SUMMARY

We generalize imaging conditions and angle decompositions for wave-equation migration of primary and converted waves. Prestack images are described as functions of space-shifts or time-shifts between source and receiver wavefields at every image location. Angle transformations allow construction of common-image gathers for both primary (P) and converted (C) waves.

IMAGING CONDITION

A conventional imaging condition for shot-record migration, often referred-to as UD imaging condition (Claerbout, 1985), consists of time cross-correlation at every image location between the source and receiver wavefields, followed by image extraction at zero time:

$$U(\mathbf{m},t) = U_r(\mathbf{m},t) \otimes U_s(\mathbf{m},t), \tag{1}$$

$$R(\mathbf{m}) = U(\mathbf{m},t=0), \tag{2}$$

where the symbol $\otimes$ denotes cross-correlation in time. Here, $\mathbf{m}$ = [x,y,z] is a vector describing the locations of image points, $U_s(\mathbf{m}, t)$ and $U_r(\mathbf{m}, t)$ are source and receiver wavefields respectively, and $R(\mathbf{m})$ denotes a migrated image. A final image is obtained by summation over shots.

For computational reasons, this imaging condition is usually implemented in the Fourier domain using the expression

$$R(\mathbf{m}) = \sum_{\omega} U_r(\mathbf{m},\omega) U_s^*(\mathbf{m},\omega). \tag{3}$$

The * sign represents a complex conjugate applied on the receiver wavefield U_s in the Fourier domain

Space-shift imaging condition

A generalized prestack imaging condition (Sava and Fomel, 2005) estimates image reflectivity using cross-correlation in space and time, followed by image extraction at zero time:

$$U(\mathbf{m},\mathbf{h},t) = U_r(\mathbf{m}+\mathbf{h},t) \otimes U_s(\mathbf{m}-\mathbf{h},t), \tag{4}$$

$$R(\mathbf{m},\mathbf{h}) = U(\mathbf{m},\mathbf{h},t=0). \tag{5}$$

Here, $\mathbf{h} = [h_x,h_y,h_z]$ is a vector describing the local source-receiver separation in the image space. Special cases of this imaging condition are horizontal space-shift (Rickett and Sava, 2002) and vertical space-shift (Biondi and Symes, 2004).

As for the conventional imaging condition, this imaging condition can be implemented in the Fourier domain using the expression

$$R(\mathbf{m},\mathbf{h}) = \sum_{\omega} U_r(\mathbf{m}+\mathbf{h},\omega) U_s^*(\mathbf{m}-\mathbf{h},\omega). \tag{6}$$

Time-shift imaging condition

Another prestack imaging condition involves shifting of the source and receiver wavefields in time, as opposed to space, followed by image extraction at zero time:

$$U(\mathbf{m},\tau,t) = U_r(\mathbf{m},t+\tau) \otimes U_s(\mathbf{m},t-\tau), \tag{7}$$

$$R(\mathbf{m},\tau) = U(\mathbf{m},\tau,t=0). \tag{8}$$

Here, τ is a time shift between the source and receiver wavefields prior to imaging. This imaging condition can be implemented in the Fourier domain using the expression

$$R(\mathbf{m},\tau)=\sum_{\omega}U_r(\mathbf{m},\omega)U_s^*(\mathbf{m},\omega)e^{2i\omega\tau}, \tag{9}$$

which simply involves a phase-shift applied to the wavefields prior to summation over frequency ω for imaging at zero time.

ANGLE TRANSFORMATION IN WAVE-EQUATION IMAGING

Using the definitions introduced in the preceding section, we can make the standard notations for source and receiver coordinates: **s**=**m**−**h** and **r**=**m**+**h**. The traveltime from a source to a receiver is a function of all spatial coordinates of the seismic experiment $t = t(\mathbf{m},\mathbf{h})$. Differentiating t with respect to all components of the vectors **m** and **h**, and using the standard notations

$$\mathbf{p}_\alpha=\nabla_\alpha t, \tag{10}$$

where $\alpha = \{\mathbf{m},\mathbf{h},\mathbf{s},\mathbf{r}\}$, we can write:

$$2\mathbf{p_m}=\mathbf{p_r}+\mathbf{p_s} \tag{11}$$

$$2\mathbf{p_h}=\mathbf{p_r}-\mathbf{p_s} \tag{12}$$

From equations (11)-(12), we can write

$$\mathbf{p_s}=\mathbf{p_m}-\mathbf{p_h}, \tag{13}$$

$$\mathbf{p_r}=\mathbf{p_m}+\mathbf{p_h}. \tag{14}$$

By analyzing the geometric relations of various vectors at an image point (Figure 1), we can write the following trigonometric expressions:

$$4|\mathbf{p_h}|^2=|\mathbf{p_s}|^2+|\mathbf{p_r}|^2-2|\mathbf{p_s}||\mathbf{p_r}|\cos(2\theta), \tag{15}$$

$$4|\mathbf{p_h}|^2=|\mathbf{p_s}|^2+|\mathbf{p_r}|^2-2|\mathbf{p_s}||\mathbf{p_r}|\cos(2\theta). \tag{16}$$

Therefore, we can write

$$4|\mathbf{p_h}|^2+4|\mathbf{p}_m|^2=2|\mathbf{p}_s|^2+2|\mathbf{p}_r|^2. \tag{17}$$

Space-shift angle decomposition for P waves

Defining $\mathbf{k_m}$ and $\mathbf{k_h}$ as location and offset wavenumber vectors, and assuming $|\mathbf{p_s}|=|\mathbf{p_r}|=s$, where $s(\mathbf{m})$ is the slowness at image locations, we can replace $|\mathbf{p_m}| = |\mathbf{k_m}|/\omega$ and $|\mathbf{p_h}| = |\mathbf{k_h}|/\omega$ in equations (15)-(16):

$$4|\mathbf{p_h}|^2=2(\omega s)^2(1-\cos 2\theta), \tag{18}$$

$$4|\mathbf{p}_m|^2=2(\omega s)^2(1+\cos 2\theta). \tag{19}$$

If we eliminate from equations (17)-(18) the dependence on frequency and slowness, and obtain an angle decomposition formulation after imaging by expressing $\tan\theta$ function of position and offset wavenumbers ($\mathbf{k_m},\mathbf{k_h}$):

$$\tan\theta=\frac{|\mathbf{k_h}|}{|\mathbf{k_m}|}. \tag{20}$$

Time-shift angle decomposition for P waves

Using the same definitions as the ones introduced in the preceding subsection, we can rewrite equation (16) as

$$4|\mathbf{p_m}|^2=2s^2(1+\cos 2\theta), \tag{21}$$

from which we can derive an expression for angle-transformation after time-shift prestack imaging:

$$\cos\theta = \frac{|\mathbf{p_m}|}{s}. \tag{22}$$

Space-shift angle decomposition for C waves

We can transform equations (15)-(16) using the notations $|\mathbf{p}_s| = s$ and $|\mathbf{p}_r| = \gamma s$, where $\gamma(\mathbf{m})$ is the v_p/v_s ratio, and s(**m**) is the slowness associated with the incoming ray at every image point:

$$4|\mathbf{p_h}|^2 = \omega^2 s^2 \left(1+\gamma^2 - 2\gamma\cos 2\theta\right), \tag{23}$$

$$4|\mathbf{p}_m|^2 = \omega^2 s^2 \left(1+\gamma^2 + 2\gamma\cos 2\theta\right). \tag{24}$$

If we eliminate ω, we obtain the expression

$$\tan^2\theta = \frac{(1+\gamma)^2|\mathbf{k_h}|^2 - (1-\gamma)^2|\mathbf{k_m}|^2}{(1+\gamma)^2|\mathbf{k_m}|^2 - (1-\gamma)^2|\mathbf{k_h}|^2} \tag{25}$$

that can be used for angle decomposition for C waves after space-shift imaging condition.

Time-shift angle decomposition for C waves

Using the same definitions for $|\mathbf{p}_s|$ and $|\mathbf{p}_r|$ as in the preceding subsection, we can transform equation (16)

$$4|\mathbf{p}_m|^2 = \omega^2 s^2 \left(1+\gamma^2 + 2\gamma\cos 2\theta\right), \tag{26}$$

therefore we can write

$$\cos^2\theta = \frac{1}{\gamma}\frac{|\mathbf{p_m}|^2}{s^2} - \frac{(1-\gamma)^2}{4\gamma}. \tag{27}$$

For P waves ($\gamma = 1$), this expression reduces to equation (22).

EXAMPLE

Figure 2 shows common-image gathers obtained from space-shift imaging (middle) and time-shift imaging (right) for the Sigsbee 2A model (Paffenholz et al., 2002). The left panel depicts a portion of the migrated image (zero offset of the prestack image). Figure 2 shows angle-domain common-image gathers obtained from space-shift imaging (middle) and time-shift imaging (right) using the angle decomposition formulas presented in this paper.

CONCLUSION

Equations (14) and (16) can be used for angle decomposition of common-image gathers obtained by space-shift and time-shift imaging conditions for P waves. Likewise, equations (19) and (21) can be used for angle decomposition of common-image gathers obtained by space-shift and time-shift imaging conditions for C waves. The formulas derived in this paper generalize similar decompositions derived by Sava and Fomel (2005a,b,c). Further research is needed to asses the applicability of such angle decompositions for real data.

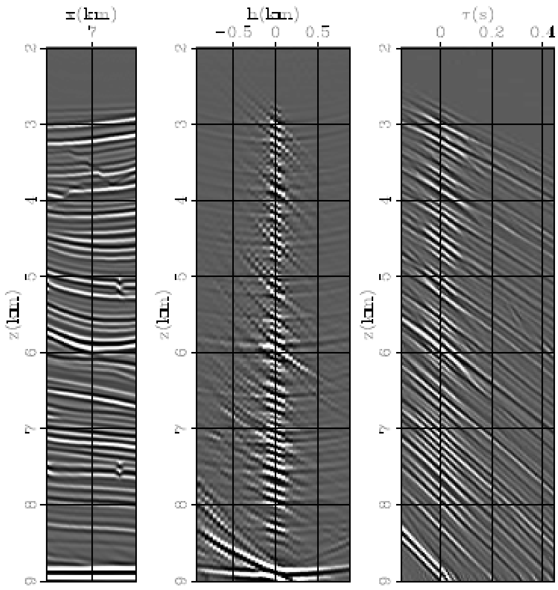

Figure 2: Offset gathers at x = 7 km: image (left), space-shift gather (middle), and timeshift gather (right)

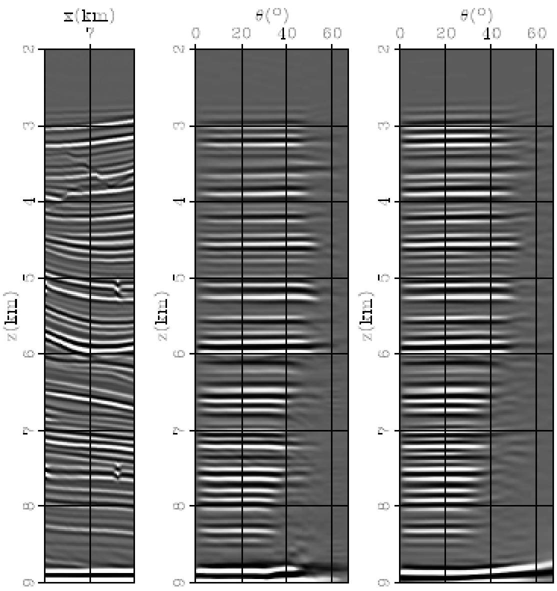

Figure 3: Angle gathers at x = 7 km: image (left), angle-gather from space-shift imaging (middle), and angle-gather from time-shift imaging (right).

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