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SEISMIC MODELING WITH RIEMANNIAN WAVEFIELD EXTRAPOLATION

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1 Abstract

Riemannian spaces are described by non-orthogonal curvilinear coordinates. We generalize one-way wavefield extrapolation to semi-orthogonal Riemannian coordinate systems, which include, but are not limited to, ray coordinate systems. We obtain one-way wavefield extrapolation methods which are not dip-limited, and which can even be used to image overturning waves. Ray coordinate systems can be initiated either from point sources, or from plane waves incident at various angles. Since wavefield propagation happens mostly along the extrapolation direction, we can use inexpensive finite-difference or mixed-domain extrapolators to achieve high angle accuracy. The main applications of our method include imaging of steeply dipping or overturning reflections.

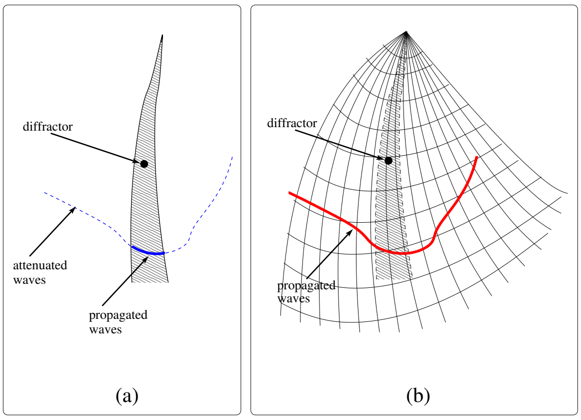
2 Motivation

Imaging complex geology is one of the main challenges of today's seismic processing. Of the many seismic imaging methods available, downward continuation (Claerbout, 1985) is accurate, robust, and capable of handling models with large and sharp velocity variations. Such methods naturally handle the multipathing which occurs in complex geology and provide a band-limited solution to the seismic imaging problem. With increased computational power, such methods are moving into the mainstream of seismic processing. This explains why one-way wave extrapolation has been a subject of extensive theoretical research in the recent years (Ristow and Ruhl, 1994; de Hoop, 1996; Huang and Wu, 1996; Thomson, 1999; Biondi, 2002).

However, migration by downward continuation imposes limitations on the dip of reflectors that can be imaged since it favors energy which is propagating mainly in the downward direction. Upward propagating energy, e.g., overturning waves, can be imaged in principle using downward continuation methods (Hale et al., 1992), although the procedure is difficult, particularly for prestack data. In contrast, Kirchhoff-type methods based on ray-traced traveltimes can image steep dips and handle overturning waves, although those methods are far less reliable in complex environments given their high-frequency asymptotic nature.

The steep-dip limitation of downward continuation techniques has been addressed in several ways: A first option is to increase the angular accuracy of the extrapolation operator, for example by employing methods from the Fourier finite-difference (FFD) family (Ristow and Ruhl, 1994; Biondi, 2002), or the Generalized Screen Propagator (GSP) family (de Hoop, 1996; Huang and Wu, 1996). A second option is to perform the wavefield extrapolation in tilted coordinate systems (Etgen, 2002), or by designing sources which favor illumination of particular regions of the image (Rietveld and Berkhout, 1994). A third possibility is hybridization of wavefield and ray-based techniques, either in the form of Gaussian beams (Červený, 2001; Hill, 2001; Gray et al., 2002), coherent states (Albertin et al., 2001), or beam-waves (Brandsberg-Dahl and Etgen, 2003). Such techniques are quite powerful, since they couple wavefield methods with multipathing and band-limited properties, with ray methods, which deliver

Figure 1: The extrapolated energy is attenuated at beam boundaries (a), but is propagated in a Riemannian coordinate system (b).



arbitrary directions of propagation, even overturning. Decoupled beams, however, may leave shadow zones in various parts of the model, which hamper their imaging abilities. Furthermore, beams have limited size, which in turn limits the extent of the diffractions created by sharp features in the model to that of any particular beam, no matter how accurate the extrapolator within each beam is. The narrow extrapolation domain also poses serious beam superposition problems, such as beam boundary effects.

The main idea of our paper is to couple the beams together and extrapolate within all of them at once. We, therefore, cannot talk about beams anymore, but instead we need to talk about continuously changing coordinate systems. We extend downward continuation in a regular Cartesian space to wavefield extrapolation in distorted coordinates, known as *Riemannian spaces*, thus the name of our method. Special cases are represented by ray or polar/spherical coordinate systems (Nichols, 1994).

In Riemmannian coordinates, the one-way wave equation is represented by

$$k_\zeta = i\frac{c_\zeta}{2c_{\zeta\zeta}} \pm \sqrt{\frac{(\omega s)^2}{c_{\zeta\zeta}} - \left(\frac{c_\zeta}{2c_{\zeta\zeta}}\right)^2 - \sum_{j=\xi,\eta}\left[\frac{c_{jj}}{c_{\zeta\zeta}}k_j^2 - i\frac{c_j}{c_{\zeta\zeta}}k_j\right] - \frac{c_{\xi\eta}}{c_{\zeta\zeta}}k_\xi k_\eta}\,, \tag{1}$$

where the solution with the positive sign corresponds to propagation in the positive direction of the extrapolation axis ζ. The coefficients c_* $(* = \xi,\eta,\zeta,\xi\xi,\eta\eta,\zeta\zeta,\xi\eta)$ are known and characterize a particular Riemannian coordinate system, s and ω represent the slowness and temporal frequency, and k_* stand for wavenumbers of the coordinate axes $(* = \xi,\eta,\zeta)$.

We can solve equation (1) numerically using finite-difference methods involving approximations similar to the ones made for one-way wave equation in Cartesian coordinates. Our method can be seen as a finite-difference solution to the acoustic wave-equation in ray coordinates. In this respect, it is related to Huygens wavefront tracing (Sava and Fomel, 2001), which is a finite-difference solution to the eikonal equation in ray coordinates.

The upside of our method is that the coordinate system may follow the waves, and can even overturn, such that we can use one-way extrapolators to image diving waves. We can also use extrapolators with small angle accuracy (e.g. 15°), since, in principle, we are never too far from the actual direction in which waves propagate. We are also not confined to the extent of any individual beam, therefore we can track diffractions for their entire spatial extent (Figure 1).

3 Examples

We illustrate our method with the Marmousi synthetic model. We use extrapolation in 2-D orthogonal Riemannian spaces (ray coordinates), and compare the results with extrapolation in Cartesian

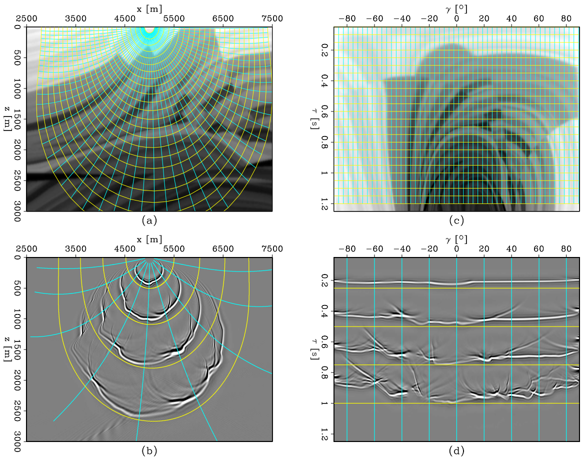


Figure 2: Marmousi model: Cartesian coordinates (a,b) and ray coordinates (c,d). velocity model with an overlay of the ray coordinate system initiated by a point source at the surface (a); image obtained by downward continuation in Cartesian coordinates with the 15° equation (b); velocity model with an overlay of the ray coordinate system (c); image obtained by wavefield extrapolation in ray coordinates with the 15° equation (d).

coordinates. We present images obtained by migration of synthetic datasets represented by events equally spaced in time. In all examples, (x,z) are the Cartesian space coordinates, and (τ,γ) are ray coordinates for point sources. γ stands for shooting angle, and τ for one-way traveltime. Figure 2 shows the velocity models mapped into the two different domains and the corresponding wavefields. We create the ray coordinate system by ray tracing in a smooth version of the model with a source at $x = 5000$ m, and extrapolate in the rough version.

In this example, the wavefields triplicate in both domains (Figure 3). Since we are using a 15° equation, extrapolation in Cartesian coordinates is only accurate for the small incidence angles, as can be seen in panels (a) and (b). In contrast, extrapolating in ray coordinates (c) does not have the same angle limitation, which can also be seen after mapping back to Cartesian coordinates (d).

4 Conclusions

We extend one-way wavefield extrapolation to Riemannian spaces which are, by definition, described by non-orthogonal curvilinear coordinate systems. We choose semi-orthogonal Riemannian coordinates which include, but are not limited to, ray coordinate systems. We define an acoustic wave-equation for semi-orthogonal Riemannian coordinates, from which we derive a one-way wavefield extrapolation equation. We use ray coordinates initiated either from a point source, or from an incident plane wave at the surface. Many other types of coordinates are acceptable, as long as they fulfill the semi-orthogonal condition of our acoustic wave equation.

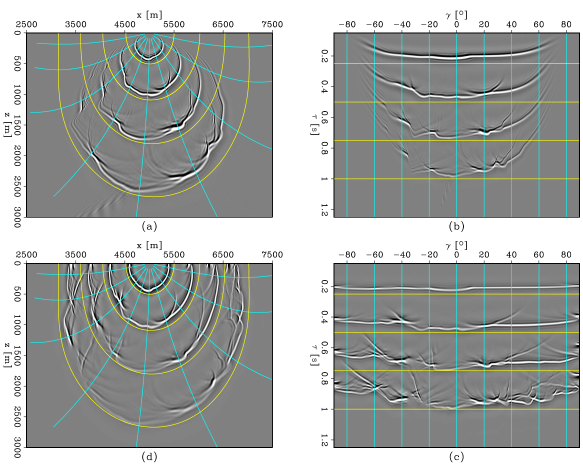


Figure 3: Marmousi model: the image obtained by downward continuation in Cartesian coordinates with the 15° equation (a); the image in panel (a) interpolated to ray coordinates (b); image obtained by extrapolation in ray coordinates with the 15° equation (c); the image in panel (c) interpolated to Cartesian coordinates (d).

Since wavefield propagation happens mostly along the extrapolation direction, we can use inexpensive 15° finite-difference or mixed-domain extrapolators to achieve high angle accuracy. If the ray coordinate system overturns, our method can be used to image overturning waves with one-way wavefield extrapolation.

5 Acknowledgment

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