Wavefield tomography without low frequency data

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Summary

Waveform inversion (FWI) requires a good starting model and/or data at very low frequency ($<$ 1Hz) for convergence. However, this is not a necessary condition, but an artifact of the objective function defined using differences of observed and simulated data. Image-domain tomographic methods using the same wavefields and wave-equations can converge to a reasonable solution from poor starting models and without long offset and/or low frequency data. Cascading image-domain and data-domain wavefield tomography eliminates the need for extremely low-frequency in the acquired data.

Introduction

In wavefield tomography (WT) (Tarantola, 1984; Pratt, 1999; Plessix, 2006; Symes, 2009), models are typically updated by matching simulated and recorded data. Not all observed data can be predicted by the assumed wave-equation, therefore significant effort is necessary to precondition the observed data before inversion. We assume that the WT objective function (OF) measuring the match between simulated and recorded data is convex, so it can be minimized using gradient-based techniques. The gradient is computed using the adjoint state method (Plessix, 2006; Symes, 2009). Given the source wavelet f_s and the observed data f_r, the WT state variables are obtained by solving a wave-equation for the source and receiver wavefields, u_s and u_s:

$$\begin{bmatrix} \mathcal{L}(\mathbf{x},\omega,m) & 0 \\ 0 & \mathcal{L}^*(\mathbf{x},\omega,m) \end{bmatrix} \begin{bmatrix} u_s(e,\mathbf{x},\omega) \\ u_r(e,\mathbf{x},\omega) \end{bmatrix} = \begin{bmatrix} f_s(e,\mathbf{x},\omega) \\ f_r(e,\mathbf{x},\omega) \end{bmatrix} . \tag{1}$$

$\mathcal{L}$ is the wave operator, m are the model parameters (e.g slowness squared), e is the experiment index, ω is the frequency, and $\mathbf{x}$ are space coordinates $\{x, y, z\}$. Different OFs can be used in the adjoint source calculation.

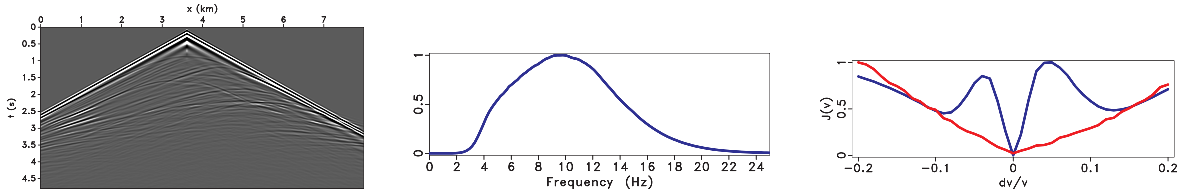
In the **data domain**, we formulate WT (dWT) as an inverse problem based on an OF defined using the difference between the source and receiver wavefields (Tarantola, 1984; Pratt, 1999): $\mathcal{J}_D = \sum_e \frac{1}{2} \| K_D (u_s(e,\mathbf{x},\omega) - u_r(e,\mathbf{x},\omega)) \|^2_{\mathbf{x},\omega}$. $K_D(e,\mathbf{x})$ is a mask operator selecting the wavefields at the receiver positions. This OF suffers from cycle skipping due to the oscillatory character of the subtracted wavefields, Figure 1(c). This problem is usually addressed by bootstrapping the frequency, thus requiring that low frequency data ($\sim$ 1Hz) are acquired in the field. If this condition is satisfied, we can define the adjoint source, $g_s(e,\mathbf{x},\omega)$, based on the source and receiver wavefields and compute the adjoint state variable, $a_s(e,\mathbf{x},\omega)$, using backward modeling,

$$g_s = K_D \overline{K_D} (u_s - u_r) \;,\; \mathcal{L}^*(\mathbf{x},\omega,m)\, a_s = g_s \;, \tag{2}$$

and then evaluate the OF gradient by correlating the state (u_s) and adjoint state (a_s) variables (Plessix, 2006): $\nabla_m \mathcal{J}_D(\mathbf{x}) = \sum_{e,\omega} \omega^2 (\overline{u_s} a_s)$.

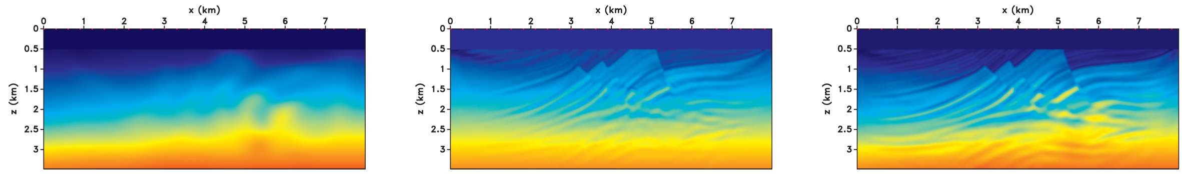
Similarly, in the **image domain** we formulate WT (iWT) using extended wave-equation imaging (Sava and Vasconcelos, 2011). The OF is $\mathcal{J}_I = \frac{1}{2} \| K_I P(\boldsymbol{\lambda},\tau)\, r(\mathbf{x},\boldsymbol{\lambda},\tau) \|^2_{\mathbf{x},\boldsymbol{\lambda},\tau}$, where $r(\mathbf{x},\boldsymbol{\lambda},\tau)$ are extended images $r = \sum_{e,\omega} \overline{T(\boldsymbol{\lambda})\; u_s(e,\mathbf{x},\omega)} T(\boldsymbol{\lambda})\; u_r(e,\mathbf{x},\omega)\, e^{2i\omega\tau}$, $T(\boldsymbol{\lambda})$ indicates space shift and $\boldsymbol{\lambda}$ and τ are cross-correlation lags. The mask $K_I(\mathbf{x})$ restricts the evaluation of the OF to some image locations, and $P(\boldsymbol{\lambda},\tau)$ is a penalty operator applied in the extended space (Symes, 2009). This OF does not suffer from the cycle-skipping problem, Figure 1(c). The source and receiver adjoint sources, $g_s(e,\mathbf{x},\omega)$ and $g_r(e,\mathbf{x},\omega)$, are used to simulate the adjoint state variables, $a_s(e,\mathbf{x},\omega)$ (backward) and $a_r(e,\mathbf{x},\omega)$ (forward) (Yang and Sava, 2011):

$$\begin{bmatrix} g_s \\ g_r \end{bmatrix} = \begin{bmatrix} K_I \overline{K_I} \sum_{\tau,\boldsymbol{\lambda}} \overline{T} \left(P \overline{P}\; r \right) T \overline{u_r} e^{-2i\omega\tau} \\ K_I \overline{K_I} \sum_{\tau,\boldsymbol{\lambda}} T \left(P \overline{P}\; r \right) \overline{T} u_s e^{-2i\omega\tau} \end{bmatrix} \;,\; \begin{bmatrix} \mathcal{L}^*(\mathbf{x},\omega,m) & 0 \\ 0 & \mathcal{L}(\mathbf{x},\omega,m) \end{bmatrix} \begin{bmatrix} a_s \\ a_r \end{bmatrix} = \begin{bmatrix} g_s \\ g_r \end{bmatrix} . \tag{3}$$



(a) (b) (c)

Figure 1: (a) A shot with short acquisition aperture (no diving waves) in the Marmousi model, (b) the data spectrum, and (c) the OF for dWT (blue) and iWT (red).



(a) (b) (c)

Figure 2: (a) iWT model starting from a $v(z)$ model, (b) dWT model starting from the same $v(z)$ model and (c) dWT model starting from the iWT model.

The OF gradient is the correlation of the state (u_s, u_r) and adjoint state (a_s, a_r) variables (Plessix, 2006): $\nabla_m \mathcal{J}_I(\mathbf{x}) = \sum_{e,\omega} \omega^2 (\overline{u_s} a_s + \overline{u_r} a_r)$.

Discussion

Both dWT and iWT use the same wavefields and wave operators, thus describing two forms of WT which differ essentially just in the definition of the OF. The image-domain OF is unimodal and allows convergence from a poor $v(z)$ starting model even if low frequencies (i.e. $<\sim$ 1Hz) are not present in the data, Figure 2(a). The data-domain OF is not unimodal and leads to less accurate results, Figure 2(b), if the starting model is inaccurate. However, this OF is more abrupt near convergence, thus producing higher resolution results, Figure 2(c), if the starting model is the one constructed using the image-domain OF, Figure 2(a). Therefore, dWT and iWT complement each-other and can lead to good model updates with inaccurate starting models, and with data recorded without low frequency, Figure 1(b), and with short offsets, Figure 1(a).

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