Introduction

Elastic reverse time migration aiming for images with physical meaning requires wave mode separation before the application of an imaging condition (Yan and Sava, 2008). Wave mode separation for isotropic media can be achieved by applying Helmholtz decomposition to the vector wavefields (Aki and Richards, 2002), which works for both homogeneous and heterogeneous isotropic media. Dellinger and Etgen (1990) extend the wave mode separation to homogeneous VTI (vertically transversely isotropic) media, and obtain P and SV modes by projecting the vector wavefields onto polarization vectors in the wavenumber domain. Yan and Sava (2009) apply this technology to heterogeneous VTI media and show that the method works if this projection is carried out in the space domain as spatial filtering, even for complex geology with large heterogeneity.

However, VTI models are only suitable for limited geological settings with horizontal layering. TTI (tilted transversely isotropic) models characterize more general geological settings like thrusts and fold belts. Many case studies have shown that TTI models are good representations of complex geology, e.g., the Canadian Foothills (Godfrey, 1991). Using the VTI assumption when imaging structures characterized by TTI anisotropy introduces image errors both kinematically and dynamically (Zhang and Zhang, 2008). Therefore, for successful application of elastic wave-equation migration, the wave mode separation algorithm needs to be adapted to TTI media.

In this abstract, we review the wave mode separation applicable in the 2D symmetry planes of TTI media, which is an extension of the space domain wave mode separation for VTI media of Yan and Sava (2009). We also show that wave mode separation can be carried out in the wavenumber domain followed by interpolation in the space domain. This procedure, which resembles the PSPI process from wave equation migration (Gazdag and Sguazzero, 1985), has the benefit of speed and accuracy.

Wave mode separation in the space domain

For plane waves propagating in a symmetry plane of a TTI medium, since qP and qSV modes are decoupled from the SH mode and polarized in the symmetry planes, we can set the crossline wavenumber $k_y = 0$ and write the Christoffel equation as (Aki and Richards, 2002; Tsvankin, 2005)

$$\begin{bmatrix} G_{11} - \rho V^2 & G_{12} \\ G_{12} & G_{22} - \rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_z \end{bmatrix} = 0 \,, \qquad (1)$$

where $\mathbf{G} = \{G_{ij}\}$ is the Christoffel matrix. The solutions to parameter V correspond to the eigenvalues of the matrix $\mathbf{G}$ and represent the phase velocities of different wave modes as functions of the wave vector $\mathbf{k}$. This equation also allows us to compute the polarization vectors $\mathbf{U}_P = \{U_x, U_z\}$ and $\mathbf{U}_{SV} = \{-U_z, U_x\}$ (the eigenvectors of the matrix) for P and SV wave modes, given the stiffness tensor at every location of the medium.

We can apply the procedure described here to heterogeneous media by computing the polarization vector components, namely U_x and U_z, at every grid point. In any symmetry plane of a TTI medium, the polarization depends on the local values of the stiffness coefficients. For each point, we pre-compute the polarization vectors as a function of the local medium parameters and transform them to the space domain to obtain the wave mode separators, L_x and L_z, which are generalizations of conventional finite difference derivatives. We assume that the medium parameters vary smoothly (locally homogeneous), but even for complex media, the localized operators work similarly to long finite difference operators used for finite difference modeling at locations where medium parameters change rapidly. Thus, wavefield separation in TI media can be achieved simply by non-stationary filtering with operators L_x and L_z.

Wave mode separation in the wavenumber domain

As discussed earlier, accurate wave mode separation for heterogeneous TI media requires non-stationary filtering with large operators in the space domain, which is computationally expensive. In this section, we show a method to separate the wavefields in the wavenumber domain at a much lower cost. The method consists of two steps: the wave modes are first separated for a number of reference models in the wavenumber domain; then the separated modes are interpolated in the space domain using the spatially variable anisotropy parameters.

In any symmetry plane of a TI model, the P- and S-wave separators in the wavenumber domain are represented by their polarization vector with components $\{U_x, U_z\}$. Using the P-mode polarization angle ν_P relative to the k_z axis we can write:

$$U_x = \sin(\nu_P)\,, \tag{2}$$
$$U_z = \cos(\nu_P)\,. \tag{3}$$

The angle ν_P depends on the anisotropy parameters ϵ and δ and on the tilt angle ν of the model, which can be approximated by the expression (Tsvankin, 2005)

$$\nu_P = \theta + D\left[\delta + 2\,(\epsilon-\delta)\sin^2(\theta-\nu)\right]\sin 2(\theta-\nu)\,. \tag{4}$$

This expression is valid for weakly anisotropic media, and $D = \frac{1}{2\left(1-V_{S0}^2/V_{P0}^2\right)}$. Here, V_{P0} and V_{S0} are P- and S-wave velocities along symmetry axis; the isotropic polarization angle θ is the angle between the phase vector $\mathbf{k}$ and the vertical axis. This equation demonstrates that the anisotropic polarization vector $(\mathbf{U})$ deviates from the isotropic polarization vector $(\mathbf{k})$ by an angle Δ with

$$\Delta = D\left[\delta + 2\,(\epsilon-\delta)\sin^2(\theta-\nu)\right]\sin 2(\theta-\nu)\,, \tag{5}$$

therefore, we can then write $\nu_P = \theta + \Delta\,(\epsilon,\delta,\nu)$. This equation is an approximation for weak anisotropy, and it tells us that the deviation angle is approximately linearly dependent on the anisotropy parameters ϵ and δ. We can also expand equations 2 and 3, assuming small Δ:

$$U_x = \sin(\theta+\Delta) = \sin\theta\cos\Delta + \cos\theta\sin\Delta \approx \sin\theta + \Delta\cos\theta = k_x + k_z\Delta\,, \tag{6}$$
$$U_z = \cos(\theta+\Delta) = \cos\theta\cos\Delta - \sin\theta\sin\Delta \approx \cos\theta + \Delta\sin\theta = k_z + k_x\Delta\,. \tag{7}$$

We can examine the dependence of Δ on ϵ and δ by fixing ν:

$$\Delta = a\,\epsilon + b\,\delta\,, \tag{8}$$

where a and b are functions of k_x and k_z (or functions of θ). If we substitute equation 8 into equations 6 and 7, we get

$$U_x(\epsilon,\delta) \approx A(k_x,k_z)\epsilon + B(k_x,k_z)\delta + C(k_x,k_z)\,, \tag{9}$$
$$U_z(\epsilon,\delta) \approx A'(k_x,k_z)\epsilon + B'(k_x,k_z)\delta + C'(k_x,k_z)\,. \tag{10}$$

Likewise, we can examine the dependence of Δ on ν by fixing ϵ and δ. Since the term in the brackets in equation 4 is small, we can write Δ approximately as

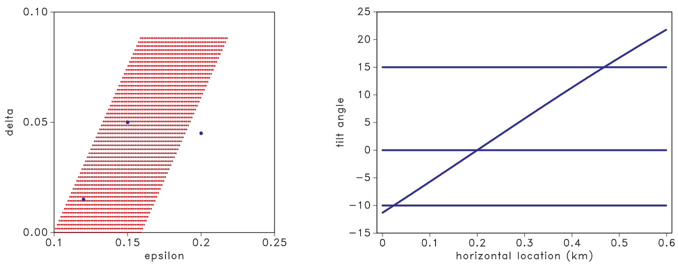
$$\Delta = c\sin 2(\theta-\nu) = c\sin(2\theta)\cos(2\nu) - c\cos(2\theta)\sin(2\nu). \tag{11}$$

If we substitute equation 11 into equations 6 and 7, we get

$$U_x(\nu) \approx E(k_x,k_z)\cos 2\nu + F(k_x,k_z)\sin 2\nu + G(k_x,k_z)\,, \tag{12}$$
$$U_z(\nu) \approx E'(k_x,k_z)\cos 2\nu + F'(k_x,k_z)\sin 2\nu + G'(k_x,k_z)\,. \tag{13}$$

Equations 9 and 10 and equations 12 and 13 tell us that the P-mode separators are approximately linear functions of ϵ and δ (with ν being constant) and linear functions of $\sin 2\nu$ and $\cos 2\nu$ (with ϵ and δ being constants).

Suppose that at a certain location in the model, the anisotropy parameters are ϵ and δ and the wavefields at this location can be separated with the polarization vector with components $U_x(\epsilon,\delta)$ and $U_z(\epsilon,\delta)$ (equations 9 and 10). Because of the linear dependence of $\{U_x, U_z\}$ on ϵ and δ, we can express $\{U_x, U_z\}$ as weighted sum of the operators at three different references. After some manipulation, P and S modes



(a) (b)

Figure 1: Schematics showing (a) cross plot of ϵ and δ. The parallelogram area corresponds to the model parameters ϵ and δ, and the three dots are the chosen references. (b) The tilted line corresponds to the tilts of the model. The three horizontal lines are the chosen ν references.

at a certain location characterized by anisotropy parameters (ϵ, δ) can also be expressed as weighted sum of P and S modes obtained with reference parameters:

$$P = w_1 P_1 + w_2 P_2 + w_3 P_3 \,, \tag{14}$$
$$S = w_1 S_1 + w_2 S_2 + w_3 S_3 \,. \tag{15}$$

The weights w_1, w_2, and w_3 are determined by the anisotropy parameters (or tilt) at every location and three reference pairs of anisotropy parameters (or cosine and sine values of the tilt). Equations 14 and 15 can be used for interpolation between ϵ and δ (Figure 1(a)), as well as for interpolation between $\cos 2\nu$ and $\sin 2\nu$ (Figure 1(b)). Note that these weights can have negative values when the anisotropy parameters falls outside the triangle formed by the three references in the model space (for example, see Figure 1(a)). It is important to ensure that the three reference points are not co-linear; otherwise the weights cannot be calculated.

We can do the wave-mode separation for heterogeneous TTI media in three steps:

(i) Find three triplets $\{\epsilon, \delta, \nu\}$ to use as reference models.

(ii) Perform separation at three reference pairs of (ϵ, δ) at a fixed tilt angle ν_1 in the wavenumber domain; then obtain the wavefields of ν_1 by interpolating (or extrapolating) wavefields among these reference pairs of (ϵ, δ) in the space domain.

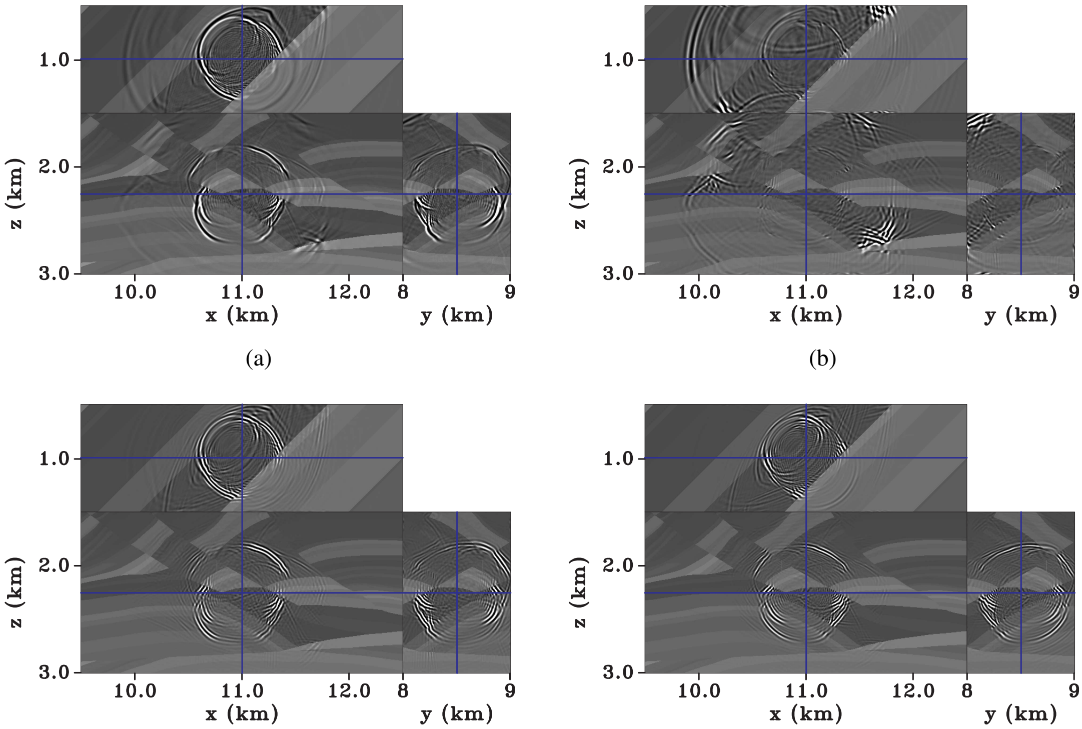
(iii) Repeat step (ii) for ν_2 and ν_3, and obtain the final wavefields by interpolating (or extrapolating) between the three references of ν.

Examples

We present the wave mode separation using a 3D version of the elastic Marmousi model. The vertical xz planes of velocities and anisotropy parameters are windowed from the 2D models. The parameters in the front face of the 3D model (Figure 2) are: V_{P0} is taken from the acoustic Marmousi velocity profile, the V_{P0}/V_{S0} ratio is a constant 2, the parameter ϵ ranges from 0.13 to 0.36, and the parameter δ ranges from 0.11 to 0.24. In the y direction, each vertical xz slice is shifted by one sample to make the next vertical slice. This makes the medium have constant azimuth angle (45°) of anisotropy symmetry axis. Figure 2(a) shows a snapshot of the z component of the elastic wavefields, and Figure 2(b)–(d) show separated P, SV, and SH modes by wavenumber domain separation followed by space domain interpolation. Here, P and SV modes are interpolated using nine references models of ϵ, δ, and ν. The interpolated separation (Figure 2(b)–(d)) indicates that the separation method in the k domain works efficiently for models with complicated geology.

Conclusions

We present a method for elastic wave mode separation applicable to complex media. In order for the operators to work for TTI models with non-zero tilt angles, we incorporate a parameter—the local tilt angle ν—in the calculation of polarization vectors, in addition to the parameters needed for the VTI operators.



(a) (b) (c) (d)

Figure 2: (a) A snapshot of the vertical component of the elastic wavefield for the 3D Marmousi II model. A displacement source located at the center of the model oriented at vector direction $\{1, 1, 1\}$ is used to simulate the wavefield. The separated (b) P, (c) SV, and (d) SH wave modes for the elastic wavefield are obtained by projection in the k domain followed by interpolation in the x domain.

Wave mode separation can be carried out by separation in the wavenumber domain at different references of the anisotropy parameters ϵ, δ, and tilt angle ν, followed by interpolation in the space domain between the wavefields separated at these references. The main advantage of applying the separation operators in the space domain constructed in this way is that they are accurate for heterogeneous media. The wave mode separation implemented in the wavenumber domain followed by interpolation in the space domain has the advantage that it is much more efficient that the straight-forward separation in the space domain. The wavenumber separation is especially beneficial for 3D models because the space domain separation becomes prohibitively expensive. For wavenumber domain separation, only three reference pairs of ϵ and δ are needed for VTI models, and nine references of ϵ, δ, and ν are necessary for TTI models.

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