Introduction

Wavefield tomography (including waveform inversion) is the method of choice for retrieving high resolution velocity models. A key advantage of these methods is that wavefields are consistent with the bandwidth of seismic data. This allows the inversion to access model wavenumbers that are constrained by the data. We can exploit the data directly, or we can remap them into extended images characterizing reflectors in the subsurface. Image defocusing provides information for tomography; we can access this information by the application of penalty functions to migrated images and then redistributing this information in the model using bandlimited wavefields. We show different ways of extracting focusing information from seismic images with either (differential) semblance or illumination penalty functions, and demonstrate that illumination plays a crucial role for robust inversion.

Method

Wavefield tomography for both data- and image-domains makes use of seismic wavefields reconstructed by solving a given wave equation:

$$\begin{bmatrix} \mathbf{L}(m,\mathbf{x},t) & 0 \\ 0 & \mathbf{L}^\top(m,\mathbf{x},t) \end{bmatrix} \begin{bmatrix} u_s(\mathbf{x},t) \\ u_r(\mathbf{x},t) \end{bmatrix} = \begin{bmatrix} f_s(\mathbf{x}_s,t) \\ f_r(\mathbf{x}_r,t) \end{bmatrix} . \tag{1}$$

Here u_s and u_r are the source and receiver wavefields depending on space and time $\{\mathbf{x},t\}$, and $\mathbf{L}$ and $\mathbf{L}^\top$ are the forward and adjoint wave extrapolators, respectively. Using the reconstructed wavefields, we can construct extended images (Sava and Vasconcelos, 2011) as a function of extension parameters $\{\lambda,\tau\}$ (space- and time-lags):

$$R(\mathbf{x},\lambda,\tau) = \sum_{e,t} u_s(e,\mathbf{x}+\lambda,t+\tau)\, u_r(e,\mathbf{x}-\lambda,t-\tau) . \tag{2}$$

The extended images are multidimensional and can be expensive to store, which is why they are often subsamples at sparse locations inside the image. One could extract the location at sparse surface positions locations, i.e. construct common image gathers (CIGs), although this introduces aliasing due to the coarse surface sampling. An alternative is to extract the information along sparse locations in the image, i.e. construct common image-point gathers (CIPs). These most meaningful locations are at reflector position (Cullison and Sava, 2011), and therefore represent a better sampling of the model.

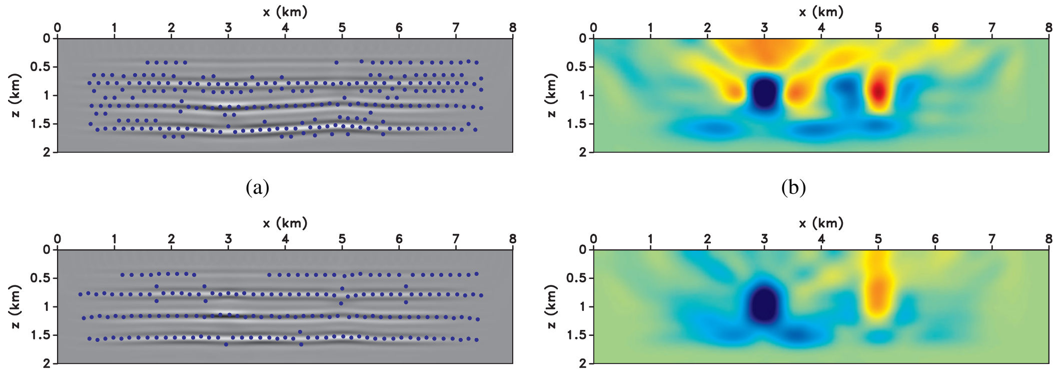
The extended image has maximum similarity at the image points if the velocity model if correct. Thus, we can design an optimization problem that minimizes the energy outside $\{\lambda=\mathbf{0},\tau=0\}$:

$$J(m) = ||P(\lambda,\tau)\,R(\mathbf{x},\lambda,\tau)|| . \tag{3}$$

The penalty operator $P_D(\lambda,\tau) = \sqrt{|\lambda|^2+(v\tau)^2}$ (Yang and Sava, 2015) highlights the defocused energy. Here, $v(\mathbf{x})$ is the local velocity at specific CIP locations. This penalty operator is isotropic in the λ,τ space and does not capture the truncations due to illumination (Yang et al., 2013) and due to physical constraints as shown by (Thomson, 2012). Alternatively, we can use a penalty operator that captures both aspects of the extended reflection response at every image point. We can simulate the response of realistic acquisition by demigrating a reference seismic image, and then migrating the obtained synthetic data (Yang et al., 2013). Mathematically, the penalty operator is computed using the image envelope (Env) as

$$P_I(\lambda,\tau) = \left[Env\left(\mathbf{M}\mathbf{M}^\top R_{ref}(\lambda,\tau)\right)\right]^{-1} \tag{4}$$

The demigration operator $\mathbf{M}^\top$ is applied to a reference image R_{ref} to produce synthetic data, which is then migrated back using operator $\mathbf{M}$ to compute the extended images at the common image point locations; the application of the envelope captures the energy of the idealized gathers. The reference image contains impulses at all CIP locations, hence, the process captures the point spread functions for



(a) (b) (c) (d)

the given imaging system (including the background model and the acquisition geometry). This penalty involves a demigration and migration of extra synthetic data at each iteration; however, this simulation and migration can be performed on a coarser grid which makes it much faster than a normal migration iteration. The objective function in equation 3 is minimized using gradient based optimization. To compute the gradient of this constrained objective function, we use the adjoint state method (Plessix, 2006).

Example

We demonstrate the method using a model with 4 horizontal reflectors (inserted in the density model) and a velocity model with two Gaussian anomalies of opposite sign. For inversion, we start from a constant background velocity with the purpose of recovering the Gaussian anomalies. Figure 1(a) shows the retrieved image using the penalty operator from (Yang and Sava, 2015), and Figure 1(b) shows the corresponding inverted model. Similarly, Figure 1(c) depicts the image from the illumination penalty framework described above, and Figure 1(d) shows the corresponding inverted model. We can observe that the illumination based inversion recovers the anomalies much better than the conventional penalty function, and that the retrieved models contain the correct sign and fewer oscillatory artifacts. Also, the image from the illumination framework is flatter in the area where the anomalies are located. This demonstrates that accounting for illumination is crucial for robust wavefield tomography.

Acknowledgments

We thank the sponsors of the Center for Wave Phenomena at Colorado School of Mines and the developers of the Madagascar open-source package (http://www.ahay.org).

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