Elastic wave-mode separation for VTI media

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The separation of wave modes from isotropic elastic wavefields is typically done using Helmholtz decomposition. However, Helmholtz decomposition using conventional divergence ($\nabla \cdot$) and curl ($\nabla \times$) operators in anisotropic media does not give satisfactory results and leaves the different wave modes only partially separated. The separation of anisotropic wavefields requires the use of more sophisticated operators which depend on local material parameters.

Dellinger and Etgen (1990) suggest separating wave modes in anisotropic media by projecting the wavefields $\mathbf{W}$ onto the directions $\mathbf{U}$ in which the P and S modes are polarized:

$$qP = i\,\mathbf{U}(\mathbf{k}) \cdot \widetilde{\mathbf{W}} = i\,U_x\,\widetilde{W}_x + i\,U_y\,\widetilde{W}_y + i\,U_z\,\widetilde{W}_z\,. \tag{1}$$

In anisotropic media, polarization vectors $\mathbf{U}(k_x, k_y, k_z)$ are different from the wave vector $\mathbf{k}$, and generally are not radial because qP waves in an anisotropic medium are not polarized in the same directions as wave vectors, except in the symmetry planes ($k_z = 0$) and along the symmetry axis ($k_x = 0$).

We use the same idea for wave-mode separation, but implement the operator ion the space-domain as opposed to the wavenumber-domain. The equivalent expression to equation 1 in the space domain is:

$$qP = \nabla_a \cdot \mathbf{W} = L_x * W_x + L_y * W_y + L_z * W_z\,, \tag{2}$$

where L_x, L_y and L_z represent the inverse Fourier transforms of $i\,U_x$, $i\,U_y$ and $i\,U_z$, and $*$ represents spatial filtering. L_x, L_y and L_z define the pseudo derivative operators in the x, y and z directions for an anisotropic medium.. We obtain the polarization vectors $\mathbf{U}(\mathbf{k})$ by solving the Christoffel equation (Tsvankin, 2005). The polarization vectors are the eigenvectors of the equation, and in VTI media, the large and small eigenvectors correspond to qP and qS modes, respectively.

The procedure described here can be applied to heterogeneous media by computing a different operator at every grid point. In the symmetry planes of VTI media, the operators are 2D and depend on the local values of the stiffness coefficients. For each point, we pre-compute the polarization vectors as a function of the local medium parameters, and transform them to the space domain to obtain the wave mode separators. If we represent the stiffness coefficients using Thomsen parameters (Thomsen, 1986), then the pseudo derivative operators L_x and L_z depend on ϵ, δ and V_{P0}/V_{S0}, which are in general spatially dependent parameters. We can compute and store the operators for each grid point in the medium, and then use those operators to separate P and S modes from reconstructed elastic wavefields. Thus, wavefield separation in VTI media can be achieved simply by non-stationary filtering with operators L_x and L_z.

We illustrate the technique with a synthetic model which is an elastic anisotropic version of the Sigsbee 2A model (Paffenholz et al., 2002). In this modified model, the P wave velocity is taken from the original model, V_{P0}/V_{S0} ranges from 1.5 to 2, the parameter ϵ ranges from 0 to 0.48 and the parameter δ ranges 0 from to 0.10. The model is isotropic in the salt and the top part of the model, and is transversely isotropic under salt. A vertical point force source is located in the middle of the model to simulate the elastic anisotropic wavefield.

Figure 1(a) shows one snapshot of the modeled elastic anisotropic wavefields using this model. Figure 1(b) illustrates the separation of the anisotropic elastic wavefields using the $\nabla \cdot$ and

$\nabla \times$ operators, and Figure 1(c) illustrates the separation using our pseudo derivative operators. Figure 1(b) shows the residual of unseparated P and S wave modes, such as the P-wave energy envelope in the qP panel and under-salt in the qS panel. In contrast, Figure 1(c) shows the qP and qS modes completely separated, demonstrating the effectiveness of the anisotropic pseudo derivative operators constructed using the local medium parameters.

To summarize, we present a method of obtaining spatially-varying pseudo derivative operators with application to wave mode separation in anisotropic media. The main idea is to utilize polarization vectors constructed in the wavenumber domain using the local media parameters at specific locations and then transform these vectors back to the space domain. The main advantage of applying the pseudo derivative operators in the space domain is that they can be used for heterogeneous media. The wave mode separators obtained using the method described in this abstract are spatially-variable filtering operators and can be used to separate wavefields in VTI media with arbitrary degree of anisotropy. This methodology is applicable for elastic RTM in heterogeneous anisotropic media.

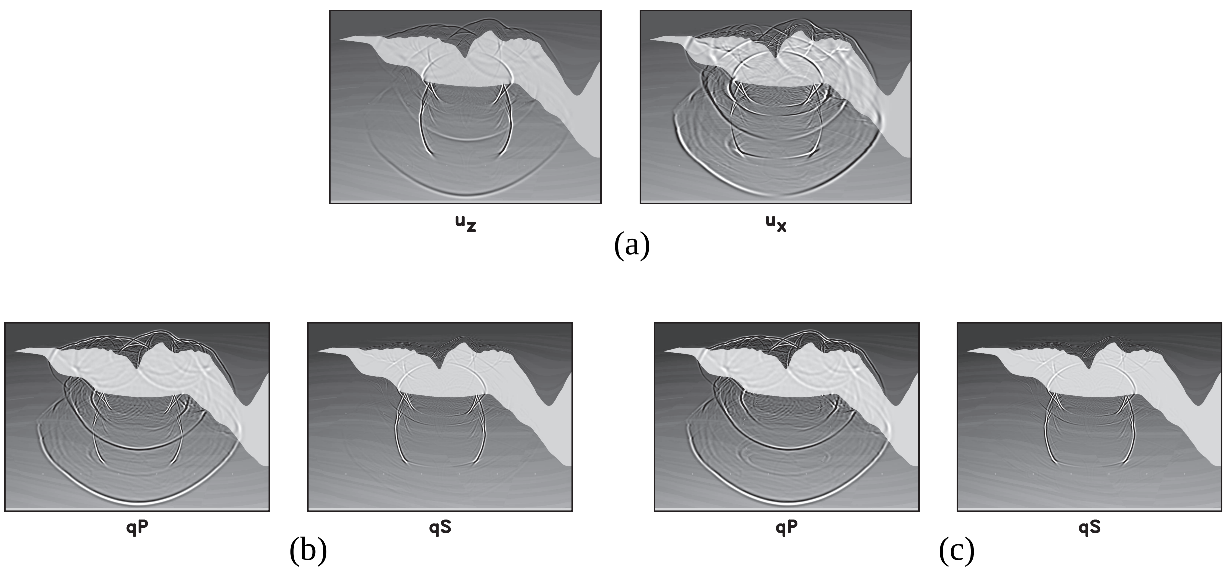

Figure 1: (a) Anisotropic wavefield modeled with a vertical point force source in the middle of the model. Anisotropic qP and qS modes separated using (b) $\nabla \cdot$ and $\nabla \times$ and (c) pseudo derivative operators for the vertical and horizontal components of the elastic wavefields shown in (a).

References

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