Wave-equation angle-domain common-image gathers for converted waves

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SUMMARY

Wavefield-extrapolation methods can produce Angle-Domain Common-Image Gathers (ADCIGs). These ADCIGs are a function of the half-aperture angle, that is, the average between the incidence angle and the reflection angle. This article presents a method to compute ADCIGs for converted-wave data (PS-ADCIGs) after imaging with wave-equation migration. The method exploits the robustness of computing 2-D isotropic single-mode ADCIGs, and incorporates both the P-to-S velocity ratio ($\gamma$), and the local image-dip field. This method also maps the final PS-ADCIGs into two different angle-domain common-image gathers, the first ADCIG is function of the P incidence angle, the second one is function of the S reflection angle. Results with both synthetic data and real data show the practical application for converted-wave angle-domain common-image gathers. The main contribution of this paper is independent of the migration algorithm implemented. The approach is valid as long as the migration algorithm is based on wavefield downward continuation, and the final prestack image is a
function of the horizontal subsurface offset.

INTRODUCTION

The imaging operator transforms the data, which is in data-midpoint position, data-offset, and time coordinates \([m_D,h_D,t]\), into an image that is in image-midpoint location, offset, and depth coordinates \([m_z,h,z]\). This image provides information about the accuracy of the velocity model. This information is present in the redundancy of the prestack seismic image, (i.e. non-zero-offset images). The subsets of this image for a fixed image point \((m_z)\) with coordinates \((z,h)\) are known as common-image gathers (CIGs), or common-reflection-point gathers (CRPs). If the CIGs are a function of \((z,h)\), the gathers are also referred as offset-domain common-image gathers (ODCIGs).

The common-image gathers can also be expressed in terms of the opening angle \(\theta\), by transforming the offset axis \((h)\) into the opening angle \(\theta\) to obtain a common-image gather with coordinates \((z,\theta)\); these gathers are known as Angle-Domain Common-Image Gathers (ADCIGs) (de Bruin et al., 1990; Prucha et al., 1999; Brandsberg-Dahl et al., 1999; Rickett and Sava, 2002; Sava and Fomel, 2003; Biondi and Symes, 2004).

There are two kinds of ODCIGs: those produced by Kirchhoff migration, and those produced by wave-equation migration. There is a conceptual difference in the offset dimension between these two kinds of gathers. For Kirchhoff ODCIGs, the offset is a data parameter \((h = h_D)\), and involves the concept of flat gathers. For wave-equation ODCIGs, the offset dimension is a model parameter \((h = h_z)\), and involves the concept of focused events. In this paper, we will refer to these gathers as subsurface offset-domain common-image gathers (SODCIGs). Imaging artifacts due to multipathing are present in ODCIGs. However, an event in an angle section uniquely determines a ray couple, which in turn uniquely locates the reflector. Hence, the image representation in the angle domain
does not have artifacts due to multipathing (Stolk and Symes, 2002; Clapp, 2005). Unlike ODCIGs, ADCIGs produced with either Kirchhoff methods or wave-equation methods have similar characteristics, since the ADCIGs describe the reflectivity as a function of the reflection angle.

This paper also presents the option to transform the PS-ADCIGs into two angle-domain common-image gathers. The first angle-gather is function of the P-incidence angle, the second one is function of S-reflection angle. We refer to these two angle gathers as P-ADCIGs and S-ADCIGs, respectively. Throughout this process, the ratio between the different velocities plays an important role in the transformation. We present the equations for this mapping and show results on a synthetic data set. We also present results on a 2-D real data set from the Mahogany field in the Gulf of Mexico. For this exercise a comparison between the PZ-ADCIGs and the PS-ADCIGs yield information to improved the PS image.

**TRANSFORMATION TO THE ANGLE DOMAIN**

The transformation to the angle domain of PS-SODCIGs follows an approach similar to the 2-D isotropic single-mode (PP) method (Sava and Fomel, 2003). Figure 1 describes the angles we use in this section. For the converted-mode case, we define the following angles:

\[
\begin{align*}
\theta & \equiv \frac{\phi + \sigma}{2}, \\
\alpha & \equiv \frac{2\alpha_x + \phi - \sigma}{2}.
\end{align*}
\]  

(1)

In definition 1 the angles $\phi$, $\sigma$, and $\alpha_x$ represent the incident, reflected, and geological dip angles, respectively. This definition is consistent with the single-mode case; notice that for the single-mode case the angles $\phi$ and $\sigma$ are the same. Therefore, the angle $\theta$ represents the reflection angle, and the angle $\alpha$ represents the geological dip (Sava and Fomel, 2003; Biondi and Symes, 2004). For the
converted-mode case, the angles $\phi$ and $\sigma$ are not the same. Hence, the angle $\theta$ is the *half-aperture* angle, and the angle $\alpha$ is the *pseudo-geological* dip.

Throughout this paper, we present a relationship between the known quantities from our image, $I(m_\xi, z_\xi, h_\xi)$, and the *half-aperture* angle ($\theta$). Appendix A presents the full derivation of this relationship. Here, we present only the final result, its explanation and its implications. The final relationship to obtain converted-mode angle-domain common-image gathers is the following (Appendix A):

\[
\tan \theta = \frac{4\gamma(m_\xi, z_\xi)\tan \theta_0 + \delta(\gamma^2(m_\xi, z_\xi) - 1)(\tan^2 \theta_0 + 1)}{\tan^2 \theta_0(\gamma(m_\xi, z_\xi) - 1)^2 + (\gamma(m_\xi, z_\xi) + 1)^2},
\]

(2)

where

\[
\tan \theta_0 = -\frac{\partial z_\xi}{\partial h_\xi},
\]

\[
\delta = -\frac{\partial z_\xi}{\partial m_\xi}.
\]

Equation 2 consists of three main components. First $\gamma(m_\xi, z_\xi)$ is the P-to-S velocity ratio. Next, $\theta_0$ is the *pseudo-opening* angle. This pseudo-opening angle is the angle obtained throughout the conventional method to transform SODCIGs into isotropic ADCIGs as described by Sava and Fomel (2003). Finally, $\delta$ is the field of local image-dips. Equation 2 describes the transformation from the subsurface-offset domain into the angle-domain for converted-wave data. This equation is valid under the assumption of constant velocity. However, it remains valid in a differential sense in an arbitrary velocity medium, by considering that $h_\xi$ is the subsurface half-offset. Therefore, the limitation of constant velocity applies in the neighborhood of the image. For $\gamma(m_\xi, z_\xi)$, it is important to consider that every point of the image is related to a point on the velocity model with the same image coordinates. Notice that for the non-physical case of $v_p = v_s$, i.e. no converted waves, $\gamma(m_\xi, z_\xi) = 1$, and
the angles $\theta_0$ and $\theta$ are the same.

**Mapping of PS-ADCIGs**

Following definition 1, and after explicitly computing the half-aperture angle with equation 2, we have almost all the tools to compute the P-incidence angle ($\phi$), and the S-reflection angle ($\sigma$). Snell’s law, and the P-to-S velocity ratio are the final two components for this procedure. The final result of this process is the mapping of the PS-ADCIGs, that are function of the half-aperture angle, into two angle-gathers. The first one is a function of the P-incidence angle, we refer to this angle gather as P-ADCIG. The second angle-gather is a function of the S-reflection angle to form an S-ADCIG. After basic algebraic and trigonometric manipulations, the final two expressions for this mapping are (Appendix B):

$$
\tan \phi = \frac{\gamma \sin 2\theta}{1 + \gamma \cos 2\theta}, \quad (3)
$$

$$
\tan \sigma = \frac{\sin 2\theta}{\gamma + \cos 2\theta}. \quad (4)
$$

Expressions 3 and 4 clearly show a non-linear relation between the half-aperture angle and both the incident and reflection angles. The main purpose of this set of equations is to observe and analyze the converted-wave angle-gathers in two different domains each one corresponding to the incidence angle and the reflection angle. The analysis of these angle-gathers might help to obtain residual moveout equations for both the P-velocity and the S-velocity; therefore, individual updates for each of the velocity models.

**METHODOLOGY**

We present a method to implement equations 2-4. First, we describe the method and then illustrate it with a simple synthetic example. Throughout this section, we will refer to two different methods:
first, the conventional method, and second, the proposed method. The conventional method consists of the transformation from SODCIGs into ADCIGs as in the single-mode case (Sava and Fomel, 2003). Figure 2 presents the flow chart for the proposed method.

The flow in Figure 2 presents the basic steps to implement and obtain the true angle-domain common-image gathers for converted-wave data (PS-ADCIGs). First, we use the final image, \( I(m_z, z, h) \), to obtain two main pieces of information: first, the pseudo-opening angle gathers, \( \tan \theta_0 \), using for example, the Fourier-domain approach (Sava and Fomel, 2003); second, the estimated image dip, \( \delta \), using plane-wave destructors (Fomel, 2002). For the second step, we combine \( \tan \theta_0 \) and \( \delta \) together with the \( \psi(m_z, z) \)-field using equation 2 to obtain true converted-wave angle-domain common-image gathers. Finally, we map these PS angle-domain common-image gathers into both the P-ADCIGs and the S-ADCIGs through equations 3 and 4, respectively.

A simple synthetic example illustrates the flow in Figure 2. The synthetic dataset consists of a single shot experiment over a 30° dipping layer. Panel (a) on Figure 3 shows the shot gather; observe that the top of the hyperbola is not at zero offset because of the reflector dip, and the polarity flip does not happen at the top of the hyperbola. Panel (b) shows the image of the single shot gather, which represents \( I(m_z, z, h) \) in the flow chart of Figure 2. The solid line in panel (b) represents the location for the CIG in study.

For this experiment the shot location is at 500 m, with the common image gather at 1000 m, and the geometry given for the reflector, the half aperture angle should be 35°. This corresponds to a value of \( \tan \theta \approx 0.7 \), that is represented with a solid line on both common image gathers at the bottom of Figure 3.

The angle-domain common-image gather in panel (c) of Figure 3 was obtained with the conventional method. Observe that the angle obtained is not the correct one. This ADCIG represents the
tangent of the pseudo-opening angle, \( \tan \theta_0 \), of flow 2. The ADCIG in panel (c) combined with the dip information, \( \delta \), and the \( \gamma(m, z) \)-field, results in the true PS-ADCIG. Panel (d) presents the result of this process. Notice the angle in the true PS-ADCIG coincides with the correct angle.

The last step for the flow chart in Figure 2 correspond to map the true PS-ADCIG into both a P-ADCIG and an S-ADCIG, each one corresponding to the P-incidence (\( \phi \)) and S-reflection (\( \sigma \)) angles, respectively. Figure 4 shows the result for this transformation. Panel (a) is the same image for the single shot gather on a 30\(^\circ\) dipping layer. Panel (b) is the corresponding true PS-ADCIG, which is taken at the location marked in the image. Panels (c) and (d) present the PS-ADCIG map into both the P-ADCIG and the S-ADCIG, respectively. Both angle gathers are obtained using equations 3 and 4 respectively.

As in the previous experiment, the computed value for the P-incidence angle is 47\(^\circ\), which corresponds to \( \tan \phi \approx 1.09 \). The computed value for the S-reflection angle is 22\(^\circ\), which corresponds to \( \tan \sigma \approx 0.4 \). Both of these values are represented by the solid lines in each of the three angle-domain common-image gathers on Figure 4.

SYNTHETIC DATA TEST

The second synthetic example illustrates the importance of computing true PS-ADCIGs. For this purpose, we use the polarity-flip characteristic of converted-wave data. In true PS-ADCIGs, the correct representation of the polarity flip should happen at zero-angle, since the zero-angle represents normal incidence, and also there is no conversion from P to S energy at normal incidence. The normal incidence location is the point in the image space that distinguish opposite particle motion; hence, the separation between positive and negative polarities.

The model depicted in Figure 5 (Baina et al., 2002) includes both gentle and steep dips [panel (a)].
Both the P-velocity and the S-velocity models, panels (b) and (c), respectively, consist of a vertical gradient for a non-constant γ value. The data was created with an analytical Kirchhoff modeling scheme. The synthetic data consists of 200 shots with a shot spacing of 50 m and 400 receivers with a receiver spacing of 25 m. Figure 6 shows a single common-shot gather for this dataset. The left panel exhibits the PP component and the right panel the PS component.

We migrated the synthetic data using a wave-equation shot-profile migration scheme. Figure 7 shows the final migration result together with four selected common-image gathers. The top panels represent the zero subsurface-offset section for the PP migration (left), and the PS migration (right). Four solid lines at 3.5, 4.5, 5.5, and 6.5 km are superimposed into both migrations. These lines represent the locations for the four common-image gathers underneath each migration result. The middle panels on Figure 7 represent the SODCIGs taken at the position indicated by the solid lines on the migration result. The bottom panels are the ADCIGs, both the PP-ADCIGs and the PS-ADCIGs were obtained with the conventional method.

The result for the PP image is accurate; all the energy is focused at zero subsurface offset, and the angle gather are completely flat. This is the expected result, since we performed the migration with the correct velocity model. The PS results are the most interesting. First the migration section at zero subsurface offset has positive and negative amplitudes along the first reflection. The flat reflector has vanished because there is no conversion from P to S energy at normal incidence. The SODCIGs are focused at zero, and the polarity changes across the zero value. The PS-ADCIGs are obtained with the conventional method; therefore, they represent only the pseudo-opening angle. We follow the method previously described and combine the PS-ADCIGs with the image dip information (Figure 8) to obtain true PS-ADCIGs.

Figure 9 shows the true PS-ADCIGs. The left panel presents the PS result, the same result that
is in Figure 7. The top-left panel is the image at zero subsurface offset, and the bottom-left panel shows the PS-ADCIGs. The right panel presents the final PS result. The top-right panel is the result of stacking in the angle domain of the true PS-ADCIGs after correcting the polarity flip (Rosales and Rickett, 2001a). The bottom-right panel shows the true PS-ADCIGs, which are taken at the locations marked by the solid lines in the final image.

Observe the areas marked with an oval in both the PS-ADCIGs and the true PS-ADCIGs in Figure 9. The marked areas are taken at different reflectors for different dip values. The CIG at 3.5 km shows the most significant change; the polarity flip is completely corrected at zero angle, and there is larger angle coverage, since equation 2 stretches the events horizontally for an accurate representation of the half-aperture angle. Although the changes in the other CIGs, with respect to the polarity flip, are not that obvious, the polarity flip is now located at zero angle. Additionally, the events are stretched horizontally and they do not have any residual moveout.

**REAL DATA TEST**

We use a portion of the 2-D real dataset from the Mahogany field, located in the Gulf of Mexico. The 2-D dataset is an OBS multicomponent line. The data was already preprocessed by CGG. The hydrophone and the vertical components of the geophone have been combined to form the PZ section. The data have also been separated into the PS section. We focus on both the PZ section and the PS section.

Figure 10 presents a typical shot gather. On the left is the PZ common-shot gather, and on the right the PS common-shot gather. The PZ shot gather has fewer time samples than the PS shot gather because a longer time is needed to observe the converted-wave events. Also, note the polarity flip in the PS common-shot gather, a typical characteristic of this type of data.
In both datasets, the PZ and the PS sections were migrated using wave-equation shot-profile migration. Both, the P and the S velocity models are unknown for this problem; therefore, we migrate the data using a simple velocity model with a vertical gradient. Figure 11 shows both velocity models, the P-velocity model in the left panel, and the S-velocity model in the right panel.

Figure 12 presents a PS image on the left, and two angle-domain common-image gathers on the right. Both common-image gathers are taken at the same location, indicated by the solid line (CIG=14500) in the image. The PS image is taken at zero subsurface-offset. This is not the ideal position, since the polarity flip destroys the image at this location. The ideal case will be to flip the polarities in the angle domain, as it was discussed in the previous section; unfortunately, we do not have the correct velocity model; therefore, we have only an approximate solution to the final PS image. However, as we will see later, through an update on the velocity model we are able to obtain an image that is more accurate. Figure 13 represents the image-dip field for this experiment, which was estimated along the zero subsurface-offset of the PS section.

The angle-domain common-image gather on panel (b) of Figure 12 represents the angle-domain common-image gathers using the conventional methodology, which will be \( \tan \theta_0 \) on the diagram flow on Figure 2. The angle-domain common-image gather on panel (c), represents the true converted-wave angle-domain common-image gather, that is obtained with the method described in this paper.

The geology for this section consists of very gentle dips, representing a sedimentary depositional system with little structural deformation; therefore, the angle gather on panel (b) has the polarity flip very close to zero angle. The true PS-ADCIG, panel (c), also preserves this characteristic. The residual curvature for the events, whether primaries or multiples on panel (b), is larger than the residual curvature of the same events in the true PS angle-domain common-image gather.

Figure 14 compiles the angle-domain common-image gathers for this dataset, all of which are
taken at the same position, CIG=14500. From left to right, the PZ-ADCIG, the true PS-ADCIG, and both the P-and-S angle-gather representation, panels (c) and (d), for the true PS-ADCIG on panel (b). Notice that most of the primary events have a residual curvature. The residual moveout is more prominent for those events that we identify as multiples.

Notice that the angle coverage in both P-and-S ADCIGs representations is smaller than for the true PS-ADCIG, since the coverage of an individual plane-wave is smaller than the combination of two plane-waves, as it is the case in converted-mode data.

The difference in the residual moveout between the single-mode PZ-ADCIG and the PS-ADCIG suggests an erroneous S-velocity model. Therefore, we decided to run a second migration for the PS section alone with a slower S-velocity model; the ratio between the first and second S-velocity model is 2.

Figure 15 presents the PZ and the updated PS results using shot-profile migration. The PZ migration was done with the P-velocity model in Figure 11. The PS migration was done with the updated S-velocity model. The left panel on Figure 15 shows the PZ migration result, the top panel shows the image at zero subsurface-offset, and the center and bottom panels are four CIGs taken at locations indicated by the solid lines in the zero subsurface image. The center panel represents SODCIGs, and the bottom panel represents the angle-domain common-image gathers. Observe that most of the events in the ADCIGs are mainly flat, which suggests that the initial linear P-velocity model is a reasonable approximation.

The right panel on Figure 15 shows the results of the PS migration. The top panel presents the stacking of the angle gathers after the polarity flip correction. Similar to the PZ results (left panels), the center panel represents four subsurface offset-domain common-image gathers. The bottom panel represents the true PS-ADCIGs that corresponds to the SODCIGs, at the locations indicated by the
solid lines in the angle-stack image. Note that most of the events in the PS-ADCIGs are approximately flat. Although there is a resemblance between the PZ and the PS images with respect to the general geology, the events do not match. There is a strong presence of multiples, due to the shallow sea bottom (120 m). These multiples are more prominent in the PS section than in the PZ section, that is because the PZ summation already eliminates the source ghost.

Finally, Figure 16 shows the same common-image gather as in Figure 14 but after the second migration for the PS section. This gather is taken at a location of 14500 m from the images on Figure 15. Panel (a) presents the PZ-ADCIG, panel (b) presents the true PS-ADCIG, and panels (c) and (d) are the PS-ADCIG representation in both P-ADCIG and S-ADCIG, respectively. Note that the events in all the different angle gathers are nearly flat, and several events in both the PZ and PS angle-gathers correlate. This suggests that some of these reflections might come from the same geological feature. Also note that there is a residual moveout at high angles in the S-ADCIG, for the first 1000 m. This information might be useful for a residual moveout analysis to compute an S-velocity perturbation, which might produce a final PS image that matches the PZ image for the main events.

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APPENDIX A

DERIVATION OF THE EQUATION FOR THE ANGLE-DOMAIN TRANSFORMATION

This appendix presents the derivation of the main equation for this paper, that is, the transformation from the subsurface offset domain into the angle domain for converted-wave data.

The derivation follows the well-known equations for apparent slowness in a constant-velocity medium in the neighborhood of the reflection/conversion point. Our derivation is consistent with those presented by Fomel (2001); Sava and Fomel (2003); and Biondi (2005).

The expressions for the partial derivatives of the total traveltime with respect to the image point coordinates are as follows (Rosales and Rickett, 2001b):

\[
\begin{align*}
\frac{\partial t}{\partial m} &= S_s \sin \beta_s + S_r \sin \beta_r, \\
\frac{\partial t}{\partial h} &= -S_s \sin \beta_s + S_r \sin \beta_r, \\
\frac{\partial t}{\partial z} &= S_s \cos \beta_s + S_r \cos \beta_r. 
\end{align*} \tag{A-1}
\]

Where $S_s$ and $S_r$ are the slowness (inverse of velocity) at the source and receiver locations. Figure ?? illustrates all the angles in this discussion. The angle $\beta_s$ is the direction of the wave propagation for the source, and the angle $\beta_r$ is the direction of the wave propagation for the receiver.

Throughout these set of equations, we obtain:

\[
\begin{align*}
-\frac{\partial z}{\partial h} &= \frac{S_r \sin \beta_r - S_s \sin \beta_s}{S_r \cos \beta_r + S_s \cos \beta_s}, \\
-\frac{\partial z}{\partial m} &= \frac{S_s \sin \beta_s + S_r \sin \beta_r}{S_s \cos \beta_s + S_r \cos \beta_r}. 
\end{align*} \tag{A-2}
\]

We define two angles, $\alpha$ and $\gamma$, to relate $\beta_s$ and $\beta_r$ as follows:
\[ \alpha = \frac{\beta_r + \beta_s}{2}, \quad \text{and} \quad \theta = \frac{\beta_r - \beta_s}{2}. \] (A-3)

Through the change of angles presented on equation (A-3), and by following basic trigonometric identities, we can rewrite equations (A-2) as follows:

\[ - \frac{\partial z_\xi}{\partial h_\xi} = \frac{\tan \theta + \delta \tan \alpha}{1 - \delta \tan \alpha \tan \theta}, \]
\[ - \frac{\partial z_\xi}{\partial m_\xi} = \frac{\tan \alpha + \delta \tan \theta}{1 - \delta \tan \theta \tan \alpha}, \] (A-4)

where,

\[ \delta = \frac{S_r - S_s}{S_r + S_s} = \frac{\gamma(m_\xi, z_\xi) - 1}{\gamma(m_\xi, z_\xi) + 1}. \] (A-5)

We introduce the following notation in equation A-4:

\[ \tan \theta_0 = - \frac{\partial z_\xi}{\partial h_\xi}, \]
\[ \delta = - \frac{\partial z_\xi}{\partial m_\xi}. \]

In this notation, \( \tan \theta_0 \) represents the pseudo-reflection angle, and \( \delta \) is field for the local step-outs.

The bases for this definition resides in the conventional PP case. For that case, the pseudo-reflection angle is the reflection angle, and the field \( \delta \) represents the geological dip (Fomel, 1996). Based on this notation, equation A-4 can be rewritten as:

\[ \tan \theta_0 = \frac{\tan \theta + \delta \tan \alpha}{1 - \delta \tan \alpha \tan \gamma}, \] (A-6)
\[ \delta = \frac{\tan \alpha + \delta \tan \theta}{1 - \delta \tan \theta \tan \alpha}. \] (A-7)
Following basic algebra, equation A-7 can be rewritten as:

\[
\tan \alpha = \frac{\delta - \delta \tan \theta}{1 - \delta \delta \tan \theta}.
\]  
(A-8)

Substituting equation A-8 into equation A-6, and following basic algebraic manipulations, we obtain equation 2 in the paper.

**APPENDIX B**

**DERIVATION OF EQUATION FOR INDEPENDENT-ANGLE TRANSFORMATION**

This appendix presents the derivation for the independent-angle transformation equations. The first element of this derivation is Snell’s law:

\[
\frac{\sin \phi}{v_p} = \frac{\sin \sigma}{v_s}.
\]  
(B-1)

From the definition of the full-aperture angle, \( \theta \), (equation 1), we obtain the following:

\[
\sigma = 2\theta - \phi. \quad \text{(B-2)}
\]

\[
\phi = 2\theta - \sigma. \quad \text{(B-3)}
\]

Introducing equations B-2 and B-3 into equation B-1, and using the P-to-S velocity ratio (\( \gamma \)), we obtain:

\[
\sin \phi = \gamma \sin(2\theta - \phi), \quad \text{(B-4)}
\]

\[
\sin \sigma = \gamma^{-1} \sin(2\theta - \sigma). \quad \text{(B-5)}
\]
Using simple trigonometric relations and basic algebra, from equations B-5 and B-5, we get, respectively,

\( (1 + \gamma \cos 2\theta) \tan \phi = \gamma \sin 2\theta, \) \hspace{1cm} (B-6)

\( (\gamma + \cos 2\theta) \tan \sigma = \sin 2\theta. \) \hspace{1cm} (B-7)

Equations B-6 and B-7 translate into equations 3 and 4, respectively.

**APPENDIX C**

**VALIDATION OF ANGLE-DOMAIN TRANSFORMATION THROUGH A KIRCHHOFF APPROACH**

In this part, we obtain the relation to transform subsurface offset-domain common-image gathers into angle-domain common-image gathers for the case of PS data. To perform this derivation, we use the geometry in Figure 17 in order to obtain the parametric equations for migration on a constant velocity medium.

Following the derivation of Fomel (1996) and Fomel and Prucha (1999), and applying simple trigonometry and geometry to Figure 17, we obtain parametric equations for migrating an impulse recorded at time \( t_D \), midpoint \( m_D \) and surface offset \( h_D \) as follows:

\[
\begin{align*}
  z_\xi &= (L_s + L_r) \frac{\cos \beta_r \cos \beta_s}{\cos \beta_r + \cos \beta_s}, \\
  2h_\xi &= 2h_D + (L_s + L_r) \frac{\sin \beta_s \cos \beta_r - \sin \beta_r \cos \beta_s}{\cos \beta_r + \cos \beta_s}, \\
  m_\xi &= m_D - \frac{(L_s + L_r) \sin \beta_s \cos \beta_r + \sin \beta_r \cos \beta_s}{2} \frac{1}{\cos \beta_r + \cos \beta_s}. \hspace{1cm} (C-1)
\end{align*}
\]

where the total path length is:
\[ t_D = S_s L_s + S_r L_r, \]
\[ z_s - z_r = L_s \cos \beta_s - L_r \cos \beta_r. \]  
\[(C-2)\]

From that system of equations, Biondi (2005) shows that the total path length is

\[ L = \frac{t_D \cos \beta_r + \cos \beta_s}{2 S_s \cos \beta_r + S_r \cos \beta_s}. \]  
\[(C-3)\]

We can rewrite system (C-1) as:

\[ z_{\xi} = \frac{(L_s + L_r) \cos^2 \alpha - \sin^2 \gamma}{2 \cos \alpha \cos \gamma}, \]
\[ 2h_{\xi} = 2h_D - (L_s + L_r) \frac{\sin \gamma}{\cos \alpha}, \]
\[ m_{\xi} = m_D - \frac{(L_s + L_r) \sin \alpha}{2 \cos \gamma}. \]  
\[(C-4)\]

where \( \alpha \) and \( \gamma \) follow the same definition as in equation (A-3). where, \( L \) in terms of the angles \( \alpha \) and \( \beta \) is:

\[ L(\alpha, \beta) = \frac{t_D}{(S_r + S_s) + (S_r - S_s) \tan \alpha \tan \gamma}. \]  
\[(C-5)\]

**Tangent to the impulse response**

Following the demonstration done by Biondi (2005), the derivative of the depth with respect to the subsurface offset, at a constant image point, and the derivative of the depth with respect to the image point, at a constant subsurface offset are given by the following:
\[
\frac{\partial z_{\xi}}{\partial h_{\xi}}\bigg|_{m_{\xi}=\bar{m}_{\xi}} = -\frac{\partial T}{\partial m_{\xi}}\bigg|_{m_{\xi}=\bar{m}_{\xi}} = \frac{\partial z_{\xi}}{\partial \alpha} \frac{\partial m_{\xi}}{\partial \alpha} + \frac{\partial z_{\xi}}{\partial \gamma} \frac{\partial m_{\xi}}{\partial \gamma}, \tag{C-6}
\]

and

\[
\frac{\partial z_{\xi}}{\partial m_{\xi}}\bigg|_{h_{\xi}=\bar{h}_{\xi}} = -\frac{\partial T}{\partial h_{\xi}}\bigg|_{h_{\xi}=\bar{h}_{\xi}} = \frac{\partial z_{\xi}}{\partial h_{\xi}} \frac{\partial m_{\xi}}{\partial h_{\xi}} + \frac{\partial z_{\xi}}{\partial \gamma} \frac{\partial m_{\xi}}{\partial \gamma}, \tag{C-7}
\]

where the partial derivatives are:

\[
\begin{align*}
\frac{\partial z_{\xi}}{\partial \alpha} &= -\frac{L}{\cos \alpha \cos \gamma} \left[ \tan \alpha (\cos^2 \alpha + \sin^2 \gamma) + \frac{(S_r - S_s) \tan \gamma (\cos^2 \alpha - \sin^2 \gamma)}{\cos^2 \alpha} \right], \\
\frac{\partial z_{\xi}}{\partial \gamma} &= -\frac{L}{\cos \alpha \cos \gamma} \left[ \tan \gamma (\cos^2 \alpha + \sin^2 \gamma) + \frac{(S_r - S_s) \tan \alpha (\cos^2 \alpha - \sin^2 \gamma)}{\cos^2 \gamma} \right], \\
\frac{\partial m_{\xi}}{\partial \alpha} &= -\frac{L}{\cos \gamma} \left[ \cos \alpha - \frac{(S_r - S_s) \sin \alpha \tan \gamma}{\cos^2 \alpha} \right], \\
\frac{\partial m_{\xi}}{\partial \gamma} &= -\frac{L \sin \alpha}{\cos^2 \gamma} \left[ \sin \gamma - \frac{(S_r - S_s) \tan \alpha \tan \gamma}{\cos \gamma} \right], \\
\frac{\partial h_{\xi}}{\partial \alpha} &= -\frac{L \sin \gamma}{\cos^2 \alpha} \left[ \sin \alpha - \frac{(S_r - S_s) \tan \alpha \sin \gamma}{\cos \alpha} \right], \\
\frac{\partial h_{\xi}}{\partial \gamma} &= -\frac{L}{\cos \alpha} \left[ \cos \gamma - \frac{(S_r - S_s) \tan \alpha \sin \gamma}{\cos^2 \gamma} \right]. \tag{C-8}
\end{align*}
\]

Figure 18 presents the analytical solutions for the tangent to the impulse response. This was done for an impulse at a PS-travel time of 2 s, and a \( \phi \) value of 2. The left panel shows the solution for equation (C-6). The right panel shows the solution for equation (C-7). The solid lines superimpose on both surfaces represents one section of the numerical derivative to the impulse response. The perfect correlation between the analytical and numerical solution validates our analytical formulations. This results supports the analysis presented with the kinematic equations (Appendix A).
APPENDIX D

ALTERNATIVE DERIVATION FOR THE ANGLE-DOMAIN TRANSFORMATION EQUATION

Sava and Fomel (2005) present an alternative derivation to transform converted-wave angle-domain common-image gathers. This appendix presents a summary for this deduction. Using the definitions introduced in the preceding section, we can make the standard notations for the source and receiver coordinates: \( s = m - h, r = m + h \) The traveltime from a source to a receiver is a function of all spatial coordinates of the seismic experiment \( t = t(m,h) \). Differentiating \( t \) with respect to \( m \) and \( h \), and making the standard notations \( \mathbf{p}_\alpha = \nabla_\alpha t \), where \( \alpha = \{m,h,s,r\} \), we can write:

\[
\begin{align*}
\mathbf{p}_m &= \mathbf{p}_r + \mathbf{p}_s, \quad (D-1) \\
\mathbf{p}_h &= \mathbf{p}_r - \mathbf{p}_s. \quad (D-2)
\end{align*}
\]

By definition, \( |\mathbf{p}_s| = s_s \) and \( |\mathbf{p}_r| = s_r \), where \( s_s \) and \( s_r \) are slownesses associated with the source and receiver rays, respectively.

For computational reasons, it is convenient to define the angles \( \theta \) and \( \delta \) using the following relations:

\[
2\theta = \theta_s + \theta_r , \quad 2\delta = \theta_s - \theta_r . \quad (D-3)
\]

Those two angles are the analogs of the image midpoint and offset coordinates. With these two angles, we can find the reflection angles \( \theta_s \) and \( \theta_r \) using the relations

\[
\theta_s = \theta - \delta , \quad \theta_r = \theta + \delta . \quad (D-4)
\]

Angle \( 2\theta \) represents the opening between and incident and a reflected ray, and angle \( \delta \) represents the deviation of the bisector of the angle \( 2\theta \) from the normal to the reflector. For P-P reflections, \( \delta = 0. \)
Reflection angle $\theta$

By analyzing the geometric relations of various $p_o$ vectors, we can write the following trigonometric expressions:

\[
|p_h|^2 = |p_s|^2 + |p_o|^2 - 2|p_s||p_o|\cos(2\theta), \quad (D-5)
\]
\[
|p_m|^2 = |p_s|^2 + |p_o|^2 + 2|p_s||p_o|\cos(2\theta), \quad (D-6)
\]
\[
p_m \cdot p_h = |p_o|^2 - |p_s|^2. \quad (D-7)
\]

We can transform this expression further using the notations $|p_s| = s$ and $|p_o| = \gamma(m, z)s$, where $s(m, h)$ is the slowness associated with the incoming ray at every image point.

Solving for $\tan \theta$ from equations D-5 and D-6, we obtain an expression, equivalent to equation 2 in the main text, for the reflection angle function of position and offset wavenumbers ($k_m, k_h$):

\[
\tan^2 \theta = \frac{(1 + \gamma(m, z))^2|k_h|^2 - (1 - \gamma(m, z))^2|k_m|^2}{(1 + \gamma(m, z))^2|k_m|^2 - (1 - \gamma(m, z))^2|k_h|^2}. \quad (D-8)
\]

REFERENCES


Fomel, S., 1996, Migration and velocity analysis by velocity continuation: SEP–92, 159–188.


LIST OF FIGURES

1 Definition of angles for the converted-mode reflection experiment. The angles $\theta$, $\phi$, $\sigma$, $\alpha_x$ represent the half-aperture, the incident, the reflection, and the geological dip angles, respectively.

2 Flow chart to transform the subsurface-offset common-image gathers into the angle domain. The flow diagram also presents the mapping into P-ADCIGs and S-ADCIGs.

3 Synthetic example that illustrates the method in Figure 2. Panel (a) is a single shot gather for a 30° dipping layer event. Panel (b) is the image of this single shot gather. Panel (c) is the pseudo-opening angle, $\tan \theta_0$. Panel (d) is the true PS-ADCIG.

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16 Compilation of angle-domain common-image gather for the 2-D Mahogany dataset; all the CIGs are taken at the same image location (CIG=14500). (a) PZ-ADCIG, (b) PS-ADCIG, (c) P-ADCIG, (d) S-ADCIG.

17 Parametric formulation of the impulse response.

18 Validation of the analytical solutions for the tangent to the impulse response, the surface represents the analytical solutions and superimpose is the cut with the numerical derivative. Left: For equation (C-6). Right: For equation (C-7) analytical solutions for the tangent of the spreading surface for different values of $\phi$
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