SUMMARY

Images of Earth structure are created through seismic wavefield extrapolation, which extends surface-recorded data to depth through application of a wave-equation operator. The one-way extrapolation operators are derived from the acoustic wave-equation dispersion relation, and are usually defined in a Cartesian coordinate system with a vertical extrapolation axis. Riemannian wavefield extrapolation (RWE) is a generalization of the downward continuation concept to coordinate systems that closely conform to the orientation of extrapolated wavefields, or to some pre-existing acquisition surface topology. In the former situation, path propagation effects are consequently modeled directly in the coordinate system enabling accurate one-way wavefield extrapolation, even in situations where wavefields overturn. In the latter, wave-equation migration from surfaces of arbitrary topology is rendered possible.

INTRODUCTION

Wave-equation imaging is computed almost exclusively on Cartesian meshes both for computational simplicity and because imaged subsurface volumes are usually rectangular parallelepipeds. However, situations exist where performing wave-equation imaging on more generalized coordinate system meshes is warranted. For example, one may with to extrapolate wavefields directly from an undulating topographic surface, to orient the extrapolation axis of lower-order operators in the direction of wavefield propagation to improve higher angle accuracy, or to image with overturning waves not currently modeled by one-way extrapolation operators. Employing non-Cartesian meshes, though, necessitates resolving these three issues: which coordinate system should be chosen? How is the preferred coordinate system generated? And what are the appropriate governing wavefield extrapolation equations for this coordinate system choice? This paper presents a method that addresses the third issue, given that the first two have been resolved, say, through ray-tracing.

Non-Cartesian wavefield extrapolation theory has advanced in recent years in the context of both global and exploration seismology. All of these methods locally transform the coordinate system and the corresponding governing propagation equations to a more appropriate reference frame linked to an underlying Cartesian grid through one-to-one mappings. For example, earthquake seismologists have formulated an orthogonal 2-D plane-wave-centric coordinate system appropriate for propagating overturning teleseismic wavefields. Some authors discuss the use of tilted coordinate systems that enable propagation of overturning waves with one-way extrapolation operators. Here, we present a method based on a Riemannian metric formulation of the wavefield propagation equations. This method specifies how to transform these equations to be applicable on any non-triplicating, semi-orthogonal computational grid (Sava and Fomel, 2004).

THEORY

The acoustic propagation of a monochromatic wavefield is governed by the Helmholtz equation, $\Delta U = -\omega^2 s^2 U$, where ω is temporal frequency, s is reciprocal of the wave propagation velocity, and *U* represents a propagating wavefield. Laplacian operator, Δ, can be defined in an arbitrary Riemannian space associated with coordinate system variables, $\vec{\xi} = \vec{\xi}\{\xi_1, \xi_2, \xi_3\}$, and used to explicitly write the Helmholtz equation,

$$\Delta U = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial \xi_i}\left(g_{ij} \sqrt{|g|} \frac{\partial U}{\partial \xi_j} \right) = -\omega^2 s^2 U \,, \tag{1}$$

where g_{ij} is a component of the conjugate metric tensor, $|g|$ is its determinant (Synge and Schild, 1978), and Einstein summation notation is assumed. Importantly, the differential geometry of any coordinate system is implicitly represented in metric tensor, g_{ij}.

One-way wave extrapolation in Riemannian spaces is greatly simplified where one coordinate is orthogonal to the other two. The metric tensor g_{ij} relates the geometry of vectors in Cartesian space $\vec{x} = \vec{x}\{\xi_1, \xi_2, \xi_3\}$ with the geometry of vectors in general coordinate system $\vec{\xi} = \vec{\xi}\{\xi_1, \xi_2, \xi_3\}$

$$[g_{ij}] = \begin{bmatrix} \frac{\partial x^k}{\partial \xi_1}\frac{\partial x^k}{\partial \xi_1} & \frac{\partial x^k}{\partial \xi_1}\frac{\partial x^k}{\partial \xi_2} & 0 \\ \frac{\partial x^k}{\partial \xi_2}\frac{\partial x^k}{\partial \xi_1} & \frac{\partial x^k}{\partial \xi_2}\frac{\partial x^k}{\partial \xi_2} & 0 \\ 0 & 0 & \frac{\partial x^k}{\partial \xi_3}\frac{\partial x^k}{\partial \xi_3} \end{bmatrix} = \begin{bmatrix} E & F & 0 \\ F & G & 0 \\ 0 & 0 & \alpha^2 \end{bmatrix}, \tag{2}$$

where Einstein summation notation over index k is assumed. In our treatment, coordinate $\xi_3 = \zeta$ is defined as the extrapolation direction, and is orthogonal to the remaining two coordinates $\xi_3 = \xi$ and $\xi_3 = \eta$. This metric allows us to write a general version of the Helmholtz equation,

$$\frac{1}{\alpha J}\left[\frac{\partial}{\partial\zeta}\left(\frac{J}{\alpha}\frac{\partial U}{\partial\zeta}\right)+\frac{\partial}{\partial\xi}\left(G\frac{\alpha}{J}\frac{\partial U}{\partial\xi}-F\frac{\alpha}{J}\frac{\partial U}{\partial\eta}\right)+\frac{\partial}{\partial\eta}\left(E\frac{\alpha}{J}\frac{\partial U}{\partial\eta}-F\frac{\alpha}{J}\frac{\partial U}{\partial\xi}\right)\right]=-\omega^2 s^2 U. \quad (3)$$

Individual differential operators in equation (3) may be replaced with corresponding Fourier domain wavenumber representations, which leads to the general wave-equation dispersion relation in a semi-orthogonal 3-D Riemannian space,

$$-c_{\zeta\zeta}k_\zeta^2 - c_{\xi\xi}k_\xi^2 - c_{\eta\eta}k_\eta^2 + ic_\zeta k_\zeta + ic_\xi k_\xi + ic_\eta k_\eta - c_{\xi\eta}k_\xi k_\eta = \omega^2 s^2 \quad (4)$$

where coefficients, c_{ij}, are functions of the coordinate system and are given by,

$$c_{\zeta\zeta}=\frac{1}{\alpha^2}\ ,\ c_{\xi\xi}=\frac{G}{J^2}\ ,\ c_{\eta\eta}=\frac{E}{J^2}\ ,\ c_{\varepsilon\eta}=\frac{F}{J^2}\ ,\ c_\zeta=\frac{1}{\alpha J}\frac{\partial}{\partial\zeta}\left(\frac{J}{\alpha}\right)\ ,$$

$$c_\xi=\frac{1}{\alpha J}\left[\frac{\partial}{\partial\xi}\left(G\frac{\alpha}{J}\right)-\frac{\partial}{\partial\eta}\left(F\frac{\alpha}{J}\right)\right]\ \text{, and } c_\eta=\frac{1}{\alpha J}\left[\frac{\partial}{\partial\eta}\left(E\frac{\alpha}{J}\right)-\frac{\partial}{\partial\xi}\left(F\frac{\alpha}{J}\right)\right].$$

Wavenumber k_ζ is used in the conventional wavefield extrapolation scheme that extends the recorded wavefield away from the acquisition surface. This involves solving a one-way wave-equation which, in discrete extrapolation steps $\Delta\zeta$, can be written,

$$U(\zeta+\Delta\zeta,\xi,\eta)=U(\zeta,\xi,\eta)exp(ik_\zeta\Delta\zeta) \quad (5)$$

This requires isolating the wavenumber in the extrapolation direction, k_ζ, and selecting the solution with the positive radical,

$$k_\zeta = i\frac{c_\zeta}{c\zeta\zeta}+\sqrt{\frac{\omega^2 s^2}{c_{\zeta\zeta}}-\left(\frac{c_\zeta}{2c_{\zeta\zeta}}\right)^2-\sum_{j=\xi,\eta}\left[\frac{c_{jj}}{c_{\zeta\zeta}}k_j^2-i\frac{c_j}{c_{\zeta\zeta}}k_j\right]-\frac{c_{\xi\eta}}{c_{\zeta\zeta}}k_\xi k_\eta}. \quad (6)$$

Figure 1 shows an example of Riemannian wavefield extrapolation. Description of the content of each panel is given in the figure caption. The bottom two panels illustrate the improved angular bandwidth afforded by the RWE method relative to that expected from comparable Cartesian-based operators.

CONCLUDING REMARKS

We extend one-way wavefield extrapolation to Riemannian spaces which are, by definition, described by non-orthogonal curvilinear coordinate systems. Choosing semi-orthogonal Riemannian coordinates, we define a generalized acoustic wave-equation, from which we derive a one-way wavefield extrapolation equation.

We use ray coordinates initiated either from a point source, or from an incident plane wave at the surface. Many other types of coordinates are acceptable, as long as they fulfill the semi-orthogonal condition of our acoustic wave equation.

Since wavefield propagation happens mostly along the extrapolation direction, we can use cheap 15^0 finite-difference or mixed-domain extrapolators to achieve high angle accuracy. If the ray coordinate system overturns, our method can be used to image overturning waves with one-way wavefield extrapolation. For situations where migration from topography is desired, one must generate a coordinate system conformal with this boundary, and then apply the procedure as above (Shragge and Sava, 2005).

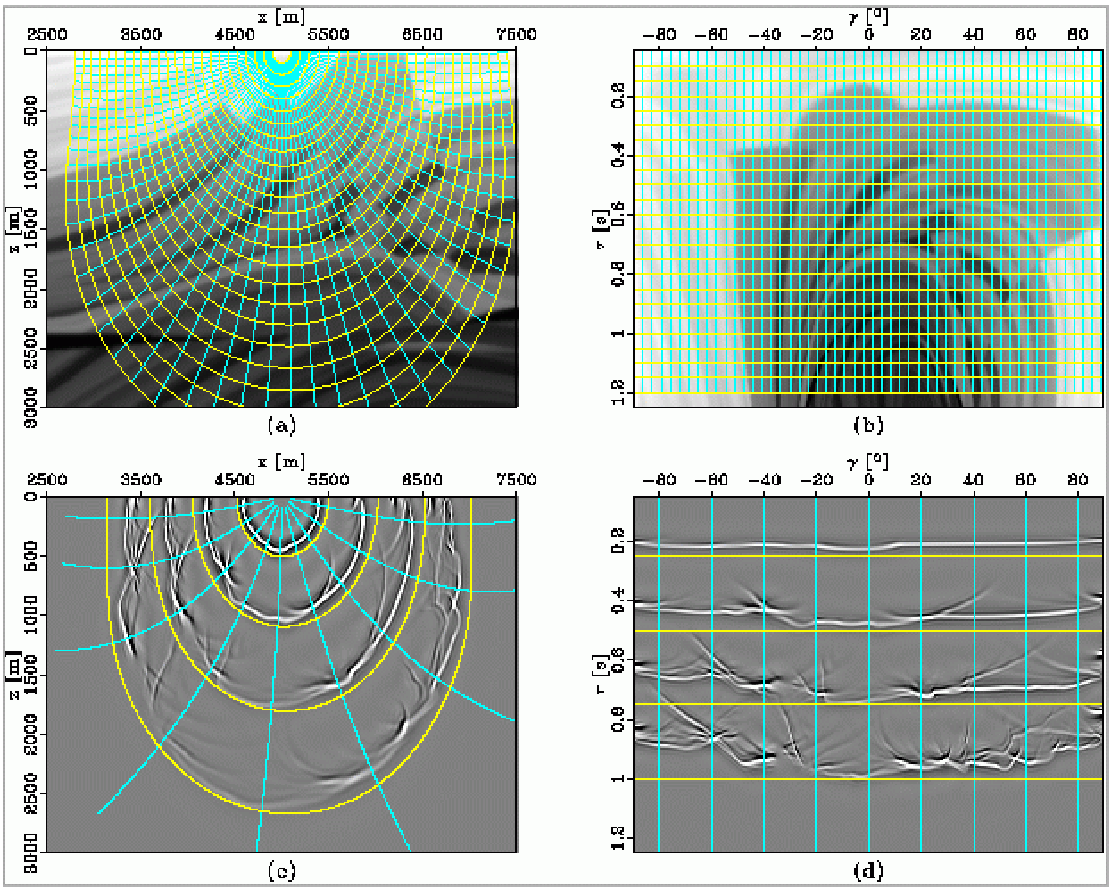

Figure 1 A demonstration of Riemannian wavefield extrapolation through the Marmousi model. (a) Cartesian coordinate view of Marmousi velocity model with an overlaid ray coordinate system; (b) Ray coordinate view of velocity model and ray coordinate system created through mappings between Cartesian and ray coordinate space; (c) Cartesian coordinate view of the wavefield extrapolated in ray coordinates in panel d); and (d) Ray coordinate view of wavefield extrapolated in ray coordinates originating with a plane wave over all angles.

REFERENCES

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