Introduction

In recent times, warping has received a lot of attention in the geophysical community as a robust tool for estimating similarity between signals in general and residuals for tomography in particular. In the context of tomography and waveform inversion, the problem of estimating model parameters (e.g., wave propagation velocities) can be formulated in both data and migrated domains. Warping allows one to define a more robust match between modelled and recorded data, or between migrated images indexed by different extension parameters (shots, offsets, etc.).

Warping involves the estimation of apparent displacements between the events in a given pair of signals. Independently on the actual implementation, warping is based on the assumption that the two signals are spatially warped versions of each other and thus contain the same number of events. The basic idea that makes warping attractive for tomography is that the estimated displacements would ideally define a one-to-one correspondence between recorded and modelled signals (in the data domain) and between reflectors in different indexed images, thus reducing the so-called cycle-skipping problem that affects matching oscillatory signals when the energy of the difference of the two signals is used as distance measure (e.g., seismic reflections in both data and migrated domain).

In the data domain, cycle-skipping appears when a given event in modelled data is shifted more than half a cycle with respect to the corresponding event in the observed data, thus making direct differencing of the waveforms an invalid matching measure; warping proved to address it effectively (Ma and Hale, 2013). The problem is also well-defined and clearly stated: the recorded data are the ground truth and we want to construct the best match between the observations and the output of our modelling for a given input model.

In the image domain, the problem is slightly different since there is no reference to match, instead a metric must be defined from mathematical/physical considerations. The problem is two-fold: how to compute the residual and thus the objective function, and in what domain we do it. For example, Differential Semblance, or DSO, (the energy of the first derivative of the image along the extension axis) gives us a more robust objective function than the stack power when we start from an inaccurate velocity model; nonetheless, we can implement DSO in different domains (shot, offset, incident angle, etc.) and they are not equivalent. In image domain tomography, image-warping represents a valid option to make the DSO objective function more robust against cycle-skipping but it does not specify in which domain the residuals should be measured (Xu et al.,2001; Stolk and Symes, 2004).

In this abstract, I describe and discuss the main issues related to the application of image-warping to the estimations of errors in the velocity model from migrated images.

Theory

Image-warping estimates the apparent shift field between two input signals. The shift estimation can be implemented in several different fashions (Hale, 2009; Hale, 2013; Baek et al., 2014). (Hale, 2008) use local correlations to measure the apparent displacement between 4D time-lapse images. The peak of the local correlations indicate the relative shift between two images at a given particular location; by penalizing the correlation lags in the direction normal to the imaged structure one can obtain a proxy for displacement and define the optimisation problem (Perrone et al., 2015)

$$J = \left\| \mathcal{P}\left(\mathbf{x}, \boldsymbol{\lambda}\right) c(\mathbf{x}, \boldsymbol{\lambda}) \right\|^2, \tag{1}$$

where $\mathcal{P}\left(\mathbf{x}, \boldsymbol{\lambda}\right)$ is an space-variant operator that penalises the correlation lags $\boldsymbol{\lambda}$ in the direction normal to the local imaged slope, and $c(\mathbf{x}, \boldsymbol{\lambda})$ are the local correlations at every point in space.

A more direct inversion of estimated displacements can be implemented by recasting the Differential Semblance residual in the shot domain using the displacement vector fields that matches images from nearby shots (Perrone and Sava, 2015):

$$R_i(\mathbf{x}) - R_j(\mathbf{x}) \approx \nabla R_i(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x}), \tag{2}$$

where $R_i(\mathbf{x})$ represents the migrated image of the i-th shot, $\nabla R_i(\mathbf{x})$ is the spatial gradient of the migrated image, and $\mathbf{u}(\mathbf{x})$ is the vector displacement field that maps the images for shot *i* and *j*. We can use the estimated displacements to construct an image perturbation (approximation of the image difference) that does not suffer from cycle-skipping since no difference is actually taken and only one image enters the construction of the residual (Perrone and Sava, 2015):

$$R_{i-1}(\mathbf{x}) - 2R_i(\mathbf{x}) + R_{i+1}(\mathbf{x}) \approx \nabla R_i(\mathbf{x}) \cdot (\mathbf{u}_i^+ + \mathbf{u}_i^-) \tag{3}$$

where the left hand side represents the standard DSO image perturbation in the shot domain (i.e. the derivative of the objective function with respect to the migrated image) and the right-hand side is the image-warping approximation; the superscripts + and - indicate the displacements are estimated forward and backward with respect to the reference image *i*. Notice that $\nabla R_i(\mathbf{x})$ comes from the linearization of the warping relationship between the two images, and its computation is independent of the extension parameter used (shot, offset, angle, etc.).

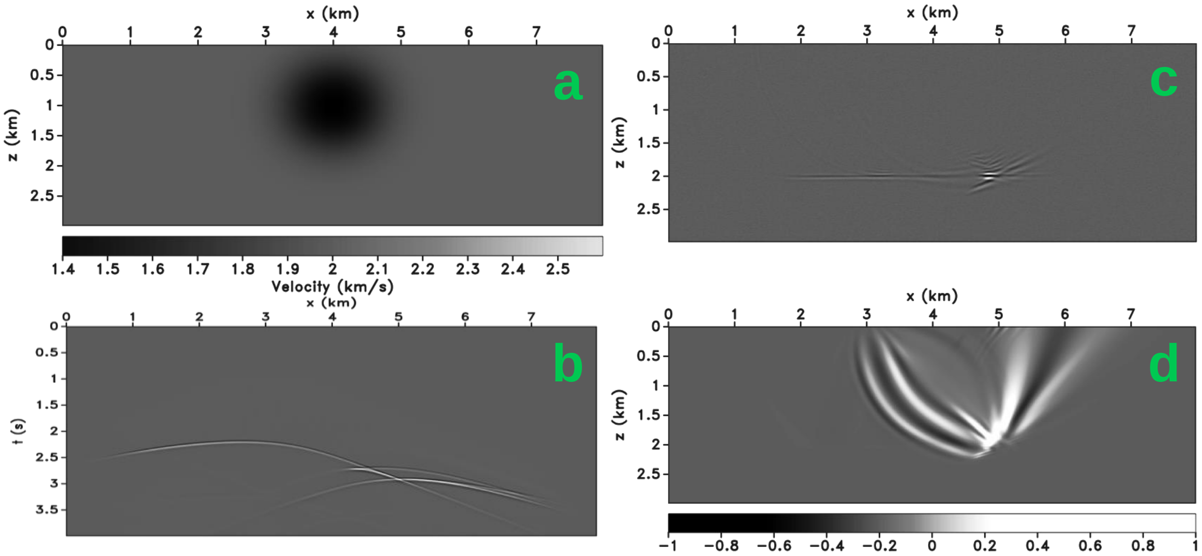

***Figure 1** Migration with a highly refractive model. The velocity model has a strong Gaussian-shaped lens (a) and the reflector is a horizontal density interface at 2km (not shown). Notice the wavefront triplications in the data (b). Migration of a single shot with the correct model gives the image in (c). Notice the spurious artefacts at x=5km. Image-warping tomography results are strongly affected by the migration artefacts (d).*

Examples

Image-domain tomography using the displacement field estimated with local image correlations has been applied on real data with limited geologic structural complexity (Perrone et al., 2015); however, it can be shown to be quite prone to the bias due to the particular construction of the residual and the differences in illumination of the different shots (Perrone and Sava, 2015). The same idea can be applied on images obtained from different surveys for the estimation of the time-lapse anomaly (Perrone and Sava, 2013). Di et al. (2013) also presented a similar approach based on dynamic image warping, which removes some of the weaknesses of local correlations (Hale, 2013).

The migrated-shot domain is strongly affected by kinematic artefacts if the velocity model is highly refractive and presents caustics (Stolk and Symes, 2004). In this case, image-warping cannot distinguish between signal and migration noise and the inversion results bias by the artefacts in the migrated images (Fig. 1). Rather than a failure of the measure of similarity, it is the domain in which the accuracy of the model is assessed that breaks the tomographic inversion.

The synthetic Marmousi model in Fig 2 shows that image-warping is much less sensitive to cycle-skipping than the direct difference that implements Differential Semblance, and Fig 2a and 2b show the difference of the gradients for an objective function based on the direct difference of migrated shot-image and on image-warping of images from nearby shots, respectively. Image warping is able to correct gross errors in the background velocity model, which are evident in the bulk shifts of the reflectors in the initial model (Fig 2c). However, tomography converges to a low-resolution model, and the final image is properly focused only in the regions where waves do not experience strong refractions and triplications (Fig 2d).

We could implement warping between images migrated in a more suitable domain, using an imaging algorithm that is not affected by kinematic artefacts (e.g., incident/reflection angle computed after wavefield focusing) (Stolk and Symes, 2004). However, in order to implement image-domain tomography we need to be able to define and construct the adjoint operator for the sequence of migration and warping (e.g., using the concept of connective function as proposed by Luo and Schuster (1991) or the adjoint state method as in Ma and Hale (2013)) from the domain of interest, which may not be straightforward or practical especially in the case of imaging algorithms based on Reverse-Time Migration and domains such as common-reflection angle. On the other hand, the time-lapse problem, i.e. image-warping applied to time-lapsed migrated images from different surveys, can represent a suitable application for the inversion of estimated displacements since in the migration kinematic artefacts that can contaminate single-shot (or single-offset) images cancel out in the full stack image, thus removing possible sources of noise for image warping.

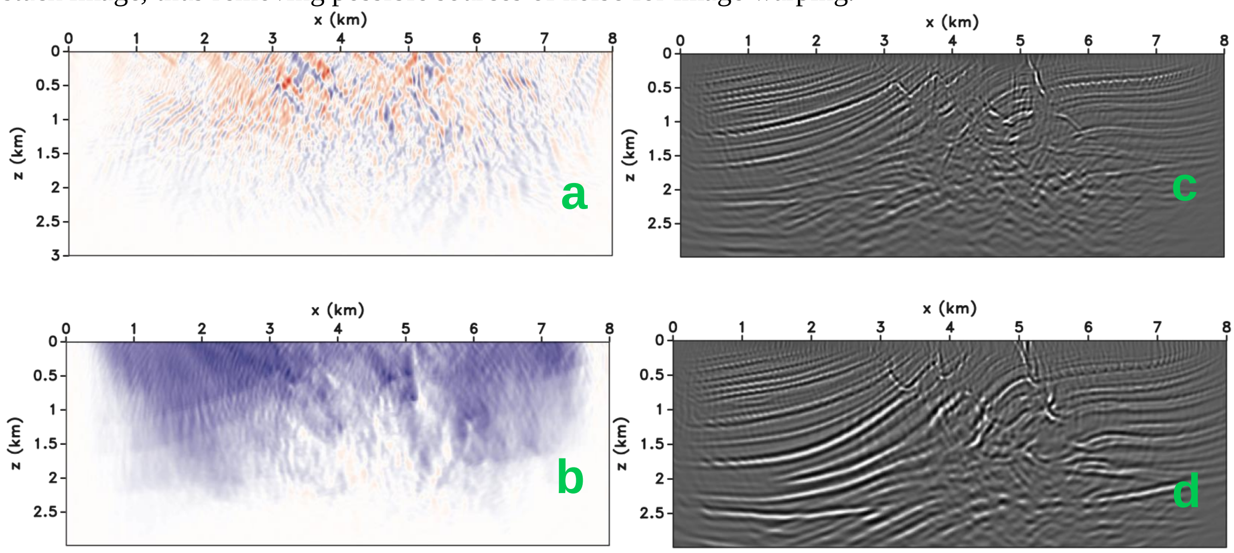

Figure 2 *Marmousi model: first gradient for image domain tomography computed for the image-difference objective function in the shot domain (a) and for the image-warping objective function (b). Migrated images using the initial model (c) and the final model obtained from image-warping tomography (d).*

Conclusions

Image-warping is effective in reducing cycle-skipping in the Differential Semblance functional in certain domains, e.g. the migrated shot domain or the full stacked migrated domain. However, the kinematic artefacts introduced in the migrated images bias the measure of similarity and the computation of the gradient, and thus they hinder the inversion. For wavefield-based tomography algorithms, the choice of the domain is also partly constrained by the ability to perform demigration using the same modelling scheme used for migration.

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