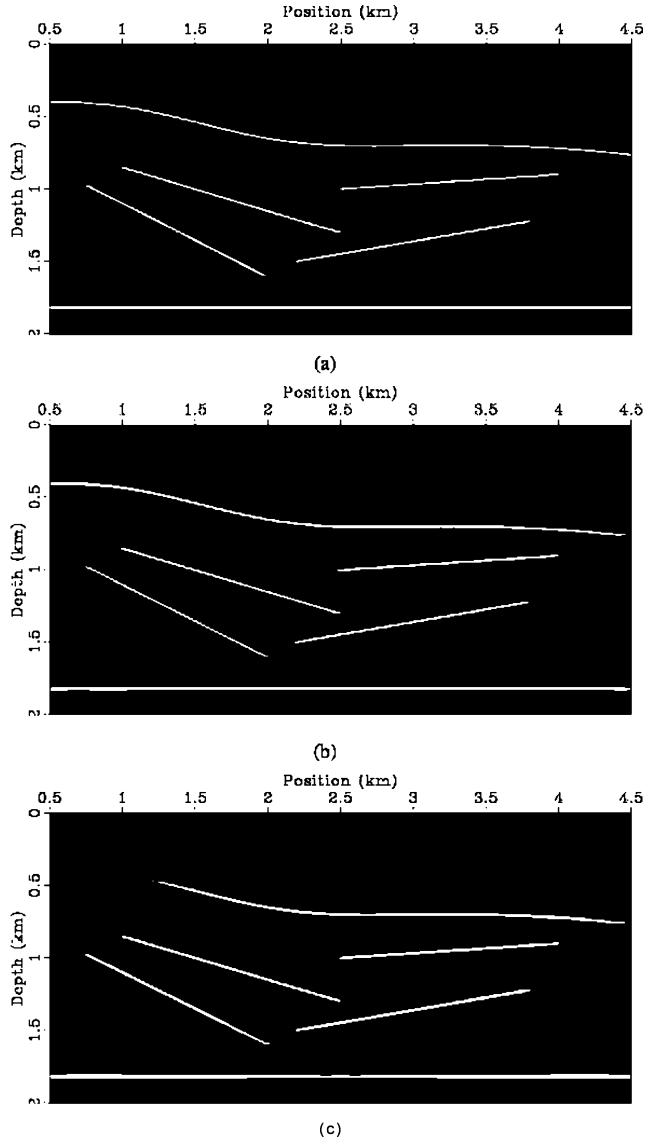

Figure 8 (a) Zero-offset wavefield extrapolation migration, (b) common-offset wavefield extrapolation migration and (c) common-offset reverse time migration.

APPLICATION TO FIELD DATA

The WEM using orthogonal isochron rays was applied in a pseudo 2.5D dataset, which consists in twenty-two common-offset gathers extracted from a 3D dataset via the following sequence. First, the input traces were organized in twenty-two groups, using as sorting criteria the source-receiver offset; second a 3D Kirchhoff time migration algorithm was applied; finally, a 2.5D Kirchhoff time demigration procedure was applied to each image. In the sorting procedure of the first step, each input trace was multiplied by an areal factor in

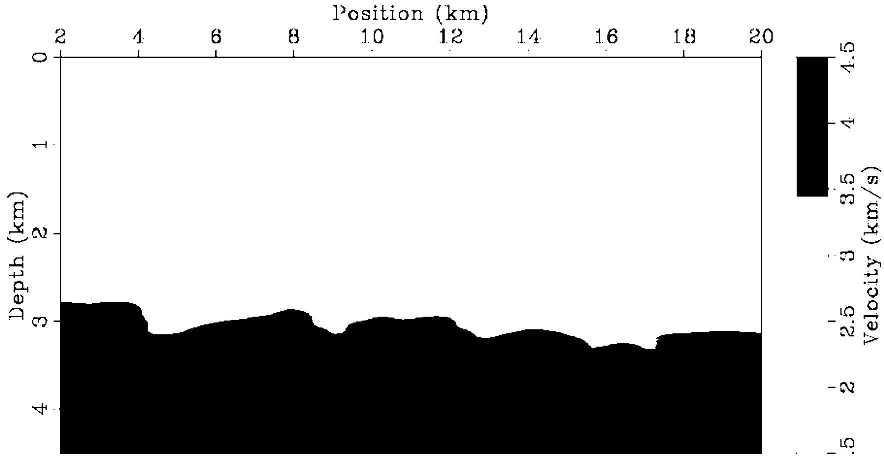

Figure 9 Velocity model used for isochron-ray and Kirchhoff migration (field-data example).

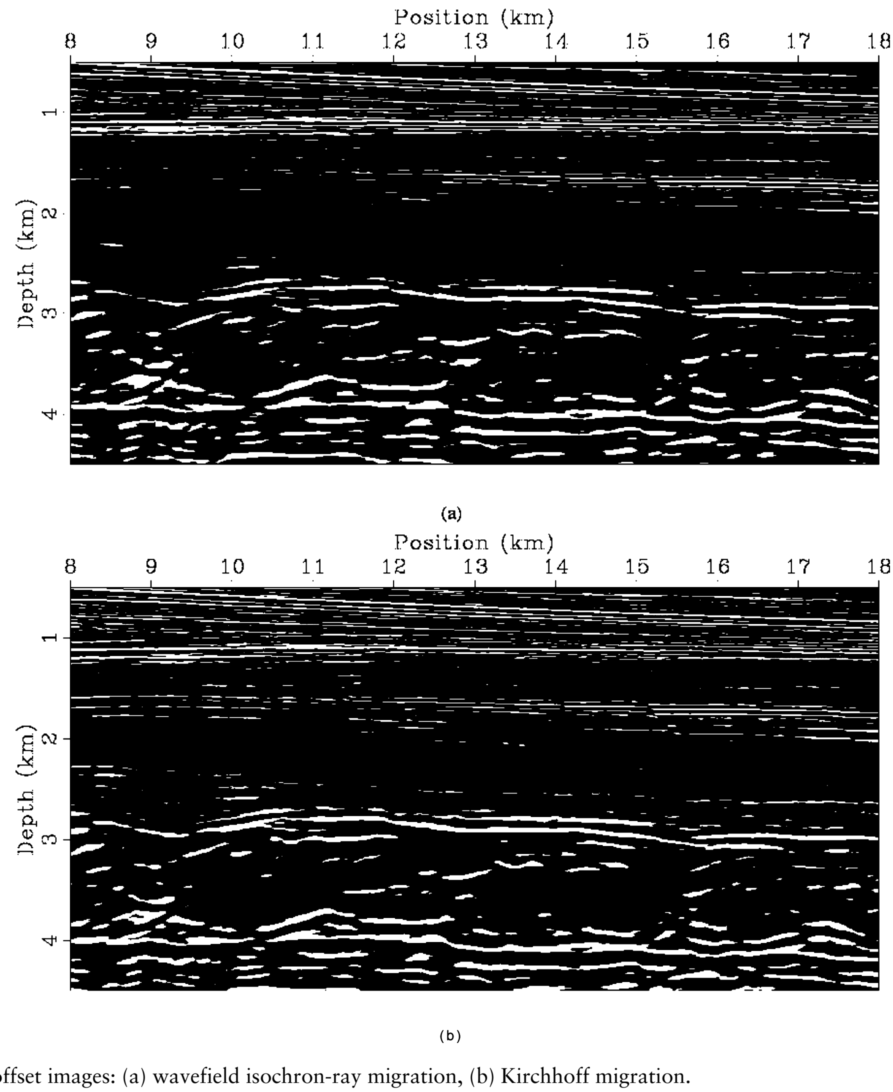

Figure 10 Common-offset images: (a) wavefield isochron-ray migration, (b) Kirchhoff migration.

order to compensate for the effect of acquisition irregularities. The weight function used in the 3D time-migration algorithm produces a true-amplitude image gather when the medium velocity is constant, i.e., the output amplitudes are proportional to the reflection coefficients. Also, the applied demigration program uses a true-amplitude weight function, which produces a 2D common-offset gather whose amplitudes are affected by a 3D geometrical spreading factor, which is correct when the medium is homogeneous.

The minimum offset is 160 m and the increment between offsets is 200 m. Each common-offset gather has 1351 traces and the distance between them is 18.75 m. The traces are 5.0 s long and the time sampling interval is 4 ms.

The same smoothed velocity model (Fig. 9) was used to migrate four common-offset sections (from 1560 m to 2160 m) using the wavefield extrapolation approach and a traditional common-offset Kirchhoff program. The WEM image from the section with the largest offset is presented in Fig. 10(a), while Fig. 10(b) shows the Kirchhoff result. The slight difference in depth positioning in the two images is probably caused by numerical errors in equivalent velocity estimation. This problem is not observed in the synthetic examples because a smoother velocity field with a denser grid is used. As expected, there are no significant differences between the images from these migration methods. The result is basically the same because we use the same velocity and the same approach to compute traveltime maps for Kirchhoff migration and to calculate the equivalent velocity in wavefield extrapolation. A similar result is observed when we stack the four migrated sections: compare Fig. 11(a) and Fig. 11(b). A significant

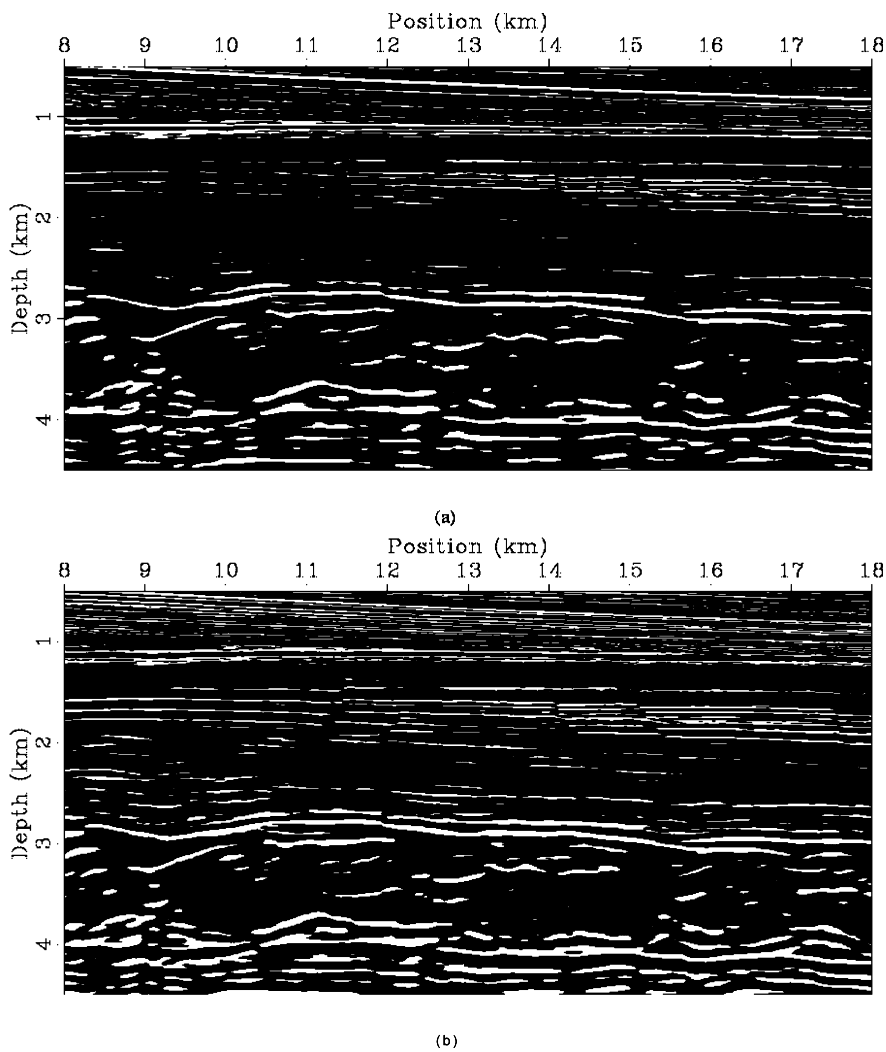

Figure 11 Stack of common-offset images: (a) wavefield isochron-ray migration, (b) Kirchhoff migration.

difference between the Kirchhoff method and the WEM isochron ray migration should be expected in the presence of caustics, only if we determine the equivalent velocity medium by means of an isochron ray-tracing algorithm and extrapolate the wavefield using an algorithm that works in isochron-ray coordinates. In this case, a better image is expected for WEM.

DISCUSSION

The computational cost of modelling or migrating an individual seismic trace using the isochron ray approach, as presented above, is close to the cost of modelling or migrating a zero-offset section. For example, the computation time to migrate a common-offset section with n traces is approximately the same of migrating n zero-offset sections with the same size. Besides the high computational cost, the described methodology is restricted to smooth velocity models, which reduces the attractiveness of this approach. The computational cost can be reduced by using beams, redatuming and limited aperture. Silva and Sava (2008) showed that the combination of these procedures can drastically decrease the processing time, especially for greater offsets. Problems due to triplication can be eliminated by representing the equivalent velocity in isochron ray coordinates.

Future extensions of this methodology include the development of an effective isochron ray tracing algorithm without making any assumption about the medium. This algorithm might be implemented by making use of the paraxial ray theory in isochron-ray coordinates. The isochron ray-tracing algorithm could be used to define equivalent velocity media in isochron ray coordinates, which can be used to extrapolate the isochron-field in this coordinate system (Sava and Fomel 2005).

CONCLUSION

We reformulate wave-equation modelling and migration using the concepts of orthogonal isochron rays and equivalent velocity media. This methodology falls into the general class of time imaging techniques and it is applicable to imaging in geologic environments characterized by smooth lateral velocity variation. Under this formalism, modelling and migration of non zero-offset data can be performed using zero-offset algorithms, thus inheriting their speed and robustness. Imaging at non-zero offset can be done with typical wavefield extrapolation or reverse-time algorithms, as demonstrated in this paper but also with beam or other types of algorithms. The main applications of this methodology are in migration velocity analysis, which inherits the speed and robustness of zero-offset imaging algorithms and in imaging irregularly sampled data, which exploits the fact that isochron-ray imaging operates on individual or groups of traces, independent of the sampling of the output grid.

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APPENDIX A

In this appendix, we show that the kinematics of the isochron 'movement' respect an equation similar to the eikonal and its right side is related to the speed of the isochron 'movement', which is referred to as equivalent velocity. In the absence of caustics, this isochron 'eikonal' equation can be used to numerically compute the equivalent velocity from traveltime maps.

For a given source S and receiver R, the equation $z = \zeta(x,R,S,\tau)$ defines a family of isochrons where each member is determined by the two-way reflection traveltime τ. The parameter τ has to be greater than the direct wave traveltime τd because it is related to a reflection path. When τ goes to τd, the corresponding isochron shrinks and collapses into a line connecting the source and receiver.

This line is the hypothetical initial isochron with initial traveltime τ_d. For the case that the source and receiver are located at the surface $z = 0$ and there is no lateral velocity variation at $z = 0$, the initial isochron is a horizontal segment and the orthogonal lines are vertical.

The two-way traveltime is $\tau = t_s + t_r$, where t_s and t_r are the one-way traveltime maps for the source and receiver locations respectively. The square of its partial derivative in relation to the spatial coordinate x_j is:

$$\left(\frac{\partial \tau}{\partial x_j}\right)^2 = \left(\frac{\partial t_s}{\partial x_j}\right)^2 + \left(\frac{\partial t_r}{\partial x_j}\right)^2 + 2\frac{\partial t_s}{\partial x_j}\frac{\partial t_r}{\partial x_j} \tag{A1}$$

The same expression represents the summation of the quadratic derivatives in Einstein's notation. As t_s and t_r obey the eikonal equation, we have that

$$\left(\frac{\partial t_s}{\partial x_j}\right)^2 = \left(\frac{\partial t_r}{\partial x_j}\right)^2 = \frac{1}{V^2} \tag{A2}$$

and

$$\frac{\partial t_s}{\partial x_j}\frac{\partial t_r}{\partial x_j} = \mathbf{p_s} \cdot \mathbf{p_r}, \tag{A3}$$

where V stands for velocity and p_s and p_r are the slowness vector for waves from the source and receiver, respectively. Inserting equations (A2) and (A3) into equation (A1), we obtain

$$\left(\frac{\partial \tau}{\partial x_j}\right)^2 = 2\left(\frac{1}{V^2} + \mathbf{p_s} \cdot \mathbf{p_r}\right). \tag{A4}$$

The isochron 'movement' satisfies equation (A4). The right side of this equation corresponds to the quadratic slowness of the isochron movement, whose inverse square root is the equivalent velocity.