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MIGRATION VELOCITY ANALYSIS BY RECURSIVE WAVEFIELD EXTRAPOLATION

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1 Abstract

We present a wave-equation method of migration velocity analysis, based on the linear relation that can be established between a perturbation in the migrated image and the corresponding perturbation in the slowness function. Our method formulates an objective function in the image space, in contrast with other wave-equation tomography techniques which formulate objective functions in the data space. We iteratively update the slowness function to account for improvements in the focusing quality of the migrated image. We overcome the major limitation of the Born approximation by analytically creating image perturbations consistent with this approximation. The image perturbation operator is computed as a derivative of prestack Stolt residual migration, thus our method directly exploits the power of prestack residual migration.

2 Introduction

Migration velocity analysis based on downward continuation methods, also known as *wave-equation migration velocity analysis* (WEMVA), is a technique designed as a companion to wave-equation migration (Biondi and Sava, 1999). The main idea of WEMVA is to use downward continuation operators for migration velocity analysis (MVA), as well as for migration. This is in contrast with other techniques which use downward continuation for migration, but traveltime-based techniques for migration velocity analysis (Clapp, 2001; Liu et al., 2001; Mosher et al., 2001).

WEMVA benefits from the same advantages over traveltime estimation methods as wave-equation migration benefits over Kirchhoff migration. The most important among them are the accurate handling of complex wavefields which are characterized by multipathing, and the band-limited nature of the imaging process, which can handle sharp velocity variations much better than traveltime-based methods (Woodward, 1992). Complex geology, therefore, is where WEMVA is expected to provide the largest benefits.

WEMVA is based on a linearization of the downward-continuation operator using the Born approximation. This approximation leads to severe limitations on the magnitude and size of the anomalies that can be handled. It, therefore, cannot operate successfully exactly in the regions of highest complexity. Other methods of linearization are possible (Sava and Fomel, 2002), but neither one allows for arbitrarily large anomalies.

In our early tests (Biondi and Sava, 1999), we construct the image perturbation using Prestack Stolt Residual Migration (PSRM) (Sava, 2000). In summary, this residual migration method provides updated images for new velocity maps that correspond to a fixed ratio (ρ) of the new velocity with respect to the original velocity map. Residual migration is run for various ratio parameters, and finally pick the best image by selecting the flattest gathers at every point.

The main disadvantage of building the image perturbation using PSRM is that for large velocity ratio parameters (ρ) the background and improved images can get out of phase. The consequence is that the image perturbation computed by the forward operator and the one computed by residual migration are fundamentally different, and can have contradictory behaviors when using the Born-linearized WEMVA operator for inversion.

We show that alternative methods can be used to create image perturbations for WEMVA (Sava and Biondi, 2003), in compliance with the Born approximation and computed directly from the background image.

3 Wavefield scattering

In migration by downward continuation, the wavefield at depth $z+\Delta z$, $\mathcal{W}(z+\Delta z)$, is obtained by phase-shift from the wavefield at depth z, $\mathcal{W}(z)$:

$$\mathcal{W}(z+\Delta z) = \mathcal{W}(z)\, e^{-ik_z \Delta z}. \tag{1}$$

Using a Taylor series expansion, the depth wavenumber (k_z) depends linearly on its value in the reference medium (k_{z_o}) and the laterally varying slowness $s(x,y,z)$ in the depth interval under consideration

$$k_z \approx k_{z_o} + \left.\frac{dk_z}{ds}\right|_{s=s_o} (s - s_o). \tag{2}$$

s_o represents the constant slowness associated with the depth slab between the two depth intervals. $\left.\frac{dk_z}{ds}\right|_{s=s_o}$ represents the derivative of the depth wavenumber with respect to the reference slowness and can be implemented in many different ways, e.g by the Fourier-domain method of Huang et al. (1999).

The wavefield downward continued through the *background* slowness $s_b(x,y,z)$ is

$$\mathcal{W}_b(z+\Delta z) = \mathcal{W}(z)\, e^{-i\left[k_{z_o} + \left.\frac{dk_z}{ds}\right|_{s=s_o}(s_b - s_o)\right]\Delta z}, \tag{3}$$

and the full wavefield $\mathcal{W}(z+\Delta z)$ depends on the background wavefield $\mathcal{W}_b(z+\Delta z)$ by

$$\mathcal{W}(z+\Delta z) = \mathcal{W}_b(z+\Delta z)\, e^{-i \left.\frac{dk_z}{ds}\right|_{s=s_o} \Delta s \Delta z}, \tag{4}$$

where Δs represents the difference between the true and background slownesses $\Delta s = s - s_b$.

Defining the *wavefield perturbation* $\Delta\mathcal{W}(z+\Delta z)$ as the difference between the wavefield propagated through the medium with correct velocity, $\mathcal{W}(z+\Delta z)$, and the wavefield propagated through the background medium, $\mathcal{W}_b(z+\Delta z)$, we can write

$$\begin{aligned} \Delta\mathcal{W}(z+\Delta z) &= \mathcal{W}(z+\Delta z) - \mathcal{W}_b(z+\Delta z) && (5)\\ &= \mathcal{W}_b(z+\Delta z)\left[e^{-i \left.\frac{dk_z}{ds}\right|_{s=s_o} \Delta s \Delta z} - 1\right]. && (6) \end{aligned}$$

Equation (5) represents the foundation of the wave-equation migration velocity analysis method. The major problem with Equation (5) is that the wavefield $\Delta\mathcal{W}$ and slowness perturbations Δs are not linearly related. For inversion purposes, we need to find a linearization of this equation around the reference slowness, s_o. Biondi and Sava (1999) linearize Equation (5) using the Born approximation ($e^{i\phi} \approx 1 + i\phi$). With this choice, the WEMVA Equation (5) becomes

$$\Delta\mathcal{W}(z+\Delta z) = \mathcal{W}_b(z+\Delta z)\left[-i \left.\frac{dk_z}{ds}\right|_{s=s_o} \Delta s \Delta z\right]. \tag{7}$$

The wavefield perturbation $\Delta\mathcal{W}$ in Equation (7) turns into an image perturbation $\Delta\mathcal{R}$ after we apply an imaging condition. The WEMVA objective function is

$$\min_{\Delta s} \|\Delta\mathcal{R} - \mathbf{L}\Delta\mathbf{s}\| \tag{8}$$

where $\mathbf{L}$ is a linear operator defined recursively from Equation (7) at every depth level and frequency.

4 Analytical image perturbation

Residual migration can be used to create image perturbations ($\Delta\mathcal{R}$). We can build $\Delta\mathcal{R}$ as a difference between an *improved* image ($\mathcal{R}$) and the *reference* image ($\mathcal{R}_b$), $\Delta\mathcal{R} = \mathcal{R} - \mathcal{R}_b$, where $\mathcal{R}$ is derived from $\mathcal{R}_b$ as a function of the parameter ρ, the ratio of the original and improved velocities (Sava, 2000). The improved velocity is unknown explicitly, but it is described indirectly by the ratio map of the two velocities.

If we define $\Delta\rho = \rho - 1$, we can also write the discrete version of the image perturbation as

$$\Delta\mathcal{R} = \frac{\mathcal{R} - \mathcal{R}_b}{\rho - 1}\Delta\rho, \tag{9}$$

equation which, in the limit, can be written in differential form as

$$\Delta\mathcal{R} = \left.\frac{d\mathcal{R}}{d\rho}\right|_{\rho=1}\Delta\rho = \left.\frac{d\mathcal{R}}{dk_z}\frac{dk_z}{d\rho}\right|_{\rho=1}\Delta\rho. \tag{10}$$

The derivative with respect to k_z can be implemented in the image space as a simple complex multiplication with depth, and the derivative with respect to ρ is explicit in Sava and Biondi (2003).

5 Example

We illustrate the theoretical analysis in the preceding sections with a synthetic example. We present the images as angle-domain common image gathers (Sava and Fomel, 2003). Figure 1 shows: (1) correct slowness; (2) correct image; (3) background slowness; (4) background image; (5) true slowness perturbation; (6) true image perturbation obtained by the forward WEMVA operator; (7) inverted slowness perturbation after one non-linear iteration and 5 linear iterations; (8) analytical image perturbation; (9) updated slowness; (10) updated image.

6 Conclusions

We present a recursive wave-equation migration velocity analysis method operating in the image domain. Our method is based on the linearization of the downward continuation operator that relates perturbations in slowness to perturbations in image. The fundamental idea is to improve the quality of the slowness function by optimizing the focusing of the migrated image.

We construct the image perturbations by a differential operator applied to the reference image. In this way, we ensure that we do not violate the inherent Born approximation made in our method.

7 REFERENCES

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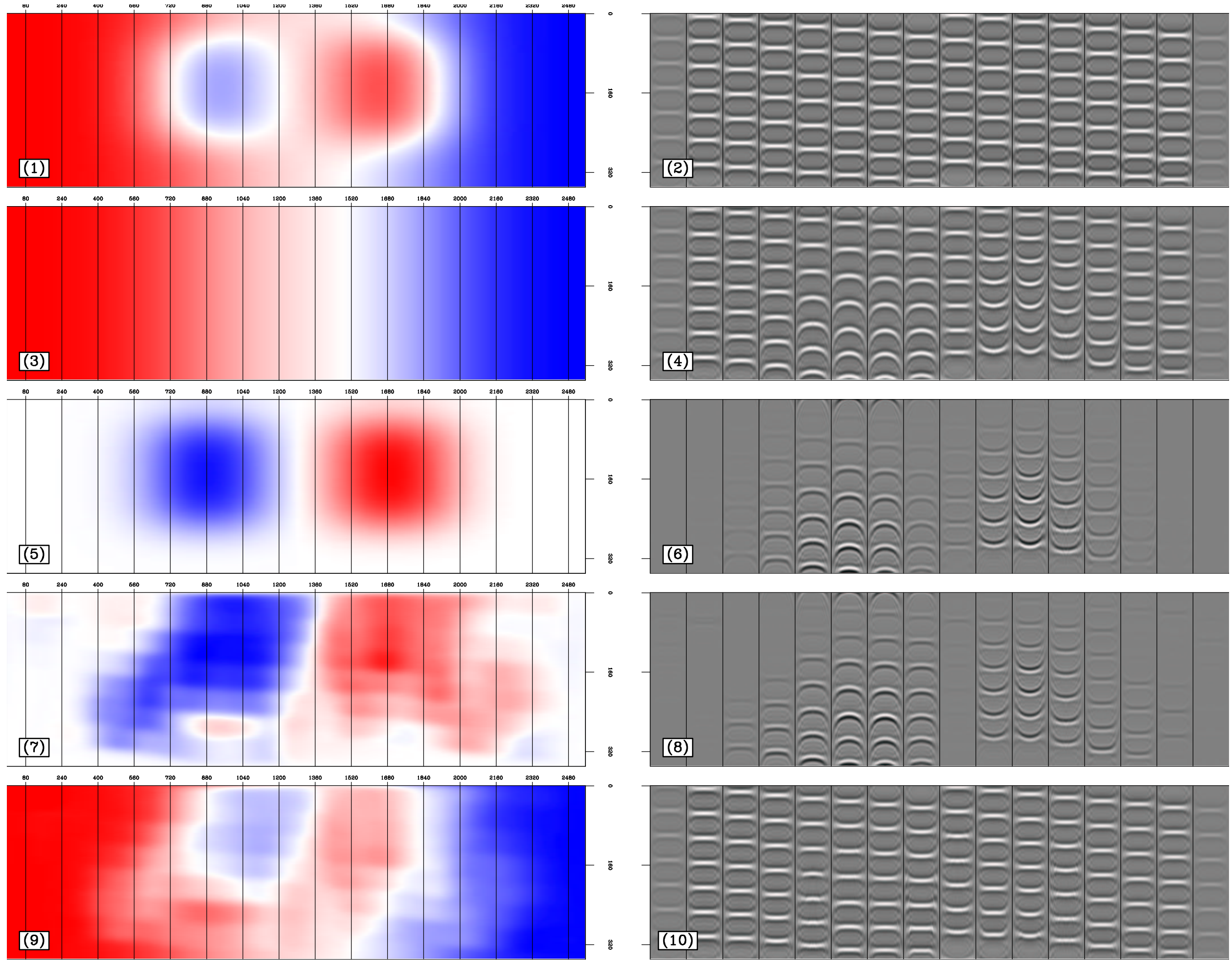

Figure 1: WEMVA example. The left column shows slowness or slowness perturbations. The right column shows images presented as angle-domain common image gathers.

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