SUMMARY

Wave-equation migration velocity analysis (WEMVA) produces wrong results if it starts from an image perturbation which is not compliant with the assumed Born approximation. Many attempts to correct this problem lead to either unreliable or hard to implement solutions. We present a new method designed to construct image perturbations that are always compliant with the Born approximation. This new method is robust, easy to implement, and produces results that are consistent with those obtained using the ideal operators.

INTRODUCTION

Migration methods using wavefield continuation techniques, commonly referred to as wave-equation methods, have enjoyed a renewed interest in the recent years, since the limitations of Kirchhoff migration became apparent (Geoltrain and Brac, 1993; Audebert et al., 1997; O'Brien and Etgen, 1998).

Migration velocity analysis based on wavefield continuation methods, also known as *wave-equation migration velocity analysis*, is a promising technique designed as a companion to wave-equation migration (Biondi and Sava, 1999; Sava and Fomel, 2002). The main idea of WEMVA is to use downward continuation operators for migration velocity analysis (MVA), as well as for migration. This is in contrast with other techniques which use downward continuation for migration but traveltime-based techniques for migration velocity analysis (Clapp, 2001; Liu et al., 2001; Mosher et al., 2001).

In our early tests (Biondi and Sava, 1999), we construct the image perturbation using Prestack Stolt Residual Migration (PSRM) (Sava, 2000). In summary, this residual migration method provides updated images for new velocity maps that correspond to a fixed ratio of the new velocity with respect to the original velocity map.

The main disadvantage of building the image perturbation using PSRM is that, if the velocity ratio parameter (γ) is too large, there is a good chance for the reference and the updated images to get out of phase. In other words, a large change in velocity violates the Born approximation. The end result is that the image perturbation computed by the forward operator and the one computed by residual migration are fundamentally different, and can have contradictory behaviors when using the Born-linearized WEMVA operator for inversion.

In this paper, we present a new method that can be used to create image perturbations for WEMVA. The two main goals here are

- to create an image perturbation that is compatible with the one computed using the forward

WEMVA operator, and

- to create the image perturbation directly from the background image, and therefore compliant with the Born approximation.

Residual migration can be used to create image perturbations ($\Delta\mathcal{R}$). In its simplest form, we can build $\Delta\mathcal{R}$ as a difference between an *improved* image ($\mathcal{R}$) and the *reference* image ($\mathcal{R}_o$)

$$\Delta\mathcal{R} = \mathcal{R} - \mathcal{R}_o, \tag{1}$$

where $\mathcal{R}$ is derived from $\mathcal{R}_o$ as a function of the parameter γ, which is the ratio of the original and improved velocities (Sava, 2000). Of course, the improved velocity map is unknown explicitly, but it is described indirectly by the ratio map of the two velocities.

If we define $\Delta\gamma = \gamma - 1$, we can also write the discrete version of the image perturbation as

$$\Delta\mathcal{R} \approx \frac{\mathcal{R} - \mathcal{R}_o}{\gamma - 1}\Delta\gamma, \tag{2}$$

equation which can be written in differential form as

$$\Delta\mathcal{R} \approx \left.\frac{d\mathcal{R}}{d\gamma}\right|_{\gamma=1} \Delta\gamma, \tag{3}$$

or, equivalently, using the chain rule, as

$$\Delta\mathcal{R} \approx \left.\frac{d\mathcal{R}}{dk_z}\frac{dk_z}{d\gamma}\right|_{\gamma=1} \Delta\gamma, \tag{4}$$

where k_z is the depth wavenumber defined for PSRM.

Equation (4) offers the possibility to build the image perturbation directly. We achieve this by computing three elements: the derivative of the image with respect to the depth wavenumber, and two weighting functions, one for the derivative of the depth wavenumber with respect to the velocity ratio parameter (γ), and the other one for the magnitude of the $\Delta\gamma$ perturbation from the reference to the improved image.

Firstly, the image derivative in the Fourier domain, $\frac{d\mathcal{R}}{dk_z}$, is straightforward to compute in the space domain as

$$\left.\frac{d\mathcal{R}}{dk_z}\right|_{\gamma=1} = -iz\mathcal{R}_o. \tag{5}$$

The derivative image is nothing but the imaginary part of the migrated image, scaled by depth.

Secondly, we can obtain the weighting representing the derivative of the depth wavenumber with respect to the velocity ratio parameter, $\left.\frac{dk_z}{d\gamma}\right|_{\gamma=1}$, starting from the double square root (DSR) equation written for prestack Stolt residual migration (Sava, 2000):

$$\begin{aligned} k_z &= {k_z}^s + {k_z}^r \\ &= \frac{1}{2}\sqrt{\gamma^2\mu^2 - |\mathbf{k}_s|^2} + \frac{1}{2}\sqrt{\gamma^2\mu^2 - |\mathbf{k}_r|^2}, \end{aligned}$$

where μ is given by the expression:

$$\mu^2 = \frac{\left[4\,({k_z}^o)^2 + (|\mathbf{k}_r| - |\mathbf{k}_s|)^2\right]\left[4\,({k_z}^o)^2 + (|\mathbf{k}_r| + |\mathbf{k}_s|)^2\right]}{16\,({k_z}^o)^2}. \tag{6}$$

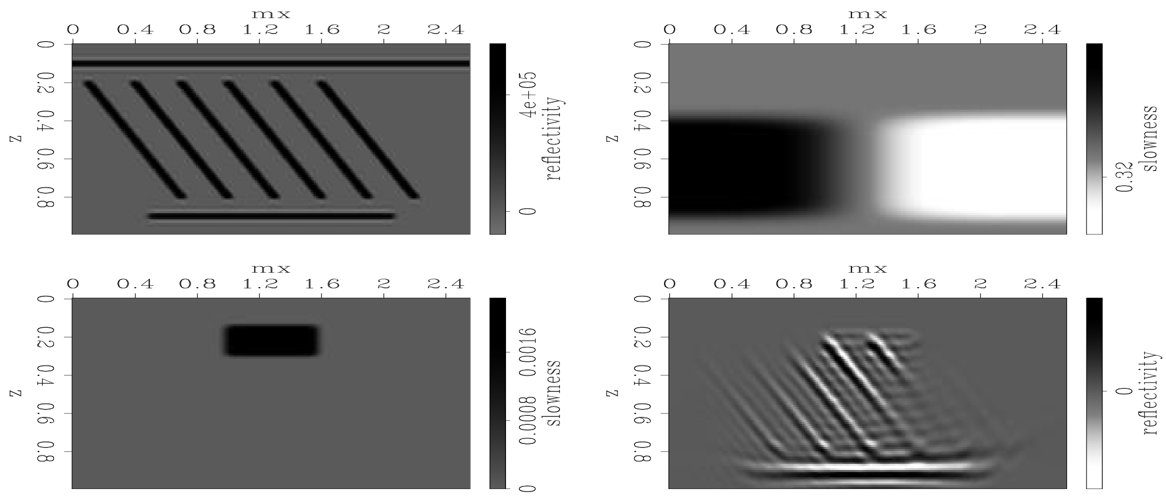

Figure 1: Model and theoretical image perturbation.

The derivative of k_z with respect to γ is

$$\frac{dk_z}{d\gamma} = \gamma \left(\frac{\mu^2}{4{k_z}^s} + \frac{\mu^2}{4{k_z}^r} \right), \tag{7}$$

therefore

$$\left.\frac{dk_z}{d\gamma}\right|_{\gamma=1} = \frac{\mu^2}{2\sqrt{\mu^2 - |\mathbf{k}_s|^2}} + \frac{\mu^2}{2\sqrt{\mu^2 - |\mathbf{k}_r|^2}}. \tag{8}$$

Finally, $\Delta\gamma$ can be picked from the set of residually migrated images at various values of the parameter γ (Sava, 2000). The main criterion that should be used is the flatness of the angle-domain image gathers, although in principle other derived parameters, such as stack power or semblance, can be used as well.

EXAMPLES

We demonstrate the method on a synthetic example. The reflectivity model (Figure 1, top left) consists of two flat reflectors surrounding a set of reflectors dipping at 45 degrees. The background velocity (Figure 1, top right) is characterized by strong lateral variation to provide a somewhat complex background wavefield. The image perturbation (Figure 1, bottom left) consists of a rectangular block in the upper part of the section, which creates perturbations in the image both on the flat and on the dipping reflectors.

We use the true slowness to model the data, and then we use the background slowness to obtain the reference image and the reference wavefield. From the slowness perturbation, we use the forward WEMVA operator to create the *ideal* image perturbation (Figure 1, bottom right). This image perturbation represents the benchmark against which we want to compare our new method.

We use Equation (4) and the $\Delta\gamma$ weight (Figure 2, left) to create the analytical image perturbation (Figure 2, right) which is comparable in shape and magnitude with the ideal perturbation (Figure 1, bottom right).

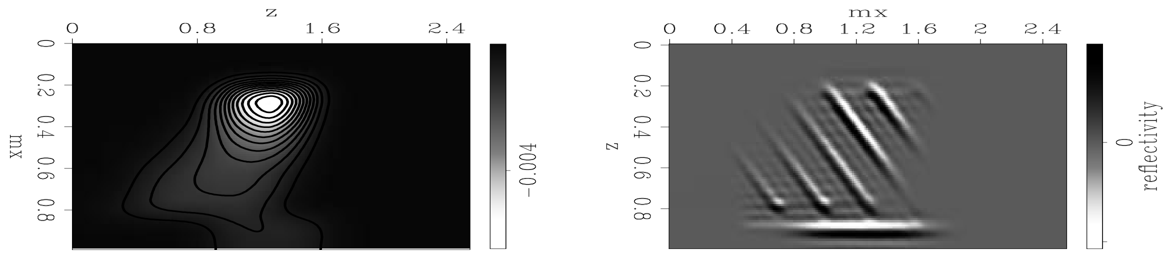

Figure 2: Analytical image perturbation.

CONCLUSIONS

We have presented a new approach to the construction of image perturbations for velocity analysis using WEMVA. This method directly constructs the image perturbation from the background image, and is always compliant with the Born approximation which is the underlying assumption of WEMVA. We show that, given correct scaling, we can obtain slowness anomalies that are fully consistent with those obtained by the application of the forward and adjoint WEMVA operators.

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