## Introduction

The elastic wave equation migration for multicomponent data can be implemented in two ways. The first approach is to separate recorded elastic data into compressional and transverse (P and S) modes, and use these separated modes for acoustic wave equation migration respectively. This acoustic imaging approach to elastic waves is more frequently used, but it is fundamentally based on the assumption that P and S data can be successfully separated on the surface, which is not always true (Etgen, 1988). The second approach is to not separate P and S modes on the surface, extrapolate the entire elastic wavefield at once, then separate wave modes prior to applying an imaging condition.

The imaging condition applied to the reconstructed vector wavefields directly determines the quality of the images. Conventional cross-correlation imaging condition does not separate the wave modes and cross-correlates the Cartesian components of the elastic wave. In general, the various wave modes (P and S) are mixed on all wavefield components and cause crosstalk and image artifacts. Yan and Sava (2008) suggest using imaging conditions based on elastic potentials, which require cross-correlation of separated modes. Potential-based imaging condition creates images that have clear physical meanings, in contrast with images obtained with Cartesian wavefield components, thus justifying the need for wave mode separation.

As the need for anisotropic imaging increases, more processing and migration are performed based on anisotropic acoustic one-way wave equations (Alkhalifah, 1998). However, much less research has been done on anisotropic elastic migration based on two-way wave equations. One of the complexities that impedes elastic wave equation anisotropic migration is the difficulty to separate anisotropic wavefields into different wave modes. However, the proper separation of anisotropic wave modes is as important for anisotropic elastic migration as is the separation of isotropic wave modes for isotropic elastic migration. The main difference between anisotropic and isotropic wavefield separation is that Helmholtz decomposition is only suitable for the separation of isotropic wavefields, and does not work well for anisotropic wavefields. In this abstract, we show how to construct wave-mode separators for VTI (vertical transverse isotropy) media applicable to models with spatially varying parameters. We apply these operators to anisotropic elastic wavefields and show that they successfully separate anisotropic wave modes, even for extremely anisotropic media.

## Separation method

Separation of scalar and vector potentials can be achieved by Helmholtz decomposition, which is applicable to any vector field. By denition, the vector waveeld can be decomposed into a curl-free scalar potential $\Theta$ and a divergence-free vector potential $\boldsymbol{\Psi}$ according to the relation:

$$\mathbf{W} = \nabla\Theta + \nabla \times \boldsymbol{\Psi} \,. \tag{1}$$

Equation 1 is not used directly in practice, but the scalar and vector components are obtained indirectly by the application of the $\nabla \cdot$ and $\nabla \times$ operators to the extrapolated elastic wavefield:

$$P = \nabla \cdot \mathbf{W} \,, \tag{2}$$
$$\mathbf{S} = \nabla \times \mathbf{W} \,. \tag{3}$$

For isotropic elastic fields far from the source, quantities $P$ and $\mathbf{S}$ describe compressional and transverse wave modes, respectively (Aki and Richards, 2002). Equations 2 and 3 allow us to understand why $\nabla \cdot$ and $\nabla \times$ pass compressional and transverse wave modes, respectively. In the space domain, we can write:

$$P = \nabla \cdot \mathbf{W} = D_x * W_x + D_y * W_y + D_z * W_z \,, \tag{4}$$

where $D_x$, $D_y$ and $D_z$ represent spatial derivatives in $x$, $y$ and $z$ directions and $*$ represents spatial convolution. In the Fourier domain, we can represent the operators $D_x$, $D_y$ and $D_z$ by $i\,k_x$, $i\,k_y$ and $i\,k_z$, therefore we can write an equivalent expression to equation 4 as:

$$P = i\,\mathbf{k} \cdot \widetilde{\mathbf{W}} = i\,k_x \widetilde{W}_x + i\,k_y \widetilde{W}_y + i\,k_z \widetilde{W}_z \,, \tag{5}$$

where $\mathbf{k} = \{k_x, k_y, k_z\}$ represents the wave vector, and $\widetilde{\mathbf{W}}(k_x, k_y, k_z)$ is the 3D Fourier transform of the wavefield $\mathbf{W}(x, y, z)$. We see that in this domain, the operator $i\,\mathbf{k}$ essentially projects the wavefield $\widetilde{\mathbf{W}}$ onto the wave vector $\mathbf{k}$, which represents the polarization direction for P waves. Similarly, the operator $\nabla\times$ projects the wavefield onto the direction orthogonal to the wave vector $\mathbf{k}$, which represents the polarization direction for S waves (Dellinger and Etgen, 1990).

Dellinger and Etgen (1990) suggest the idea that wave mode separation can be extended to anisotropic media by projecting the wavefields onto the directions in which the P and S modes are polarized. This requires that we modify the wave separation equation 5 by projecting the wavefields onto the true polarization directions $\mathbf{U}$ to obtain *quasi*-P ($q$P) waves:

$$qP = i\,\mathbf{U}(\mathbf{k}) \cdot \widetilde{\mathbf{W}} = i\,U_x\,\widetilde{W}_x + i\,U_y\,\widetilde{W}_y + i\,U_z\,\widetilde{W}_z\,. \tag{6}$$

In anisotropic media, $\mathbf{U}(\mathbf{k})$ is different from $\mathbf{k}$, that is the polarization vectors are not radial because $q$P waves in an anisotropic medium are not polarized in the same directions as wave vectors, except in the symmetry planes ($k_z = 0$) and along the symmetry axis ($k_x = 0$).

Dellinger and Etgen (1990) demonstrate wave mode separation in the wave number domain using projection of the polarization vectors, as indicated in equation 6. However, for heterogeneous media, this equation does not work because the polarization vectors are spatially varying. We can write an equivalent expression to equation 6 in the space domain as:

$$qP = \nabla_a \cdot \mathbf{W} = L_x * W_x + L_y * W_y + L_z * W_z\,, \tag{7}$$

where $L_x$, $L_y$ and $L_z$ represent the inverse Fourier transforms of $i\,U_x$, $i\,U_y$ and $i\,U_z$, and $*$ represents spatial convolution. $L_x$, $L_y$ and $L_z$ define the pseudo derivative operators in the $x, y$ and $z$ directions for an anisotropic medium, and they change from location to location according to the material parameters.

We obtain the polarization vectors $\mathbf{U}(\mathbf{k})$ of a VTI medium by solving the Christoffel equation in the symmetry plane (Tsvankin, 2005):

$$\begin{bmatrix} c_{11}k_x^2 + c_{55}k_z^2 - \rho V^2 & (c_{13} + c_{55})\,k_x k_z \\ (c_{13} + c_{55})\,k_x k_z & c_{55}k_x^2 + c_{33}k_z^2 - \rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_z \end{bmatrix} = 0\,, \tag{8}$$

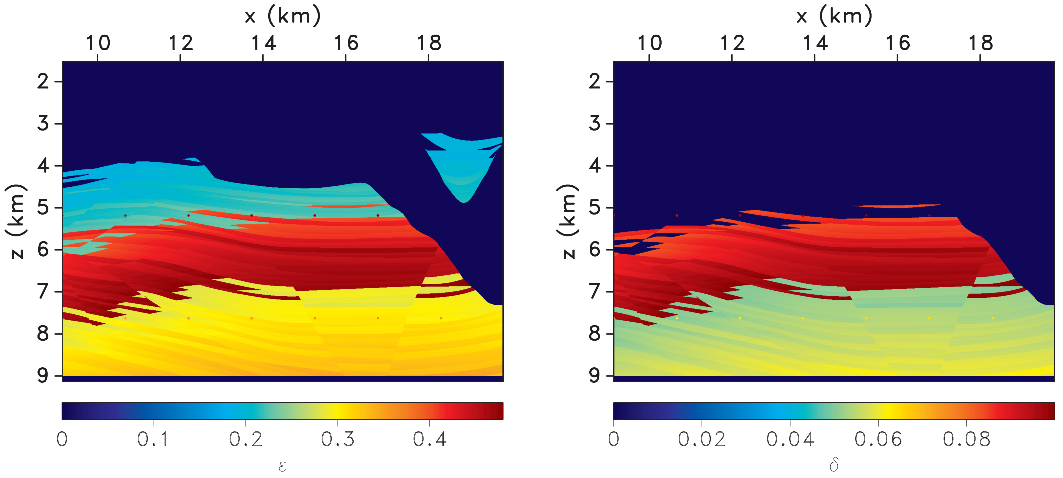
which allows us to compute the components of the polarization vector $\mathbf{U}$ (the eigenvectors of the Christoffel matrix) of a certain wave mode given the stiffness tensor at every location of the medium. We thus extend the procedure to heterogeneous media by computing a different operator at every grid point.

**Operator properties**

In the symmetry planes of VTI media, the operators are 2D and depend on the local values of the stiffness coefficients. For each point, we pre-compute the polarization vectors as a function of the local medium parameters, and transform them to the space domain to obtain the wave mode separators. We can compute and store the operators for each grid point in the medium, and then use those operators to separate P and SV modes from reconstructed elastic wavefields at different time steps. Thus, wavefield separation in VTI media can be achieved simply by non-stationary filtering with operators $L_x$ and $L_z$.

Depending on the order of accuracy used in modeling of the wavefield, the orders of the separation operators should vary accordingly. The anisotropic separators need also be high order of accuracy when the modeling is done with high order in space. As shown in Figure 2, the $8^{th}$ order operators degenerate into traditional difference operators when the medium is isotropic, and become bigger when the medium gets anisotropic. The size of the operators depends on Thomsen parameters $\epsilon$, $\delta$ and $V_P V_S$ ratio, which are in general spatially varying.

Because the size of the anisotropic derivative operators is usually large in space, it is natural that one would truncate the operators to save computation. Too much truncation reduces the effectiveness of wave-mode separation. This is because the truncation changes the direction of the polarization vectors, thus projecting the wavefield displacement onto the wrong direction. Therefore, it is safer to leave the operator big in size, which makes the P and SV wave-mode separation for heterogeneous anisotropic media more expensive than the separation for isotropic media.




(a) (b)

Figure 1: Elastic anisotropic Sigsbee 2A model: (a) parameter $\epsilon$ ranges from 0 to 0.48 and (b) parameter $\delta$ ranges 0 from to 0.10.

## Examples

We illustrate the anisotropic wave mode separation with an elastic anisotropic version of the Sigsbee 2A model (Paffenholz et al., 2002). In our modified model, the P wave velocity is taken from the original model, the $V_P V_S$ ratio ranges from 1.5 to 2, the parameter $\epsilon$ ranges from 0 to 0.48 (Figure 1(a)) and the parameter $\delta$ ranges 0 from to 0.10 (Figure 1(b)). The model is isotropic in the salt and the top part of the model. Figure 2 includes a subset of the spatially varying anisotropic wave-mode separators at the cross sections of $x = 12, 15, 17$ km and $z = 3, 5, 7$ km. A vertical point force source is located at coordinates $x = 14.5$ km and $z = 5.3$ km to simulate the elastic anisotropic wavefield. Figure 3(a) shows one snapshot of the modeled elastic anisotropic wavefields using the model shown in Figure 1. Figure 3(b) illustrates the separation of the anisotropic elastic wavefields using the $\nabla \cdot$ and $\nabla \times$ operators, and Figure 3(c) illustrates the separation using our pseudo derivative operators. Figure 3(b) shows the residual of unseparated P and S wave modes, such as at coordinates $x = 13$ km and $z = 7$ km in the $q$P panel and at $x = 11$ km and $z = 7$ km in the $q$S panel. The residual of S waves in the $q$P panel of Figure 3(b) is very significant because of strong reflections from the salt bottom. This extensive residual can be harmful to under-salt elastic or even acoustic migration, if not removed completely. In contrast, Figure 3(c) shows the $q$P and $q$S modes completely separated, demonstrating the effectiveness of the anisotropic pseudo derivative operators constructed using the local medium parameters. These wavefields composed of well separated and modes are essential to producing clean seismic images.

## Conclusions

We present a method of obtaining spatially-varying pseudo derivative operators with application to wave mode separation in anisotropic media. The main idea is to utilize polarization vectors constructed in the wavenumber domain using the local medium parameters and then transform these vectors back to the space domain. The main advantage of applying the pseudo derivative operators in the space domain constructed in this way is that they are suitable for heterogeneous media. The wave mode separators obtained using this method are spatially-variable filtering operators and can be used to separate waveelds in VTI media with arbitrary degree of anisotropy. This methodology is applicable for elastic RTM in heterogeneous anisotropic media.

## Acknowledgment

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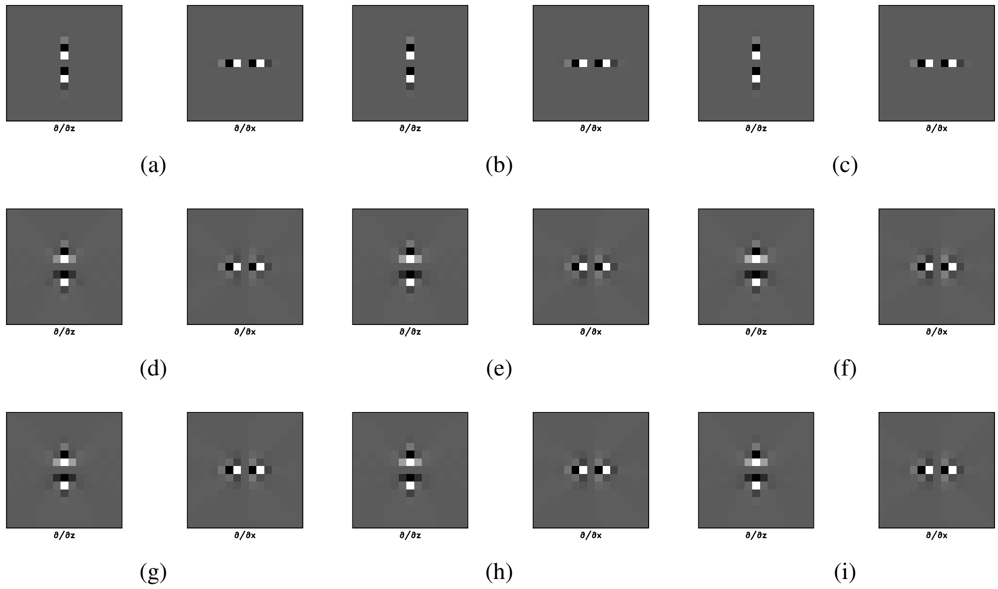


Figure 2: A subset of the spatially varying anisotropic wave-mode separators in the $z$ and $x$ directions at the cross sections of $x = 12, 15, 17$ km and $z = 3, 5, 7$ km for the elastic Sigsbee 2A model.

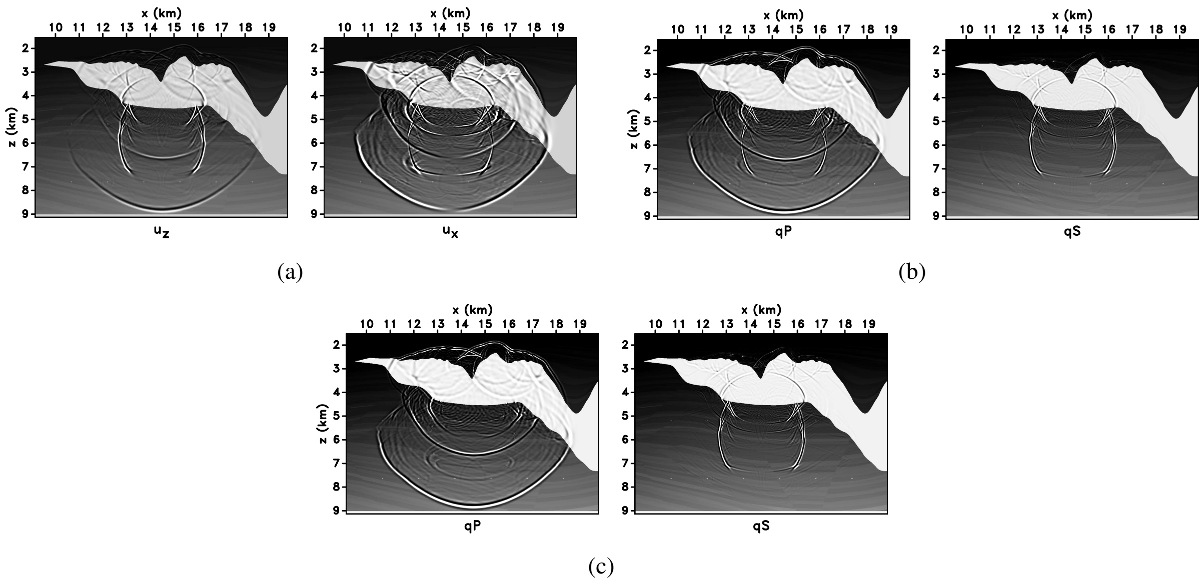


Figure 3: (a) A snapshot of the elastic wavefield. (b) Attempt separation of P and SV modes with $\nabla \cdot$ and $\nabla \times$ . (c) Separated P and SV wave modes with anisotropic separators $\nabla_a \cdot$ and $\nabla_a \times$.


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