Migration velocity analysis using a transversely isotropic medium with tilt normal to the reflector dip

Tariq Alkhalifah, KACST, and Paul Sava, CSM

February 16, 2010

Abstract

A transversely isotropic model in which the tilt is constrained to be normal to the dip (DTI model) allows for simplifications in the imaging and velocity model building efforts as compared to a general TTI model. Though this model, in some cases, can not be represented physically like in the case of conflicting dips, it handles all dips with the assumption of symmetry axis normal to the dip. It provides a process in which areas that meet this feature is handled properly. We use efficient downward continuation algorithms that utilizes the reflection features of such a model. For lateral inhomogeneity, phase shift migration can be easily extended to approximately handle lateral inhomogeneity, because unlike the general TTI case the DTI model reduces to VTI for zero dip. We also equip these continuation algorithms with tools that expose inaccuracies in the velocity. We test this model on synthetic data of general TTI nature and show its resilience even couping with complex models like the recently released anisotropic BP model.

Introduction

Migration velocity analysis (MVA), despite the many developments in recent years, is still a challenging process especially in complex media. MVA is even more of a challenge in anisotropic media in which the medium is described by more than one parameter, all of which can change as a function of position. Anisotropy introduces flexibility to the model to better simulate the Earth subsurface, but it also introduces null space to the parameter estimation process or MVA. As a result, we need to use anisotropy to allow for more freedom up to the point where data seize to control such freedom. This anisotropy null-space trade-off has recently guided us to using a transversely isotropic (TI) medium with a tilt in the axis of symmetry (TTI). To avoid the null space such tilt is assumed to be in the direction of the dip (Alkhalifah and Bednar, 2000; Audebert et al., 2006). In transversely isotropic (TI) with vertical symmetry axis (VTI) media, the acoustic problem can be described by three parameters (Alkhalifah and Tsvankin, 1995), the vertical velocity, the NMO velocity, and an anisotropy parameter that relates the NMO velocity to the horizontal velocity referred to as η. For TI media with a tilt in the axis of symmetry two additional parameters, θ and α that describe the tilt in 3-D, are needed to fully characterize acoustic wave propagation. Alkhalifah and Sava (2010) introduce the concept of using the tilt in the direction of the dip as a constrain to develop simplified and explicit representations for moveout in extended images, for angle gathers and for migration. They refer to the model as dip-constrained TI (DTI).

In this abstract, we use the explicit formulations provided by the DTI model to perform downward continuation, and discuss the potential TTI parameter estimation. In this context, many of the familiar tools developed for the isotropic case apply with little or no modifications.

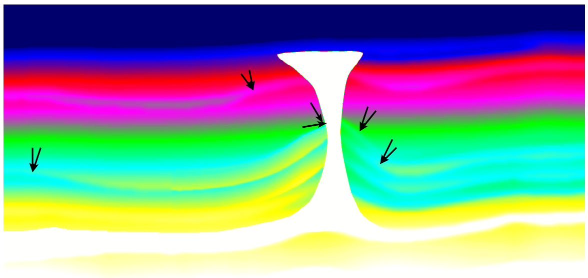

Figure 1: Part of BP anisotropic velocity model that contains a salt body. The abrupt change in velocity magnitude can be interpreted as reflections and the arrows point to examples of the possible directions of TI symmetry tilt to accommodate a DTI model.

We utilize this to build a framework for imaging and velocity model building in DTI models and illustrate the method with synthetic data.

Dip-constrained TTI: Not a physical model

One question that arises is how does the DTI constraint be imposed on a model? Specifically, what happens when we have conflicting dips? For the equations developed here and especially those of Alkhalifah and Sava (2010), based on a plane wave representation, the DTI constraint is explicitly handled in the formulations. This implies that even for conflicting dips (at a point) the tilt is always normal to the reflector dip, as if the dips were handled in separate planes. This can not be represented in the physical space, and thus, it is a process. If conflicting dips exist, then the dip that physically adheres to the constraint is handled properly. Figure 1 shows the BP anisotropic model with arrows pointing to conflicting dips. However, the axis of symmetry is single valued and thus the DTI model handles the reflections that adhere to this assumption. Nevertheless, conflicting dips are truly conflicting (cross at a point) if the true velocity is used in imaging. Otherwise, their conflict is in an inaccurate position and thus do not reflect the physical behavior of the reflectors.

Downward Continuation

Downward continuation, with the double-square-equation in the DTI framework, utilizes the equal incidence and reflection angles imposed by the constraint. As shown by Alkhalifah and Sava (2010), the downward continuation process includes two steps: determining the offset wavenumber corresponding to a particular angle gather and then using the offset wavenumber to determine the required phase shift based on the double-square-root (DSR) equation. The two steps are given by explicit equations that can be implemented using the algorithm suggested by Alkhalifah and Sava (2010).

Considering that angle gather extraction is a localized process relying on the plane wave behavior around the scattering point, it is applicable, within the limits of high frequencies compared to medium variations, to complex media. To allow the DSR-based downward continuation to honor lateral inhomogeneity at least approximately, we may use the phase-shift-plus-interpolation method Gazdag and Sguazzero (1984). In this case, we downward continue multiple times for various vertical velocity fields and then interpolate the wavefields. However, the split-step approach of Stoffa et al. (1990) lends itself to this approach as we maintain angle-gather information in the space domain needed for a good zero-dip $v(x,z)$ correction.

The phase shift operation uses the depth wavenumber k_z which is defined by the DSR formula. If we use, in 2D, the phase velocity $v(x,z,\theta)$, where θ is the angle gather, at angle zero as the reference correction in the space domain, we will have the following DSR form to apply in the midpoint wavenumber domain:

$$k_z = \sqrt{\frac{\omega^2}{v_p^2(\theta)} - (k_m - k_h)^2} + \sqrt{\frac{\omega^2}{v_p^2(\theta)} - (k_m + k_h)^2} - 2\sqrt{\frac{\omega^2}{v_p^2(z,\theta)} - k_h^2}\,. \tag{1}$$

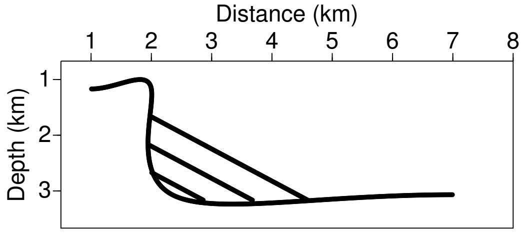

Figure 2: A reflector model depicting a Salt flank with three parallel reflectors laying along side the flank. The angle of symmetry is normal to the three reflectors at 30 degrees from the vertical.

where ω is the angular frequency, and k_m and k_h are the midpoint and offset wavenumbers, respectively. To use k_z in this form we need to evaluate k_h for a given θ in the downward continuation process. To do that we use [Alkhalifah and Sava (2010)]

$$k_h^2(\theta) = (2\omega s(\theta)\sin\theta)^2 - k_m^2 \tan\theta^2. \tag{2}$$

This isotropic-like form can not be obtained for general TTI as we can not isolate the locally fixed angle of symmetry, whereas the DTI model reduces to a VTI one in the zero-dip formulation.

Synthetic Example

In the following example we use for simplicity a vertically inhomogeneous model, although nothing in the development of processes for DTI requires that. We consider the reflector model in Figure 2, which is made up of a salt-flank like reflector in addition to three parallel reflectors depicting sediments laid over the salt flank. The TTI in this model has a symmetry axis normal to the three reflectors at 30 degrees angle from vertical. This model allows us to test the DTI concept for these three reflectors by analyzing the angle gather at 2.5 km location. It also allows us to observe the errors for reflectors that do not adhere to the constraint, like the salt flank and semi-horizontal reflector. For a velocity of 1.5 km/s at the surface and increasing at a gradient of 0.6 s^{-1}, and $\eta = 0.2$, we generate the prestack synthetic dataset. We use Kirchhoff modeling to obtain the synthetic data.

Conventional phase shift downward continuation requires that no lateral velocity variation is present. Since the synthetic model has no lateral velocity variation, we use the phase-shift approach to migrate the data. However, prior to applying the zero-lag imaging condition we map the offset wavenumbers to angle, and thus, obtain angle gathers. Figure 3 shows the isotropically migrated section at near zero angle. It also shows on the right hand side the angle gather for isotropic angle gather mapping. Clearly, the angle gather includes residuals resulting from ignoring anisotropy. They include predominantly nonhyperbolic errors associated with anisotropy with some second order errors associated with the dip [Alkhalifah and Tsvankin (1995)]. The top plot in Figure 3 is a 2 km depth slice section and includes some residual error information spanning other angle gathers.

On the other hand, the phase shift migration under the DTI assumption results in the image and angle gather shown in Figure 4. While the reflections and diffractions associated with angle not normal to the axis of symmetry show clear errors, the three parallel reflections show accurate placement and no residuals in the angle gather. This implies that the parameters used (velocity and η) are accurate within the DTI model that was appropriate to these reflections.

This synthetic test shows a preliminary example of usefulness of the DTI model for analysis of key reflections. Further analysis that includes the anisotropic BP dataset will be presented at the workshop.

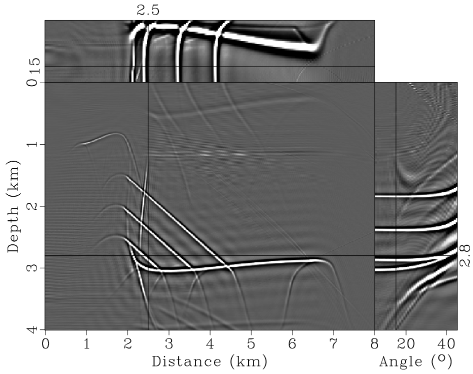

Figure 3: Migrated section after an isotropic migration with velocity of 2 km/s of the TTI synthetic data. The angle gathers obtained using an isotropic mapping at 2.5 km location is displayed on the right, and the top section shows a depth slice as a function of angle gather at depth 2 km.

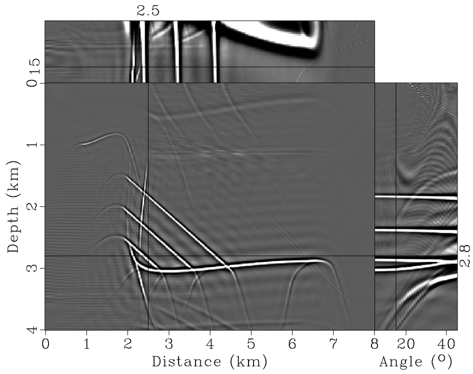

Figure 4: Migrated section after DTI-based migration with velocity of 2 km/s and η=0.2 of the TTI synthetic data. The angle gathers obtained, now, using the VTI mapping at 2.5 km location is displayed on the right. The top section shows a depth slice as a function of angle gather at depth 2 km.

Conclusions

Constraining the tilt of a transversely isotropic medium normal to the reflector dip (DTI) allows for explicit formulation of plane waves around the scattering point. These formulations form the basis for angle decomposition and simplified downward continuation. As a result, DTI is a convenient model for anisotropy parameter estimation in media in which such models are applicable. This model also allows us to use the general TTI assumption in a simplified form that better fits the information embedded in the recorded data. A simple synthetic example demonstrated the potential features of this model.

Acknowledgments

We thank KAUST and the Center of Wave Phenomena at Colorado School of Mines for their support.

REFERENCES

Alkhalifah, T., and J. Bednar, 2000, Building a 3-D anisotropic model: Its implications to traveltime calculation and velocity analysis: 70th Ann. Internat. Mtg, Soc. of Expl. Geophys., 965–968.

Alkhalifah, T., and P. Sava, 2010, A transversely isotropic medium with a tilted symmetry axis normal to the reflector: Geophysics, accepted.

Alkhalifah, T., and I. Tsvankin, 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Audebert, F. S., A. Pettenati, and V. Dirks, 2006, TTI anisotropic depth migration - which tilt estimate should we use?: EAGE, Expanded Abstracts, P185.

Gazdag, J., and P. Sguazzero, 1984, Migration of seismic data by phase-shift plus interpolation: Geophysics, **49**, 124–131.

Stoffa, P. L., J. T. Fokkema, R. M. de Luna Freire, and W. P. Kessinger, 1990, Split-step Fourier migration: Geophysics, **55**, 410–421.