Moveout analysis of wave-equation extended images

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ABSTRACT

Conventional velocity analysis applied to images produced by wave-equation migration with a cross-correlation imaging condition makes use either of moveout information from space-lags or of focusing information from time-lags. However, more robust velocity estimation methods can be designed to simultaneously take advantage of the semblance and focusing information provided by the migrated images. Such a velocity estimation requires characterization of the moveout surfaces defined jointly for space- and time-lag common-image gathers (CIG). The analytic solutions to the moveout surfaces can be derived by solving the system of equations representing the shifted source and receiver wavefields. The superposition of the surfaces from many experiments (shots) is equivalent to the envelope for the family of the individual CIG. The envelope forms a shape which can be characterized as a cone in the extended space of depth, space- and time-lags. When imaged with the correct velocity, the apex of the cone is located at the correct reflection depth and at zero space- and time-lags. When imaged with the incorrect velocity, the apex of the cone shifts both in the depth direction and along the time-lag axis. The characteristics of the cones are directly related to the quality of the velocity model. Thus, their analysis provides a rich source of information for
velocity model building. Synthetic examples are used to verify the derived formulae characterizing the moveout surfaces. The analytic formulae match the numeric experiments well, thus demonstrate the accuracy of the formulae. Based on all the information provided by the extended imaging condition, future application can benefit from the robustness of the depth focusing analysis and of the high resolution of the semblance analysis.
INTRODUCTION

A key challenge for imaging in complex geology is an accurate determination of the velocity model in the area under investigation. Migration velocity analysis is based on the principle that image accuracy indicators are optimized when data are correctly imaged. A common procedure for migration velocity analysis is to examine the alignment of images created with data from many complementary experiments. An optimal choice for image analysis in complex areas is the angle domain gathers which is free of complicated artifacts present in surface offset gathers (Stolk and Symes, 2004). If images constructed by illuminating a point from various directions are flat, then the velocity model used for imaging is said to be accurate. This idea is usually referred to as the semblance principle (Yilmaz, 2001) and it represents the foundation of most velocity analysis methods in use today.

Often, semblance analysis is performed in the angle domain. Several methods have been proposed for angle decomposition (Sava and Fomel, 2003; Fomel, 2004; Yoon and Marfurt, 2006; Higginbotham and Brown, 2009). Most of these procedures require decomposition of extrapolated wavefields or of migrated images in components that are related to the reflection angles. This imaging procedure requires the application of an extended imaging condition (Sava and Fomel, 2006) which implements a point-by-point comparison of the source and receiver wavefields extrapolated from the surface. In general, the comparison is done using simple image processing procedures applied at every location in the subsurface. If the source and receiver wavefields match each other kinematically, then their cross-correlation maximizes at zero lag in space and time; otherwise, their cross-correlation does not maximize at zero lag indicating wavefield reconstruction error which may have different causes, e.g., velocity model inaccuracy.

The source and receiver wavefields used for imaging are 4D objects, functions of spatial coordinates and time (or frequency). For simplicity, we discuss in this paper only imaging conditions in
the time-domain, although our analysis applies equally well to imaging conditions in the frequency-domain. For such 4D objects, the images obtained by extended imaging conditions are characterized in general by a 3D space-lag vector and a 1D time-lag. The images constructed with space- and time-lags can be decomposed into functions of reflection angles using geometric relations between incident and reflected rays (Sava and Fomel, 2006).

Conventional migration velocity analysis exploits separately either space-lag information, by semblance analysis (Sava and Biondi, 2004a,b; Shen et al., 2005; Schleicher, 2008; Xia et al., 2008), or time-lag information, by depth focusing analysis (Faye and Jeannot, 1986; MacKay and Abma, 1992, 1993; Nemeth, 1995, 1996; Sava and Fomel, 2006). The semblance analysis is based on the space-lag gathers, or more specifically, the horizontal space-lag gathers. It may suffer from the fact that the gathers become less sensitive for steeply dipping reflectors, and thus the quality of the common-image gather is degraded, as discussed in Biondi and Shan (2002) and Biondi and Symes (2004). However, there is no such problem for the depth focusing analysis based on the time-lag gathers. Therefore, we claim that depth focusing is superior in robustness to semblance analysis although its resolution is smaller, as indicated by the analysis of Sava and Fomel (2006).

If we consider the velocity estimation as an inverse problem, we suggest here that a more robust velocity analysis approach optimizes migrated images using all available information provided by the space and time-lags. In this way, we can have more constraints on the inversion and leverage at the same time the robustness of depth focusing analysis and the high resolution of semblance analysis.

In this paper, we analyze the moveout function for common-image gathers constructed by extended imaging conditions applied after conventional wavefield extrapolation. We first derive the analytic expression of the moveout function for extended images under the homogeneous media
assumption. Next, we focus on the common-image gathers in multi-shot experiments and quantitatively analyze their characteristics, especially the features related to velocity model error. Finally, different synthetic examples are used to verify the derived analytic moveout functions and to illustrate the application of the analysis to complex geologic models.

**WAVE-EQUATION IMAGING CONDITIONS**

Under the single scattering assumption, the seismic migration procedure consists of two main steps: wavefield reconstruction and imaging condition. Wavefield reconstruction involves constructing solutions to a wave-equation with recorded seismic data as initial and boundary conditions. Various numeric solutions for the acoustic wave-equation can be chosen depending on the specific requirements of cost and accuracy. However, regardless of the specific implementation, the reconstruction of both source and receiver wavefields is similar. In a known background velocity model, we forward propagate and backward propagate in time, to obtain the source and receiver wavefields from the source wavelet and recorded seismic data, respectively. The reconstructed source and receiver wavefields can be defined as four-dimensional functions of spatial location $\mathbf{x} = (x, y, z)$ and time $t$,

$$u_s = u_s (\mathbf{x}, t) ,$$  \hspace{1cm} (1)

$$u_r = u_r (\mathbf{x}, t) ,$$  \hspace{1cm} (2)

where $u_s$ and $u_r$ stand for the source and receiver acoustic wavefields, respectively.

An imaging condition is designed to extract the locations where reflections occur in the subsurface from these reconstructed wavefields. The image $r (\mathbf{x})$ is obtained by exploiting the space and time coincidence of the reconstructed source and receiver wavefields at every location in the subsur-
A conventional imaging condition (Claerbout, 1985) forms an image as the cross-correlation of the source and receiver wavefields evaluated at zero lag:

\[ r(x) = \sum_t u_r(x,t) u_s(x,t). \]  

(3)

An alternative extended imaging condition (Rickett and Sava, 2002; Sava and Fomel, 2006) generalizes the conventional imaging condition by preserving the information from non-zero cross-correlation lags in the output image:

\[ r(x, \lambda, \tau) = \sum_t u_s(x - \lambda, t - \tau) u_r(x + \lambda, t + \tau). \]  

(4)

The quantities \( \lambda \) and \( \tau \) represent the spatial and temporal cross-correlation lags between the source and receiver wavefields. Like the conventional imaging condition, the extended imaging condition also exploits the space and time coincidence of the wavefields, but it preserves in the output the information corresponding to non-zero space- and time-lags. Due to the existence of the lags, the output images are "hypercubes" characterized by different lags at each subsurface location \( x \). We refer to these hypercubes as wave-equation extended images. Such hypercubes can help us analyze the accuracy of reconstructed wavefields. If the local cross-correlation between the source and receiver wavefields is maximized at zero lag for all four dimensions, those wavefields are extrapolated correctly. If this is not true, we may conclude that the wavefield reconstruction is incorrect, indicating incorrect velocity or incorrect wavefield extrapolation or irregular illumination, or the failure of the single scattering assumption, e.g., due to the presence of multiples. In this paper, we consider that the errors in wavefield reconstruction are caused by the incorrect velocity model only.
The characteristics of extended images can be studied by analyzing common-image gathers (CIG).

A reflection event analyzed in CIG is represented by a multi-variable function \( z = z(\lambda, \tau) \), and the geometric shape of this function is often referred to as “moveout” by analogy with surface seismic data. Therefore, extended images are characterized by moveout surface in CIG. Understanding of the moveout surface in the case of correct and incorrect velocity is essential for the purpose of migration velocity analysis (MVA). How the extended images can be used for MVA is discussed in another publication (Yang and Sava, 2009).

Consider the reflection geometry depicted in Figure 1: the unit vector \( \mathbf{n} = \{n_x, n_y, n_z\} \) and the distance \( d \) identify the position of a reflection plane relative to the seismic source \( S \); the vector \( \mathbf{c} = \{c_x, c_y, 0\} \) identifies the fixed horizontal position of CIG relative to the source position, the vector \( \mathbf{z} = \{0, 0, z\} \) represents the depth of the image constructed by the imaging condition. We consider here the case where the extended imaging condition involves the time-lag \( \tau \) and only the horizontal space-lag \( \lambda = \{\lambda_x, \lambda_y, 0\} \), but the same logic applies to a more general case where the space-lag \( \lambda \) is three-dimensional.

Under the assumption of homogeneous media, a wavefield characterizing wave propagation from a point source can be represented by a cone in space-time. The source wavefield is represented by a cone with the origin at zero time and at the source location on the surface, as shown in Figure 2(a). Likewise, we construct the receiver wavefield as the mirror image of the source wavefield relative to the reflector indicated by the black line, as shown in Figure 2(b). Using this description of the seismic wavefields, we can represent the source and receiver wavefields in space-time by the
analytic expressions:

\[ \| \mathbf{c} + \mathbf{z} \| = vt , \]  

\[ \| \mathbf{c} + \mathbf{z} - 2d\mathbf{n} \| = vt , \]  

where \( v \) is the velocity of the medium, \( t \) is the propagation time of the wavefield, \( d\mathbf{n} \) characterizes the position of the reflector, \( \mathbf{z} = \{0, 0, z\} \) with \( z \) representing the depth of the image. As discussed before, the imaging conditions identify the position of the reflector by exploiting the time and space coincidence of source and receiver wavefields. In other words, an image forms at the spatial positions where the source and receiver wavefields intersect. Mathematically, this condition is equivalent to identifying the positions which are solutions to the system given by equations 5-6, i.e. by solving the system for the reflection depth \( z \) at coordinates \( \{c_x, c_y\} \). Figures 2(a)-2(c) illustrate the procedure. Figure 2(a) and Figure 2(b) represent the source and receiver wavefields respectively, and the cones are symmetric relative to the reflector. The intersections of the cones occur at different times. However, the locations of the intersections are consistent with the position of the reflector since their projection on the \( x - z \) plane matches the reflector perfectly.

Likewise, the extended imaging condition seeks to find the intersections between the source and receiver wavefields. However, the procedure is different because both wavefields are shifted by the space- and time-lags in the cross-correlation. The shifted wavefields are functions of the space quantity \( \mathbf{\lambda} = \{\lambda_x, \lambda_y, 0\} \) and time quantity \( \tau \). Thus, the extended imaging condition is represented by the system

\[ \| \mathbf{c} + \mathbf{z} + \mathbf{\lambda} \| = v(t + \tau) , \]  

\[ \| \mathbf{c} + \mathbf{z} - 2d\mathbf{n} - \mathbf{\lambda} \| = v(t - \tau) . \]  

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The application of the extended imaging condition in equation 4 is equivalent to solving equations 7-8. The solution represents the moveout function \( z = z(\lambda, \tau) \) at fixed CIG coordinates \( c = \{c_x, c_y, 0\} \). This moveout function describes how the depth of the image \( z \) changes with the space- and time-lags.

A formal solution to the system of equations 7-8 leads to the following expression:

\[
\begin{align*}
  z(\lambda, \tau) &= (dn_z)K + v\tau \sqrt{K^2 + \frac{||c + \lambda||^2}{(dn_z)^2 - (v\tau)^2}}, \\
  \text{where} \\
  K &= 1 - \frac{(c \cdot n) d - (n_x^2 + n_y^2) d^2 + (c - dn) \cdot \lambda}{(dn_z)^2 - (v\tau)^2}.
\end{align*}
\]

Equation 9 represents the moveout function characterizing the shape of the extended images.

To better understand the characteristics of the moveout function, we analyze two special cases of the extended images. The first case corresponds to imaging with space-lags only, which is the slice of the moveout surface at \( \tau = 0 \). Since the square-root term vanishes owing to zero time-lag, we obtain a linear moveout function. The coefficient depends on the reflection angles, which justifies the angle decomposition methods based on slant-stacks applied to space-lag CIG (Sava and Fomel, 2003; Biondi and Symes, 2004; Fomel, 2004). The second case corresponds to imaging with time-lag only, which is the slice of the moveout surface at \( \lambda = 0 \). For this special case, the moveout function is still nonlinear.

As discussed before, the goal of our research is to understand the features of extended images related to the velocity model error. A quantitative analysis of the influence of the velocity error on the extended images is required. When incorrect velocity is used for wavefield reconstruction, the wavefields are incorrectly extrapolated. Consequently, applying the imaging condition produces
distorted images. As a result, we first must understand the influence of an incorrect velocity model on the reconstructed wavefields because analytic descriptions of source and receiver wavefields are the key step for deriving the moveout function. To simplify the problem, we denote the migration velocity \( v_m = \rho v \), where \( \rho \) is a constant factor by which the migration velocity differs from the correct velocity.

The source wavefield is reconstructed as in the preceding situation, except that we use an incorrect migration velocity. The wavefield is represented by the cone with radii different from the case of correct velocity. Figure 3(a) shows the source wavefield reconstructed with incorrect velocity. The wavefield is described by the equation

\[
\| \mathbf{c} + z \| = v_m t. \tag{11}
\]

The situation of the receiver wavefield is more complicated. Unlike the source wavefield, the receiver wavefield is reconstructed by backward propagation of the recorded data. In other words, we reconstruct the cone representing the source wavefield from its origin while we reconstruct the cone representing the receiver wavefield from its depth slice on the surface. If the correct velocity is used, the cone for the receiver wavefield obtained is exactly the mirror image of the cone for the source wavefield, and both cones are symmetric in space and generated at the same time. In contrast, if an incorrect velocity is used, the cone representing the receiver wavefield has an incorrect radius, as for the source wavefield. Furthermore, the origin of the cone is shifted from its true position in space and time, and the symmetry axis between the two cones deviates from its true spatial location. In summary, the receiver wavefield reconstructed using an incorrect velocity is represented by a cone with incorrect radius, origin and symmetry axis.

As the receiver wavefield shifts in time, the reconstructed source and receiver wavefields are not
triggered at the same time. It is necessary to introduce a new variable to describe such a deviation
in time. Using the concepts of focusing depth \( d_f \) and migration depth \( d_m \) (MacKay and Abma, 1992), we have \( d_f = d/\rho_v, d_m = d \rho_v \). We thus define the deviation as focusing error \( t_d \), which is
quantified by the following formula:

\[
 t_d = \frac{d_f - d_m}{v_m} = \frac{d(1 - \rho_v^2)}{v \rho_v^2} .
\] (12)

If velocity \( v_m \) is correct, the focusing depth and migration depth are identical and equal to the true
depth of the reflection, and the focusing error \( t_d \) vanishes. Depending on the ratio between the
migrated velocity and true velocity, the term \( t_d \) can be positive or negative.

As the receiver wavefield also shifts in space, symmetry between both wavefields is maintained
but the symmetry plane changes. The plane defined by \( d n \) in the case of correct velocity now
becomes \( d_f n_m \), where \( d_f \) is the focusing depth of the reflection point, and \( n_m \) is a new normal
vector which is a function of source-receiver location, correct normal \( n \) and the migrated velocity
\( v_m \). Given this notation, the receiver wavefield is described by

\[
\| c + z - 2d_f n_m \| = v_m (t + 2t_d) .
\] (13)

Figure 3(b) shows the receiver wavefield in the case of a horizontal reflector when \( v_m \) is smaller
than \( v \).

Solving the system of equations 11-13, we obtain the coordinates of the image when the in-
correct velocity is used for imaging, as shown in 3(c). This solution is equivalent to applying the
conventional imaging condition and finding the intersections between the incorrectly reconstructed
source and receiver wavefields.
Likewise, we introduce the space and time-lags and obtain the expression for the shifted source
and receiver wavefields for the case of imaging with incorrect velocity:

\[ \| c + z + \lambda \| = v_m (t + \tau), \]  
(14)

\[ \| c + z - 2d_f n_m \| = v_m (t + 2t_d - \tau). \]  
(15)

Solving this system gives the expression for the moveout function of space-lag \( \lambda \) and time-lag \( \tau \) for
incorrect velocity

\[ z(\lambda, \tau) = (d_f n_m z) K + v_m (\tau - t_d) \sqrt{K^2 + \frac{||c + \lambda||^2}{(d_f n_m z)^2 - v_m^2 (\tau - t_d)^2}}, \]  
(16)

where quantity \( K \) is defined by

\[ K = 1 - \frac{(c \cdot n_m) d_f - (n_{mx}^2 + n_{my}^2) d_f^2 + (c - d_f n_m) \cdot \lambda}{(d_f n_m z)^2 - v_m^2 (\tau - t_d)^2}. \]  
(17)

Comparing the moveout function in equation 16 to the moveout function in equation 9, we
observe that the equations share a similar form, although the formula corresponding to the incorrect
velocity is more complicated. The complexity arises from the additional term \( t_d \), as well as from
the fact that \( d, v \) and \( n \) are replaced by \( d_f, v_m \) and \( n_m \). Owing to the existence of \( t_d \), the square-root
term is preserved when \( \tau = 0 \); the space-lag moveout function thus has a nonlinear dependence on
the variables.

**MOVEOUT ANALYSIS FOR EXTENDED IMAGES – PLANE-WAVES**

The analytic results discussed in the preceding section have complicated forms and correspond to
single-shot experiment, which is not how MVA procedures are implemented in practice. Moreover,
the moveout functions are derived based on the assumption of homogeneous media. Therefore, we
must reduce the complexity of moveout functions and generalize the analysis to inhomogeneous
media.

Figure 4 illustrates a seismic reflection occurring in an inhomogeneous medium. The wave
propagation is arbitrary due to the inhomogeneity, as indicated by the curved wave paths. The
corresponding wavefield can also have arbitrary geometric shape rather than a regular cone, thus
we cannot describe the wavefields using analytic formulas and derive analytic moveout functions.
However, if we restrict the observation to the immediate vicinity of the reflection point, which
means we consider the moveout surface in a small range of lags, we can approximate the irregular
wavefront by a plane. Although the shapes of wavefronts are arbitrary in heterogeneous media,
they can be approximated as plane-waves in the vicinity of the reflection point. Using the same
geometry shown in Figure 1, the source and receiver plane-waves are described by:

\[ \mathbf{p}_s \cdot \mathbf{x} = vt , \]  
\[ \mathbf{p}_r \cdot (\mathbf{x} - 2d\mathbf{n}) = vt , \]  

where \( \mathbf{p}_s \) and \( \mathbf{p}_r \) are the unit direction vectors of the source and receiver plane-waves, respectively,
and \( \mathbf{x} \) is the vector sum of \( \mathbf{c} \) and \( \mathbf{z} \). \( v \) is defined as the velocity in the locally homogeneous medium
around the reflection point, and thus be identical for both the wavefields.

We can also obtain the shifted source and receiver plane-waves by introducing the space- and
time-lags

\[ \mathbf{p}_s \cdot (\mathbf{x} + \lambda) = v(t + \tau) , \]  
\[ \mathbf{p}_s \cdot (\mathbf{x} - 2d\mathbf{n} - \lambda) = v(t - \tau) . \]
Solving the system of equations 20-21 leads to the expression

\[ (p_s - p_r) \cdot x = 2v\tau - (p_s + p_r) \cdot \lambda - 2d p_r \cdot n, \]  \hspace{1cm} (22)

which characterizes the moveout function of space- and time-lags at a common-image point.

Furthermore, we have the following relations for the reflection geometry:

\[ p_s - p_r = 2n \cos \theta, \]  \hspace{1cm} (23)
\[ p_s + p_r = 2q \sin \theta, \]  \hspace{1cm} (24)

where \( n \) and \( q \) are unit vectors normal and parallel to the reflection plane, and \( \theta \) is the reflection angle. Combining equations 22-24, we obtain the moveout function for plane-waves:

\[ z(\lambda, \tau) = d_0 - \tan \theta (q \cdot \lambda) + \frac{v\tau}{n_z \cos \theta}. \]  \hspace{1cm} (25)

The quantity \( d_0 \) is defined as

\[ d_0 = \frac{d - (c \cdot n)}{n_z}, \]  \hspace{1cm} (26)

and represents the depth of the reflection corresponding to the chosen CIG location. This quantity is invariant for different plane-waves, thus assumed constant here.

When incorrect velocity is used for imaging, based on the analysis in the preceding section, we can obtain the moveout function

\[ z(\lambda, \tau) = d_{0f} - \frac{\tan \theta_m (q_m \cdot \lambda)}{n_{nz}} + \frac{v_m (\tau - t_d)}{n_{nz} \cos \theta_m}, \]  \hspace{1cm} (27)
where $d_{0f}$ is the focusing depth of the corresponding reflection point, $v_m$ is the migration velocity, $t_d$ is the focusing error, $\mathbf{n}_m$ and $\mathbf{q}_m$ are vectors normal and parallel to the migrated reflector, respectively.

In the analysis above, we derive the moveout functions describing extended images for a single seismic experiment. However, typical imaging employs multi-shot seismic experiments for better illumination of subsurface and imaging redundancy which indicates the velocity accuracy. Thus, it is important to understand the characteristics of extended images in such complete seismic reflection experiments.

Since the wave equation is a linear partial differential equation, its solutions comply with the linear superposition principle. This is also true for extended images. Thus, the extended images in multi-shot experiments are a linear superposition of extended images from all single-shot experiments. The extended images from one shot (plane-wave) at each subsurface location can be considered as a surface in the extended space $z - \lambda - \tau$. The extended images constructed from many shots constitute a family of surfaces. This is a one-parameter family with the reflection angle $\theta$ as the parameter because one reflection angle $\theta$ corresponds to one shot (plane-wave). By definition, the envelope of a family of surfaces is a surface tangent to each member of the family at some points. Therefore, the extended images in multi-shot experiments are equivalent to the envelope for the family consisting of the surfaces represented by the extended images from all single-shot experiments. Based on the formula for the extended images from one plane-wave, we can derive the envelope formula by solving the following system of equations:
\[ G(\theta, (z, \lambda, \tau)) = 0, \quad (28) \]
\[ \frac{\partial G}{\partial \theta}(\theta, (z, \lambda, \tau)) = 0, \quad (29) \]

where \( G \) represents the implicit definition of the moveout function \( 25 \) for correct velocity and \( 27 \) for incorrect velocity, \( \theta \) is the reflection angle and also the parameter for the family of surfaces. Solving the system yields the following solutions:

\[ z(\lambda, \tau) = d_0 + \frac{v \tau}{n_z} \sqrt{1 - \left( \frac{n_z (q \cdot \lambda)}{v \tau} \right)^2} \quad (30) \]

for correct velocity, and

\[ z(\lambda, \tau) = d_0 + \frac{v_m (\tau - t_d)}{n_{m,z}} \sqrt{1 - \left( \frac{n_{m,z} (q_m \cdot \lambda)}{v_m (\tau - t_d)} \right)^2} \quad (31) \]

for incorrect velocity.

Analyzing the envelope functions for the cases of correct and incorrect velocities, we note that both envelope functions share a similar form; so they should have similar properties. The envelope functions become singular when \( \tau = 0 \) or \( \tau = t_d \), because at these special time-lags, all the individual surfaces corresponding to various experiments intersect at the same location. Mathematically, the envelope function is equivalent to a singular delta function at this \( \tau \). Also, the square-root term in both the formulas contains a subtraction, we must ensure that the quantity under the square-root is non-negative; otherwise, the formula fails. This failure implies that the range of space-lag \( \lambda \) is limited, which suggests that we must restrict the range of \( \lambda \) when we measure the moveout of reflections.
Given the envelope functions shown in equations 30-31, we conclude that the envelope surfaces form cones regardless of the dipping angle and velocity model used for imaging, as shown in Figures 5(a) - 5(d). However, the shapes of the cones change with both velocity and reflector dip. The cones are incomplete due to the limitations of acquisition aperture. When the velocity used for imaging is correct, the apex of the cone is located at zero lags and at the correct depth of the reflection point. In contrast, when the velocity is incorrect, the cone is shifted in both depth and in the time-lag directions. The shift in time-lag is exactly the focusing error \( t_d \) defined before and the location of the shifted apex is the focusing depth \( d_{0f} \).

If we slice the cone at negative time-lags, the slices correspond to upper half of the cone and thus curve downward. In contrast, the slices correspond to lower half of the cone curve upward. The events present in the zero time-lag slice in the case of incorrect velocity are characterized by the residual moveout used in conventional migration velocity analysis (Sava and Biondi, 2004a,b; Shen and Symes, 2008), as indicated by the thick line in the Figures 5(b) and 5(d). Based on the analysis presented here, we can evaluate the accuracy of the velocity model by examining the position of the apex of the cone. If the apex occurs at zero space- and time-lag, the velocity model is correct. If the apex shifts to non-zero time-lags, then the migration velocity is incorrect. Thus, the position of the apex of the cone can be used as an indicator of velocity error.

To summarize, in inhomogeneous media, no analytic moveout function exists to describe moveout surfaces for extended images. However, by restricting our analysis to the vicinity of the reflection point, and by assuming that the velocity change above the image points is relatively uniform, we can use a plane-wave approximation to derive the analytic functions characterizing extended images. The parameters describing the moveout functions are effective parameters which represent the velocity errors accumulated along wave propagation paths just as the travelt ime errors used in conventional traveltime tomography. These parameters can be transformed into local medium
parameters through a tomographic procedure which we do not discuss here.

EXAMPLES

We illustrate the validity of the moveout functions derived in the preceding sections with several synthetic models. The first model consists of a horizontal reflector embedded in a constant velocity medium, while the second model consists of a dipping reflector embedded in a constant velocity medium. We use the first model to verify the accuracy of the moveout function for point sources and plane-wave sources, and use the second model to verify the accuracy of the envelope functions for plane-wave sources. Extended images are generated for both correct and incorrect velocities. The incorrect velocity is obtained by scaling the correct velocity with a constant factor.

The migrated images corresponding to the horizontal reflector are shown in Figures 6(a) and 6(b) for correct and incorrect velocities, respectively. The extended images are constructed at CIG location $c_x = 0.5$ km, as indicated by the vertical line. Thus, for a source located at $x = 3$ km, the CIG analyzed are located at $x = 3.5$ km. To verify the accuracy of the moveout function, we overlay the analytic moveout functions on extended images at either fixed time-lags or at fixed horizontal space-lags.

Figures 7(a)-7(c) and 7(d)-7(f) depict space-lag extended images corresponding to the chosen CIG location for correct and incorrect velocities, respectively. From left to right, the panels correspond to slices at $\tau = \{-0.20, 0, +0.20\}$ s. In each column, the upper panels correspond to correct velocity while the lower panels correspond to incorrect velocity. The dashed lines overlain correspond to the analytic functions 9 and 16 while the solid lines correspond to the analytic functions 25 and 27. Notice that, at $\tau = 0$ the moveout event is linear for correct velocity and nonlinear for incorrect velocity, as expected. Comparing the analytic functions derived for point and plane
sources, we can observe that the point source formulae accurately describe the moveout curves in this example. In contrast, since the plane-wave formulae are approximations to the point source formulae, they are only accurate at small lags and become less accurate for large lags. Finally, we mention that the mismatch at large lag values between the formulae and moveout curves shown in Figure 7(c) and 7(f) is caused by the diffractions due to truncation of the acquisition array. Such effects are not properly characterized by our formulae which apply strictly to reflection but not to diffraction. Figures 8(a)-8(c) and 8(d)-8(f) depict time-lag extended images for correct and incorrect velocities, respectively. From left to right, the panels correspond to \( \lambda_x = \{-0.3, 0, +0.3\} \) km. In each column, the upper panels correspond to correct velocity while the lower panels correspond to incorrect velocity. The dashed line overlain on each panel corresponds to the analytic functions 9 and 16 while the solid lines correspond to the analytic functions 25 and 27. In both cases, the analytic formulae of point source accurately describes the moveout surface characterizing extended images. Likewise, the formulae of plane-waves are also good approximations to the point source ones at small range of lags. This illustrates the accuracy of the analysis of the moveout functions for the extended images.

Figures 9(a) and 9(b) show the migrated images of the dipping reflector corresponding to correct and incorrect velocities, respectively. To obtain the images, we use 50 plane-wave sources equally spaced in horizontal slowness and stack the images from all individual experiments. As discussed before, the stacked moveout surfaces correspond to the envelope of the surfaces obtained from individual shots. We choose \( x = 3.0 \) km as the CIG location. Figures 14(a)-10(f) depict the moveout surfaces at different time-lags. From left to right, the upper panels display the slices at \( \tau = \{-0.15, 0.0, +0.15\} \) s. The lower panels correspond to the same slices but overlain with the derived analytic envelope function \( z(\lambda_x, \tau) \) given by equation 30 for various \( \tau \). Since the correct velocity is used for imaging, the apex of the cone should be located at zero space- and time-
lags. As expected, all events intersect at the same location at zero space-lag and a well focused image can be observed in the panel at $\tau = 0$. For incorrect velocity, the cones shift so that their apex is not located at zero time-lag. The slice at zero time-lag therefore shows a curved event.

Figures 11(a)-11(f) depict envelopes of moveout surfaces for different time-lags. From left to right, the upper panels display the slices of the cone at $\tau = \{-0.15, 0.0, +0.15\}$ s. The lower panels correspond to the same slices but overlain with the derived analytic envelope function $z(\lambda_x, \tau)$ given by equation 31 for various $\tau$. The slice at zero time-lag shows a curved event rather than a focused point, which demonstrates the shift of the apex of the cone. The slice at $\tau = 0.15$ s shows an event curved in the opposite direction, which means that the slice is passing to the apex of the cone. For both correct and incorrect velocities, the analytic functions match the experiments well, which demonstrates the accuracy of the envelope functions.

Finally, we use the Sigsbee model (Paffenholz et al., 2002) to illustrate the application of our analysis to inhomogeneous media. Figures 12 show the velocity profile of the model. The sources are distributed over the left area of the model, thus they mainly illuminate the left side of the image. Figures 13(a) and 13(b) show the image migrated with correct and incorrect velocities respectively.

Figures 14(a) and 14(b) depict the moveout surfaces at different time-lags for the case of imaging with correct velocity. The panels correspond to slices of the cone at different time-lags. Figure 14(a) shows the slice at $\tau = -0.15$ s. The events in the panel curve downward since the slice is cut at negative $\tau$ and corresponding to the upper half of the cone. Figure 14(b) shows the slice at $\tau = 0$ s, which is cut at the origin of the cone. The events in the panel appear focused at zero time-lag and space-lag, indicating correct velocity.

Figures 15(a) and 15(b) depict moveout surfaces for different time-lags for the case of imaging with incorrect velocity. The panels correspond to slices of the cone at different time-lags. Because
a higher migration velocity is used, the origin of the cone is expected to shift to negative $\tau$. Figure 15(a) shows the slice at $\tau = -0.15$ s. The shallow events in the panel focus at zero space-lag, which means that the slice is cut at the origin of the cone for those events. Deeper events have the focus of the cone at other values of $\tau$. Figure 15(b) show the slices at $\tau = 0$ s. The events in the panels curve upward because the slice is cut away from the focus of the cone.

DISCUSSION

Extended imaging conditions have been used in the past as sources of information for migration velocity analysis. For example, Biondi and Sava (1999); Shen et al. (2003); Sava and Biondi (2004b); Shen et al. (2005); Shen and Symes (2008) use space-lags extensions for MVA, while Higginbotham and Brown (2008); Brown et al. (2008); Yang and Sava (2009) use time-lag extensions for MVA. Among the interesting questions one can ask based on the analysis presented in this paper are what is the connection between the two sets of extensions and is the information provided by space- and time-lags redundant or complementary? As indicated in the preceding sections, the space- and time-lag extensions are not independent on one-another. By simply observing reflectors in space-lag gathers (at $\tau = 0$) or in time-lag gathers (at $\lambda = 0$), we are exploring subsets of the same object, as seen in Figures 5(a)-5(d). Therefore, the depth-$\lambda$-$\tau$ gathers capture more completely the behavior of events in the extended space and provide access to more robust information to be used for velocity update. It is easier to evaluate the behavior of the “cones” characterizing a reflection event by observing them in their entirety, rather than by observing subsets.

On the other hand, using space-time extended gathers we can better formulate the optimization process that could be used for velocity model update. For example, conventional wavefield-based MVA based on differential semblance optimization (Shen et al., 2005; Symes, 2009) indicates that
velocity can be optimized by minimizing the objective function of space-lags at zero time-lag:

\[
\min \frac{1}{2} \left\| r (z, \lambda_x, \lambda_y) \sqrt{\lambda_x^2 + \lambda_y^2} \right\|^2,
\]

(32)

where \( \lambda_x \) and \( \lambda_y \) are the horizontal components of the space-lag vector \( \lambda \) and \( r (z, \lambda_x, \lambda_y) \) represents an extended image gather. This objective function corresponds to the case when we penalize reflector energy outside zero space-lag, but not the reflector energy at zero space-lag. From the analysis presented in this paper, it is apparent that this type of objective function is partial and that what we really need to do is to penalize the entire defocused events at all lags (including zero space-lag) by the same amount dependent on how far the apex of the respective event departs from zero time-lag. An objective function formulated this way includes both the information at zero time-lag (the semblance information), as well as the information at zero space-lag (the focusing information), thus being more robust and effective for migration velocity analysis. This topic is discussed in Yang and Sava (2009) and we do not elaborate further on it in this paper.

Finally, an important consideration for practical application of this methodology is that of computational cost. Computing extended images function of both space- and time-lags is costlier than computing extended images function of space-lag or time-lag separately. On the other hand, there is more information in extended images mixing space- and time-lags. Therefore, what we need to do is balance the cost and benefits of the extended images by evaluating them at relatively sparse locations in the image along the in-line and cross-line directions, and by restricting the range of space- and time-lags to the extent necessary to capture the character of the reflection events.
CONCLUSIONS

An extended imaging condition offers the possibility to design robust migration velocity analysis methods that simultaneously exploit conventional semblance analysis and depth focusing analysis. The analytic moveout functions provide quantitative descriptions of the shapes of events in extended images. The envelope of the moveout function characterizing extended images constructed from multiple experiments form cones in the lags-depth domain. The apex of the cone represents a well focused image of the reflector. If velocity is correct, the apex appears at zero time-lag and correct depth, otherwise the apex shifts to nonzero time-lags and to an incorrect depth. Such a characteristic can be used as an indicator of velocity error for tomographic techniques. Synthetic examples verify the validity of the analytic moveout functions and demonstrate that the analysis for properties of the extended images hold even for complex media.

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