Conventional velocity analysis applied to images produced by wave-equation migration with a cross-correlation imaging condition makes use either of moveout information from space-lags or of focusing information from time-lags. However, more robust velocity estimation methods can be designed to take advantage simultaneously of the moveout and focusing information provided by the migrated images. Such joint velocity estimation requires characterization of the moveout surfaces defined jointly for space- and time-lag common-image gathers. Such surfaces for single events (shots) are non-planar but they reduce naturally to the conventional space-lag and time-lag moveout functions. The superposition of those surfaces for many experiments (shots) in a common-image gather forms a shape which can be characterized as a cone in the lag space. When imaged with correct velocity, the focus of the cone is located at the correct image depth and at zero space- and time-lags. When imaged with incorrect velocity, the focus of the cone shifts both in depth and along the time-lag axis. The characteristics of the cones are directly related to the quality of the velocity model, thus their analysis provides a rich source of information for velocity model building. Joint migration velocity analysis technique exploiting the entire information provided by the
extended imaging condition has the potential to benefit from the combination of the robustness of
depth focusing analysis and of the high resolution of conventional semblance analysis.
INTRODUCTION

A key challenge for imaging in complex geology is an accurate determination of the velocity model in the area under investigation. Migration velocity analysis is based on the principle that image accuracy indicators are optimized when data are correctly imaged. A common procedure for migration velocity analysis is to examine the alignment of images created with data from many complementary experiments. An optimal choice for image analysis in complex areas is the angle domain gather which is free of complicated artifacts present in surface offset gathers (Stolk and Symes, 2004). If images constructed by illuminating a point from various directions are aligned, then the velocity model used for imaging is said to be accurate. This idea is usually referred to as the semblance principle (Yilmaz, 2001) and it represents the foundation of most velocity analysis methods in use today.

Often, semblance analysis is performed in the angle domain. Several methods have been proposed for angle decomposition (Sava and Fomel, 2003a,b; Fomel, 2004; Biondi and Symes, 2004). All these procedures require decomposition of extrapolated wavefields or of migrated images in components that are related to the reflection angles. This imaging procedure requires the application of an extended imaging condition (Sava and Fomel, 2006) which implements a point-by-point comparison of the source and receiver wavefields extrapolated from the surface. In general, the comparison is done using simple image processing procedures applied at every location in the subsurface. If the source and receiver wavefields match each-other, then their cross-correlation maximizes at zero lag in space and time; otherwise, their cross-correlation does not maximize at zero lag indicating wavefield reconstruction error which may have different causes, e.g., velocity inaccuracy.

The source and receiver wavefields used for imaging are 4D objects, functions of space coordinates and time (or frequency). For simplicity, we discuss in this paper only time-domain imaging,
although our analysis applies equally well to frequency domain imaging. For such 4D objects, the images obtained by extended imaging conditions are characterized in general by a 3D space-lag vector and a 1D time-lag. The images constructed with space- and time-lags can be decomposed into functions of reflection angles using geometric relations between incident and reflected rays (Sava and Fomel, 2006).

Conventional migration velocity analysis exploits separately either space-lag information, by semblance analysis (Sava and Biondi, 2004a,b; Shen et al., 2005), or time-lag information, by depth focusing analysis (Faye and Jeannot, 1986; MacKay and Abma, 1992, 1993; Nemeth, 1995, 1996; Sava and Fomel, 2006). However, we suggest here that a more robust velocity analysis approach combines the information provided by the space and time-lags and optimize migrated images using all available information. In this way, we can leverage at the same time the robustness of depth focusing analysis and the high resolution of semblance analysis.

In this paper, we analyze the moveout function for common-image gathers constructed by extended imaging conditions applied after conventional wavefield extrapolation. We first derive the analytic expression of the moveout function for space- and time-lag extended imaging conditions under the homogeneous media assumption. Next, we focus on the common-image gathers in multi-shot experiments and quantitatively analyze their characteristics, especially the features related to velocity error. Finally, different synthetic examples are presented to verify the derived analytic moveout functions and to illustrate the application of the analyses to complex geologic models.

**WAVE-EQUATION IMAGING CONDITIONS**

Under the single scattering assumption, the seismic migration procedure consists of two main steps: wavefield reconstruction and imaging condition. Wavefield reconstruction involves constructing so-
olutions to a wave-equation with recorded seismic data as initial and boundary conditions. Various numeric solutions for the acoustic wave-equation can be chosen depending on the specific requirements of cost and accuracy. However, regardless of the specific implementation, the reconstruction of both source and receiver wavefields is similar. In a known background velocity model, we forward propagate from the source location to obtain the source wavefield, and backward propagate the recorded seismic data to obtain the receiver wavefield. The reconstructed source and receiver wavefields can be defined as four-dimensional objects functions of spatial location $x = (x, y, z)$ and time $t$, 

\begin{align}
    u_s & = u_s (x, t) , \\
    u_r & = u_r (x, t) ,
\end{align}

where $u_s$ and $u_r$ stand for the source and receiver acoustic wavefields, respectively.

An imaging condition is designed to extract from these reconstructed wavefields the locations where reflectors occur in the subsurface. The image $r (x)$ is obtained by evaluating the match between the reconstructed source and receiver wavefields at every location in the subsurface. A conventional imaging condition (Claerbout, 1985) forms an image as the cross-correlation of the source and receiver wavefields evaluated at zero lag:

$$r (x) = \sum_t u_r (x, t) u_s (x, t) .$$

An alternative extended imaging condition (Rickett and Sava, 2002; Sava and Fomel, 2006) generalizes the conventional imaging condition by preserving the information from non-zero cross-
correlation lags for the output image:

\[
r(\mathbf{x}, \lambda, \tau) = \sum_{t} u_s(\mathbf{x} - \lambda, t - \tau) u_r(\mathbf{x} + \lambda, t + \tau).
\] (4)

The quantities \(\lambda\) and \(\tau\) represent the spatial and temporal cross-correlation lags between the source and receiver wavefields. Like the conventional imaging condition, the extended imaging condition also evaluates the match between the wavefields by their cross-correlation, but it preserves in the output the information corresponding to non-zero cross-correlation space- and time-lags. Due to the existence of the lags, the resulting images are in fact series of images obtained for different lags at each subsurface location \(\mathbf{x}\). We refer to those image series as wave-equation extended images. Such image hypercubes can help us analyze the accuracy of reconstructed wavefields. If the local cross-correlation between the source and receiver wavefields is maximized at zero lag for all four dimensions, those wavefields are extrapolated correctly. If this is not true, we may conclude that the wavefield reconstruction is incorrect, indicating incorrect velocity or incorrect wavefield extrapolation or irregular illumination, or the failure of the single scattering assumption, e.g., due to the presence of multiples. In this research, we consider that the errors in wavefield reconstruction are caused by incorrect velocity only.

**MOVEOUT ANALYSIS FOR EXTENDED IMAGES – POINT SOURCE**

The characteristics of extended images can be studied by analyzing common-image gathers (CIG). A reflection event analyzed in a CIG is represented by a multi-variable function \(z = z(\lambda, \tau)\), and the geometric shape of this function is often referred to as “moveout” by analogy with surface seismic data. Therefore, extended images are characterized by moveout surface in a CIG. Understanding the moveout surfaces for the case of correct and incorrect velocity is essential for the purpose of
migration velocity analysis and the application of subsequent tomographic procedure.

Consider the reflection geometry depicted in Figure 1: the unit vector $n = \{n_x, n_y, n_z\}$ and the distance $d$ identifies the position of a reflection plane relative to the seismic source $S$; the vector $c = \{c_x, c_y, 0\}$ identifies the fixed horizontal position of a CIG relative to the source position, the vector $z = \{0, 0, z\}$ represents the depth of the image constructed by the imaging condition. We consider here the case where extended imaging condition are computed as functions of horizontal space-lag $\lambda = \{\lambda_x, \lambda_y, 0\}$ and time-lag $\tau$, but the same logic applies to the more general case when the space-lag $\lambda$ is three-dimensional.

Under the assumption of homogeneous media, a wavefield characterizing wave propagation from a point source can be represented by a cone in space-time. The source wavefield is represented by a cone with the origin at zero time and at the source location on the surface, as shown in Figure 2(a). Likewise, we construct the receiver wavefield as the mirror image of the source wavefield relative to the reflector indicated by the black line, as shown in Figure 2(b). Using this description of the seismic wavefields, we can represent the source and receiver wavefields in space-time by the analytic expressions:

\[
\|c + z\| = vt, \quad (5) \\
\|c + z - 2dn\| = vt, \quad (6)
\]

where $v$ is the velocity of the medium, $t$ is the propagation time of the wavefield, $z = \{0, 0, z\}$ with $z$ representing the depth of the image. As discussed before, the imaging conditions identify the position of the reflector by evaluating the match between source and receiver wavefields. In other words, an image forms at the spatial positions where the source and receiver wavefields intersect. Mathematically, this condition is equivalent to identifying the positions which are solutions to the
system given by equations 5-6, i.e. by solving the system for the reflection depth $z$ at coordinates \( \{c_x, c_y\} \). Figures 2(a)-2(c) illustrate this procedure. Figure 2(a) represents the source wavefield, Figure 2(b) represents the receiver wavefield, and the cones are symmetric relative to the reflector.

The intersection of the cones occurs at different times. However, the locations of the intersections are consistent with the position of the reflector since their projection on the \( x-z \) plane matches the reflector perfectly.

Likewise, the extended imaging condition seeks to find the intersections between the source and receiver wavefields. However, this procedure is different because both wavefields are shifted by quantities corresponding to the space- and time-lags of the cross-correlation. Since we construct the extended images for the same subsurface location, we keep the CIG coordinate \( \mathbf{c} \) fixed. The shifted wavefields are functions of the space quantity \( \lambda = \{\lambda_x, \lambda_y, 0\} \) and time quantity \( \tau \). Thus, the extended imaging condition is represented by the system

\[
\| \mathbf{c} + \mathbf{z} + \lambda \| = v(t + \tau), \tag{7}
\]
\[
\| \mathbf{c} + \mathbf{z} - 2dn - \lambda \| = v(t - \tau). \tag{8}
\]

The application of the extended imaging condition in equation 4 is equivalent to solving equations 7-8. The solution represents the moveout function \( z = z(\lambda, \tau) \) at fixed CIG coordinates \( \mathbf{c} = \{c_x, c_y, 0\} \). This moveout function describes how the depth of the image $z$ changes with the variation of the space- and time-lags.

A formal solution to the system of equations 7-8 leads to the following expression of the depth $z$ at the CIG coordinates \( \{c_x, c_y\} \) function of space- and time-lags:

\[
z(\lambda, \tau) = (dn_x)K + v\tau \sqrt{K^2 + \frac{||\mathbf{c} + \lambda||^2}{(dn_x)^2 - (v\tau)^2}}, \tag{9}
\]
\[ K = 1 - \frac{(c \cdot n) d - (n_x^2 + n_y^2) d^2 + (c - d n) \cdot \lambda}{(dn_z)^2 - (v \tau)^2}. \]  \hspace{1cm} (10)

Equation 9 represents the moveout function characterizing the shape of the extended images.

To better understand the characteristics of the moveout function, we analyze two special cases of the extended images. The first case corresponds to imaging with space-lags only \((\tau = 0)\), which is the slice of the moveout surface at zero time-lag. Since the square-root term vanishes owing to zero time-lag, we obtain a linear moveout function of space-lag parameters. The coefficient depends on the reflection angles, which justifies the angle decomposition methods based on slant-stacks applied to space-lag CIGs (Sava and Fomel, 2003a; Biondi and Symes, 2004; Fomel, 2004). The second case corresponds to imaging with time-lag only \((\lambda = 0)\), which is the slice of the moveout surface at zero space-lags. For this special case, the moveout function is still nonlinear.

As discussed before, the goal of our research is to understand the relationship between features of extended images and velocity error. A quantitative analysis of the influence of the velocity error on the extended images is required to achieve this goal. When incorrect velocity is used for wavefield reconstruction, the wavefields are incorrectly extrapolated. Consequently, applying the imaging condition produces distorted images. As a result, we first must understand the influence of an incorrect velocity model on the reconstructed wavefields because analytic descriptions of source and receiver wavefields are the key step for deriving the moveout function. To simplify the problem, we denote the migration velocity \(v_m = \rho_v v\), where \(\rho_v\) is a constant factor by which the migration velocity differs from the correct velocity.

The source wavefield is reconstructed as in the preceding situation, except that we use an incorrect migration velocity. The wavefields are represented by cones with radii different from those used in the case of correct velocity. Figure 3(a) shows the source wavefield reconstructed with incorrect
velocity. The wavefield is described by the equation

$$\|e + z\| = v_m t.$$  \hspace{1cm} (11)

For the receiver wavefield, the situation is more complicated. Unlike the source wavefield, the receiver wavefield is reconstructed by backward propagation of the recorded data. In other words, we reconstruct the cone representing the source wavefield from its origin while we reconstruct the cone representing the receiver wavefield from its depth slice recorded on the surface. If the correct velocity is used, the cone for the receiver wavefield obtained is exactly the mirror image of the cone for source wavefield, which means both cones are symmetric in space and have the same explosive time. In other words, the two wavefields are generated at the same time. In contrast, if an incorrect velocity is used, the cone representing the receiver wavefield has an incorrect radius, as for the source wavefield. Furthermore, the origin of the cone is shifted from its true position in space and time, and the symmetry axis between the two cones deviates from its true spatial location. In summary, the receiver wavefield reconstructed using an incorrect velocity is represented by a cone with incorrect radius, origin and symmetry axis.

As the receiver wavefield shifts in time, the reconstructed source and receiver wavefields are not triggered at the same time. It is necessary to introduce a new variable to describe such a deviation. Using the concepts of focusing depth $d_f$ and migration depth $d_m$ (MacKay and Abma, 1992), we have $d_f = d/\rho v$, $d_m = d\rho v$, where $\rho v$ is the constant velocity scaling factor. We thus define the deviation as focusing delay $t_d$, which is quantified by the following formula:

$$t_d = \frac{d_f - d_m}{v_m} = \frac{d(1 - \rho^2 v^2)}{v \rho^2 v}.$$  \hspace{1cm} (12)

If velocity $v_m$ is correct, the focusing depth and migration depth are identical and equal to the true
depth of the reflection, and the focusing delay \( t_d \) vanishes.

As the receiver wavefield also shifts in space, symmetry between both wavefields is maintained but the symmetry plane changes. The plane defined by \( d n \) in the case of correct velocity now becomes \( d_f n_m \), where \( d_f \) is the focusing depth of the reflection point, and \( n_m \) is a new normal vector which is a function of source-receiver location, correct normal \( n \) and the migrated velocity \( v_m \). Given this notation, the receiver wavefield is described by

\[
\| c + z - 2d_f n_m \| = v_m (t + 2t_d).
\]

(13)

Figure 3(b) shows the receiver wavefield in the case of a horizontal reflector when \( v_m \) is smaller than \( v \).

Solving the system of equations 11-13, we obtain the coordinates of the image when the incorrect velocity is used for imaging, as shown in 3(c). This solution is equivalent to applying the conventional imaging condition and finding the intersections between the incorrectly reconstructed source and receiver wavefields:

\[
z(c) = (d_f n_{mz}) K - v_m t_d \sqrt{K^2 + \frac{\| c \|^2}{(d_f n_{mz})^2 - (v_m t_d)^2}},
\]

(14)

where quantity \( K \) is defined by

\[
K = 1 - \frac{(c \cdot n_m) d_f - (n_{mx}^2 + n_{my}^2) d_f^2}{(d_f n_{mz})^2 - (v_m t_d)^2}.
\]

(15)

Depending on the ratio between the migrated velocity and true velocity, the term \( t_d \) can be positive or negative value.
Likewise, we may introduce the space and time-lags and obtain the expression for the shifted
source and receiver wavefields for the case of imaging with incorrect velocity:

$$\|c + z + \lambda\| = v_m (t + \tau), \quad (16)$$

$$\|c + z - 2d_f n_m - \lambda\| = v_m (t + 2t_d - \tau). \quad (17)$$

Solving this system gives the expression for the moveout function of space-lag $\lambda$ and time-lag $\tau$ for
incorrect velocity

$$z (\lambda, \tau) = (d_f n_m z) K + v_m (\tau - t_d) \sqrt{K^2 + \frac{\|c + \lambda\|^2}{(d_f n_m z)^2 - v_m^2 (\tau - t_d)^2}}, \quad (18)$$

where quantity $K$ is defined by

$$K = 1 - \frac{(c \cdot n_m) d_f - (n_{mx}^2 + n_{my}^2) d_f^2 + (c - d_f n_m) \cdot \lambda}{(d_f n_m z)^2 - v_m^2 (\tau - t_d)^2}. \quad (19)$$

Comparing the moveout function in equation 18 to the moveout function in equation 9, we
observe that the equations share a similar form, although the formula corresponding to the incorrect
velocity model is more complicated. The complexity arises from the additional term $t_d$, as well as
from the fact that $d$, $v$ and $n$ are replaced by $d_f$, $v_m$ and $n_m$. Owing to the existence of $t_d$, the
square-root term is preserved when $\tau = 0$; the space-lag moveout function thus has a nonlinear
dependence on the space-lags. This characteristic may provide information about the accuracy of
the velocity model.
The analytic results discussed in the preceding sections have complicated forms, which prevents
the formulas from being useful in practice. Moreover, the moveout functions are derived based
on the assumption of homogeneous media. Therefore, we must reduce the complexity of moveout
functions and generalize the analysis to inhomogeneous media.

Figure 4 illustrates a seismic reflection occurring in an inhomogeneous medium. The wave
propagation is arbitrary due to the inhomogeneity, as indicated by the curved wave paths. The
wavefield in such a medium can also have arbitrary geometric shape rather than a simple cone, thus
we cannot describe the wavefields using analytic formulas and derive analytic moveout functions.
However, if we restrict the observation to the immediate vicinity of the reflection point, which
means we consider the moveout surface in a small range of lags, we can approximate the irregular
wavefront by a plane. In other words, for inhomogeneous media, although the shapes of wavefronts
are arbitrary, they can be approximated as plane-waves in the vicinity of the reflection point. Using
the same geometry shown in Figure 1, the plane-waves are described by:

\[
\begin{align*}
\mathbf{p}_s \cdot \mathbf{x} &= vt, \\
\mathbf{p}_r \cdot (\mathbf{x} - 2d\mathbf{n}) &= vt,
\end{align*}
\]  

(20)

(21)

where \( \mathbf{p}_s \) and \( \mathbf{p}_r \) are the unit direction vectors of the source and receiver plane-waves, respectively,
and \( \mathbf{x} \) is the vector sum of \( \mathbf{c} \) and \( \mathbf{z} \). We can also obtain the shifted source and receiver planes by
introducing the space- and time-lag variables

\[
\begin{align*}
\mathbf{p}_s \cdot (\mathbf{x} + \lambda) &= v(t + \tau), \\
\mathbf{p}_r \cdot (\mathbf{x} - 2d\mathbf{n} - \lambda) &= v(t - \tau).
\end{align*}
\]  

(22)

(23)
Solving the system of equations 22-23 leads to the expression

\[(p_s - p_r) \cdot x = 2v\tau - (p_s + p_r) \cdot \lambda - 2d p_r \cdot n, \quad (24)\]

which characterizes the moveout at an image point function of space- and time-lags.

Furthermore, we have the following relations for the reflection geometry:

\[p_s - p_r = 2n \cos \theta, \quad (25)\]
\[p_s + p_r = 2q \sin \theta, \quad (26)\]

where \(n\) and \(q\) are unit vectors normal and parallel to the reflection plane, and \(\theta\) is the reflection angle. Combining equations 24-25-26, we obtain the simplified moveout function for plane-waves:

\[z(\lambda, \tau) = d_0 - \tan \theta \left(\frac{q \cdot \lambda}{n_z}\right) + \frac{v\tau}{n_z \cos \theta}. \quad (27)\]

The quantity \(d_0\) is defined as

\[d_0 = \frac{d - (c \cdot n)}{n_z}, \quad (28)\]

and represents the depth of the reflection corresponding to the chosen CIG location. This quantity is invariant for different plane-waves, thus assumed constant here.

When incorrect velocity is used for imaging, based on the analysis in the preceding section, we can obtain the moveout function

\[z(\lambda, \tau) = d_0 f - \frac{\tan \theta_m (q_m \cdot \lambda)}{n_{mz}} + \frac{v_m (\tau - t_d)}{n_{mz} \cos \theta_m}, \quad (29)\]
where $d_{0f}$ is the focusing depth of the corresponding reflection point, $v_m$ is the migration velocity, $t_d$ is the focusing delay, $\mathbf{n}_m$ and $\mathbf{q}_m$ are vectors normal and parallel to the reflector, respectively.

In the analysis above, we derive the moveout functions describing extended images for a single seismic experiment. However, typical imaging employs multi-shot seismic experiments for better illumination of subsurface and imaging redundancy which indicates the velocity accuracy. Thus, it is important to understand the characteristics of extended images in such complete seismic reflection experiments.

Since the wave equation is a linear partial differential equation, its solutions comply with the linear superposition principle. This is also true for extended images. Thus, the extended images in multi-shot experiments are a linear superposition of extended images from all single-shot experiments. In this case, the moveout surface represents the stack from the superposition of the surfaces characterizing those single-shot experiments.

By definition, the envelope for a family of curves is obtained by setting both the implicit definition of the family and derivative with respect to index parameter of the family equal to zero, then solving the resulting system of equations. Therefore, the envelope representing the extended images in multi-shot experiments with space-lag $\lambda$ and time-lag $\tau$ is given by the system

$$ G(z, \lambda, \tau, \theta) = 0, \quad (30) $$
$$ \frac{\partial G(z, \lambda, \tau, \theta)}{\partial \theta} = 0, \quad (31) $$

where $G$ represents the implicit definition of the moveout function 27 for correct velocity and 29 for incorrect velocity. Solving the system yields the following solutions:
\[ z(\lambda, \tau) = d_0 + \frac{v \tau}{n_z} \sqrt{1 - \left( \frac{n_z (q \cdot \lambda)}{v \tau} \right)^2} \]  
\[ (32) \]

for correct velocity, and

\[ z(\lambda, \tau) = d_{0f} + \frac{v_m (\tau - t_d)}{n_{mz}} \sqrt{1 - \left( \frac{n_{mz} (q_m \cdot \lambda)}{v_m (\tau - t_d)} \right)^2} \]  
\[ (33) \]

for incorrect velocity.

Analyzing the envelope functions for the cases of correct and incorrect velocities, we note that both envelope functions share a similar form; so they should have similar properties. The envelope functions become singular when \( \tau = 0 \) or \( \tau = t_d \), because at these special time-lags, all the individual surfaces corresponding to various experiments intersect at the same location. Mathematically, the envelope function is equivalent to a singular delta function at this \( \tau \). Also, the square-root term in both the formulas contains a subtraction, we must ensure that the quantity under the square-root is non-negative; otherwise, the formula fails. This failure implies that the range of space-lag \( \lambda \) is limited, which suggests that we must restrict the range of \( \lambda \) when we measure the moveout of reflections.

Given the envelope functions shown in equations 32-33, we conclude that the envelope surface forms cones, as shown in Figures 5(a) and 5(b), corresponding to correct and incorrect velocities respectively, for a horizontal reflector and in Figures 5(c) and 5(d) for a dipping reflector. The shapes of the cones change with both velocity and reflector dip. The cones are incomplete due to the limitations of acquisition aperture. When velocity used for imaging is correct, the origin of the cone is located at zero lags and at the correct depth of the reflection point. In contrast, when velocity is incorrect, the cone is shifted in both depth and time-lag directions. The shift in time-lag is exactly
the focusing delay $t_d$ defined above and the location of the shifted origin is the focusing depth $d_{0f}$.

If we slice the cone at negative time-lags, the slices correspond to upper half of the cone and thus curve downward. In contrast, the slices corresponds to lower half of the cone and thus curve upward. The events present in the zero time-lag slice in the case of incorrect velocity are characterized by the residual moveout used in more conventional migration velocity analysis (Sava and Biondi, 2004a,b), as indicated by the thick line in the plot 5(b) and 5(d). By examining the position of the origin of the cone, we can evaluate the accuracy of the velocity model. If the origin occurs at zero time-lag, the velocity model is correct. If the origin shifts to non-zero time-lags, then the migration velocity is incorrect. Thus, the position of the origin of the cone can be used as an indicator of velocity error.

To summarize, for inhomogeneous media, no analytic moveout function exists to describe moveout surfaces for extended images. However, by restricting our analysis to the vicinity of the reflection point, and by assuming that the velocity change above the image points is relatively uniform, we can use a plane-wave approximation to describe the analytic functions characterizing extended images. The parameters describing the moveout functions, though, represent a combination of the velocity errors accumulated along wave propagation paths, like the traveltime errors used in conventional travelt ime tomography.

EXAMPLES

We illustrate the validity of the moveout functions derived in the preceding sections with several synthetic models. The first model consists of a horizontal reflector embedded in a constant velocity medium, while the second model also consists of a constant velocity medium, but with a dipping reflector embedded. We use the first model to verify the accuracy of the moveout function for point sources, and use the second model to verify the accuracy of the moveout functions for plane-wave
sources. Extended images are generated for both correct and incorrect velocities. The incorrect velocity is obtained by scaling the correct velocity with a constant factor.

The image corresponding to the correct and incorrect velocities are shown in Figures 6(a) and 6(b), respectively. The extended images are constructed at CIG location $c_x = 0.75$ km. Thus, for a source located at $x = 4.5$ km, the CIG analyzed are located at $x = 5.25$ km. To verify the accuracy of the moveout function, we overlay the analytic moveout function on extended images at either fixed time-lags or at fixed horizontal space-lags.

Figures 7(a)-7(c) and 7(d)-7(f) depict space-lag extended images corresponding to the chosen CIG location for correct and incorrect velocities, respectively. From left to right, the panels correspond to slices at $\tau = \{-0.20, 0, +0.20\}$ s. In each column, the upper panel corresponds to correct velocity while the lower panel corresponds to incorrect velocity. The dashed line overlain on each panel corresponds to the analytic functions $z(\lambda_x, \tau)$ derived earlier. Notice that, at $\tau = 0$ the moveout event is linear for correct velocity and nonlinear for incorrect velocity, as expected.

Figures 8(a)-8(c) and 8(d)-8(f) depict time-lag extended images corresponding to the chosen CIG location for correct and incorrect velocities, respectively. From left to right, the panels correspond to $\lambda_x = \{-0.4, 0, +0.4\}$ km. In each column, the upper panel corresponds to correct velocity while the lower panel corresponds to incorrect velocity. The dashed line overlain on each panel corresponds to the analytic functions $z(\lambda_x, \tau)$ derived earlier. In both cases, the analytic formula accurately describes the moveout surface characterizing extended images, which illustrates the accuracy of the moveout function for both correct and incorrect velocities.

Figures 9(a) and 9(b) show the migrated images of the dipping reflector corresponding to correct and incorrect velocities, respectively. To obtain the images, we use 50 shots equally spaced on the surface and stack the images from all individual shots. As discussed before, the moveout surfaces
correspond to the superposition of the surfaces obtained for individual shots. We choose $x = 4.5 \text{ km}$ as the CIG location. Figures 10(a)-10(f) depict the moveout surfaces at different time-lags. From left to right, the upper panels display the slices at $\tau = \{-0.15, 0.0, +0.15\}$ s. The lower panels correspond to the same slices but overlain with the derived analytic envelope function $z(\lambda_x, \tau)$ given by equation 32 for various $\tau$. As expected, all events intersect at the same location at zero space-lag and a well focused image can be observed in the panel at $\tau = 0$ since the correct velocity is used for imaging. For incorrect velocity, the cones shift so that their origin is not located at zero time-lag. The slice at zero time-lag is therefore a curved event. Figures 11(a)-11(f) depict envelopes of moveout surfaces for different time-lags. From left to right, the upper panels display the slices of the cone at $\tau = \{-0.15, 0.0, +0.15\}$ s. The lower panels correspond to the same slices but overlain with the derived analytic envelope function $z(\lambda_x, \tau)$ given by equation 33 for various $\tau$. The slice at zero time-lag shows a curved event rather than a focused point, which demonstrates the shift of the origin of the cone. The slice at $\tau = 0.15 \text{ s}$ shows a more focused image, which means that the slice is closing to the origin of the cone. For both correct and incorrect velocities, the analytic functions match the experiments well, which demonstrates the accuracy of the formulas describing the envelopes.

Finally, we use the Sigsbee model to illustrate the application of our analyses to inhomogeneous media. Figures 12 show the velocity profile of the model. The sources are distributed over the left area of the model, thus they illuminate mainly the left side of the image. Figures 13(a) and 13(b) show the image migrated with correct and incorrect velocities respectively.

Figures 14(a) and 14(b) depict the moveout surfaces at different time-lags for the case of imaging with correct velocity. The panels correspond to slices of the cone at different time-lags. Figure 14(a) shows the slice at $\tau = -0.15 \text{ s}$. The events in the panel curve downward since the slice is cut at negative $\tau$ and corresponding to the upper half of the cone. Figure 14(b) shows the slice
at $\tau = 0$ s, which is cut at the origin of the cone. The events in the panel appear focused at zero
time-lag and space-lag, indicating correct velocity.

Figures 15(a) and 15(b) depict moveout surfaces for different time-lags for the case of imaging
with incorrect velocity. The panels correspond to slices of the cone at different time-lags. Because
a higher migration velocity is used, the origin of the cone is expected to shift to negative $\tau$. Fig-
ure 15(a) shows the slice at $\tau = -0.15$ s. The shallow events in the panel focus at zero space-lag,
which means that the slice is cut at the origin of the cone for those events. Deeper events have the
focus of the cone at other values of $\tau$. Figure 15(b) show the slices at $\tau = 0$ s. The events in the
panels curve upward because the slice is cut away from the focus of the cone.

**CONCLUSIONS**

An extended imaging condition offers the possibility to design robust migration velocity analysis
methods that simultaneously exploit conventional semblance analysis and depth focusing analy-
sis. The moveout functions characterizing extended images are non-planar analytic surfaces that
naturally reduce to the conventional space-lag and time-lag moveout functions.

The analytic moveout functions provide quantitative descriptions of the shapes of events in
extended images. The envelope of the moveout function characterizing common image gathers
constructed from multiple seismic experiments form cones in the lags-depth domain. The origin of
the cone represents a well focused image of the reflector. If velocity is correct, the origin appears at
zero time-lag and correct depth, otherwise the origin shifts to nonzero time-lags and to an incorrect
depth. Such a characteristic can be used as an indicator of velocity error for tomographic techniques.
Synthetic examples verify the validity of the analytic moveout functions and demonstrate that the
analyses for properties of the extended images hold even for complex media.
ACKNOWLEDGMENTS

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REFERENCES


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