Amplitude-preserved common image gathers by wave-equation migration

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SUMMARY

We present two methods to compute angle-domain common image gathers (ADCIG) by downward-continuation migration, and we analyze their amplitude response versus reflection angle (AVA). A straightforward implementation of the two methods leads to contradictory, and thus obviously inaccurate, amplitude responses. The amplitude problem is related to the fact that downward continuation migration is the adjoint of upward-continuation modeling, but it is a poor approximation of its inverse. We derive the weighting operators, diagonal in the frequency-wavenumber domain, that makes migration a good approximation to the inverse of modeling. After weighting, the ADCIGs computed by the two methods become consistent. Other important factors degrading the accuracy of AVA in practical situation are the limited sampling and offset range, and the bandlimited nature of seismic data.

INTRODUCTION

Traditionally, migration velocity analysis and AVO employ offset-domain common-image gathers, since most of the relevant information is not described by the zero-offset images. However, it is difficult to produce these gathers with wave-equation migration because the offset dimension of the downward continued data shrinks with depth. A solution to this problem is to use angle-gathers instead of offset-gathers. angle-domain common image gathers (ADCIG) obtained by wave-equation migration are very powerful at measuring the accuracy of the velocity model. However, a still unsolved issue for such angle gathers is that of amplitude variation as a function incidence angle. ADCIGs are also attractive because they provide more straightforward information for amplitude analysis, that is, amplitude versus angle (AVA) instead of the more common amplitude versus offset (AVO) analysis.

Angle-domain common-image gathers are representations of the seismic images sorted by the incidence angle at the reflection point. Angle-gathers can be obtained using wave-equation techniques either for shot-profile migration, as described by de Bruin et al. (1990), or for shot-geophone migration, as described by Prucha et al. (1999). In both cases, angle-gathers are evaluated using slant-stacks on the downward continued wavefield, prior to imaging. Decomposing the downward continued wavefield before imaging produces angle-gathers as a function of offset ray-parameter instead of the true reflection angle. Angle-domain gathers can also be computed by slant stacking the image, instead of the downward continued wavefield. We show that this alternative procedure directly produces angle gathers as a function of the reflection angle. In both cases the slant stack transformation can be easily performed by a radial-trace transform in the frequency-wavenumber domain (Ottolini, 1982).

For AVA analysis it is important that the procedure used for computing ADCIGs preserves the amplitude of the reflections as a function of angle. It is thus puzzling that straightforward implementations of both methods for computing ADCIGs produce contradictory amplitudes, and that for neither method the migration is a good approximation of the corresponding upward-continuation modeling. To solve the puzzle we must take into account the weighting function that is introduced in the migration process by the imaging step. This weighting is well approximated by a diagonal operator in the frequency-wavenumber domain. Since the two methods for computing ADCIGs perform a slant stack at different stages (before imaging for one and after imaging for the other) the corresponding weighting functions are different. Once the weights are taken into account, the AVA responses produced by the two methods are consistent, and migration can be made an approximate inverse of modeling. According to the physical model for reflection data, the weights can be set to make migration a good approximation of a linearized inversion. We adopt the physical model proposed by Stolt and Benson (1986), and define the appropriate weights for both methods used to compute ADCIGs. If needed for an iterative estimation procedure, modeling and migration can also be easily made pseudo-unitary, by applying the square root of the weights during both modeling and migration.

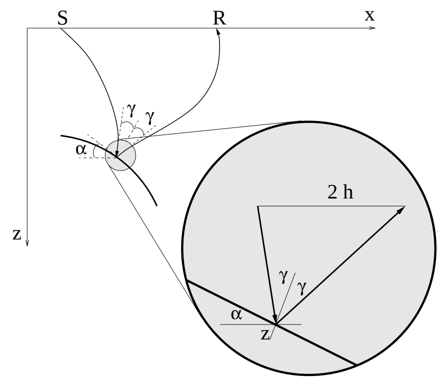

Figure 1: A scheme of reflection rays in an arbitrary-velocity medium.

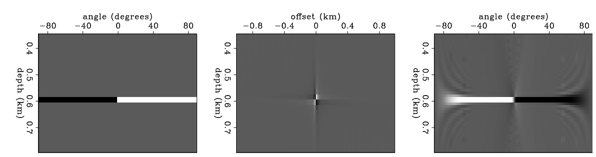

Figure 2: Synthetic example of conversion between the angle and offset domains in the image space. Left panel: synthetic angle gather. Middle panel: conversion from angle to offset. Right panel: conversion back to the angle domain.

TWO METHODS TO COMPUTE ADCIGS

Angle-gathers can be conveniently formed in the frequency domain. If we consider that in constant velocity media t is the traveltime from the source to the reflector and back to the receiver at the surface, $2h$ is the offset between the source and the receiver, z is the depth of the reflection point, α is the geologic dip, and γ is the reflection angle (Figure 1), we can write

$$\frac{\partial t}{\partial h} = \frac{2\cos\alpha\sin\gamma}{v} \tag{1}$$

and

$$-\frac{\partial t}{\partial z} = \frac{2\cos\alpha\cos\gamma}{v}. \tag{2}$$

From Equations (1) and (2) we obtain that

$$\tan\gamma = -\left.\frac{\partial z}{\partial h}\right|_{t,x}. \tag{3}$$

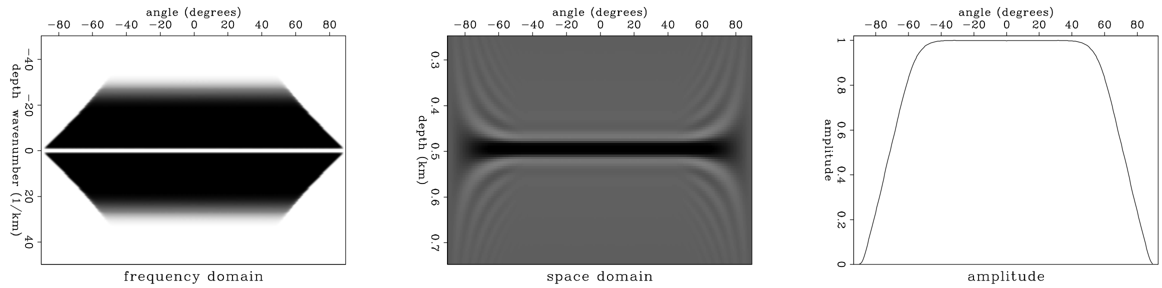

Figure 3: Angle gather created from an ideal offset gather.

Equation (3) is derived in constant velocity media, but it remains perfectly valid in a differential sense in any arbitrary velocity media if we consider h to be the effective offset at the reflector depth and not the surface offset (Figure 1).

In the frequency-wavenumber domain, formula (3) takes the trivial form

$$\tan \gamma = -\frac{k_h}{k_z}. \quad (4)$$

This equation indicates that angle-gathers can be conveniently formed with the help of frequency-domain migration algorithms (Stolt, 1978). Furthermore, wave-equation migration is ideally suited to compute angle-gathers using such a method, since the migration output is precisely described by the offset at the reflector depth, which is a model parameter, and not by the surface offset, which is a data parameter (Biondi, 1999).

We can recognize that Equation (1) describes nothing else but the ray parameter of the propagating wave at the incidence with the reflector. Using the definition

$$p_h = \frac{\partial t}{\partial h} \quad (5)$$

it follows that we can write a relation similar to (4) to evaluate the offset ray parameter in the Fourier-domain:

$$p_h = \frac{k_h}{\omega}. \quad (6)$$

Both Equations (4) and (6) can be used to compute image gathers through radial trace transforms in the Fourier domain. The major difference is that Equation (4) operates in the space of the migrated image, while Equation (6) operates in the data space.

The two methods are also different in three other ways:

1. Firstly, the image-space method is completely decoupled from migration, therefore conversion to reflection angle can be thought of as a post-processing after migration. Such post processing is interesting because it allows conversion from the angle domain back to the offset domain without re-migration (Figure 2), which is, of course, not true for the data-space method, where the transformation is a function of the data frequency.

2. Secondly, from Equation (1), it follows that offset ray parameter (p_h) is also a function of the structural dip (α), which is not true for the reflection angle (γ) estimated in the image space. The angles we obtain using (4) are geometrical measures, completely independent on the structural dip. For AVA purposes, it is also very convenient to have the amplitudes as a function of angle and not offset or offset ray parameter.

3. Thirdly, both methods require accurate knowledge of the imaging velocity, either explicitly, in the case of p_h, or implicitly, in the case of γ. The difference is that, inaccuracies in the velocity model at depths lower than that of the image point do not influence the accuracy of the angle gathers in the case of the data space method, but they can influence the quality of the gathers for the image space method.

AMPLITUDE VS. ANGLE

Figure 2 shows a synthetic example of conversion from the angle domain to offset domain and back. The original model is characterized by a polarity reversal at zero incidence angle. After conversion to offset and back to angle, the image gather displays the same polarity reversal within numerical precision. The obvious question that remains to be addressed is how reliable is the amplitude vs. angle estimate for images obtained using wave-equation migration.

Figure 3 shows a simple synthetic model on which we can observe the transformations that occur at different stages. If we take an ideal offset gather and convert it to the angle domain, we obtain the image depicted in Figure 3. For reference, the left panel contains the representation of the gather in the frequency domain. As expected, the amplitude response of the angle-gather is flat for a wide range of angles, after which it strongly decays.

The explanation for the amplitude decay at large angles arises after a simple analysis of the operations we perform in the Fourier domain. Equation (4) indicates that the conversion from offset to angle is a radial trace transform (RTT) on the depth and offset wavenumbers k_z and k_h. The offset gathers have limited spatial sampling, therefore they also have finite spatial bandwidth. The offset gather is transformed to the Fourier domain, and after RTT we obtain the left panel of Figure 3. The limits on the spatial bandwidth also limit the angle range on which we can reliably compute the RTT (Figure 8). This is limitation is given by the acquisition and processing parameters, and there is not much we can do to alleviate the situation. We can, however, have a measure of the angular range on which the amplitude values are reliable.

Figures 4 and 6 show the angle gathers computed from an image obtained by modeling the original synthetic image (Figure 3), and migrating the resulted data. Not surprisingly, since we have used the same velocity for both modeling and migration, the angle gathers are perfectly flat. However, the amplitude response is far from what we expect: both angle gathers, regardless of the method used to compute them, do not show constant amplitudes, not even for the inner angles. The explanation is that we applied a succession of the forward (modeling) and adjoint (migration) of a linear but not unitary operator **L**. To make **L** unitary we need to take into account the weights introduced by the imaging step during migration. In the frequency-wavenumber domain, these weights are well approximated by a diagonal operator and they are computed by evaluating

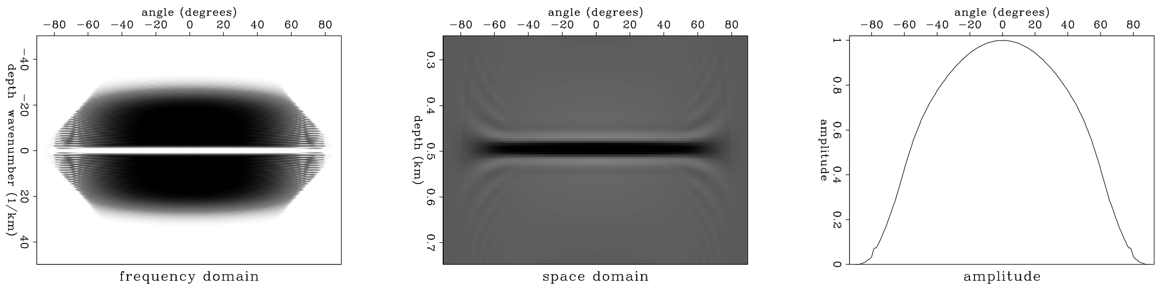

Figure 4: Angle gather computed in the image space. The amplitudes are distorted by the non-unitary nature of the migration operator.

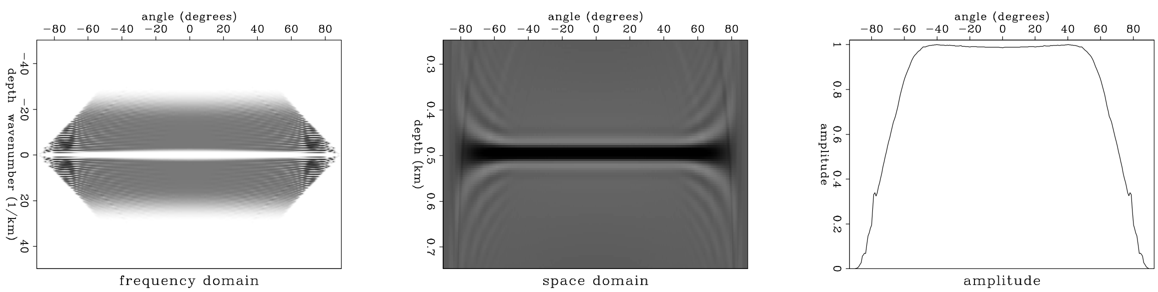

Figure 5: Angle gather computed in the image space. The weighting factors restore correct amplitudes.

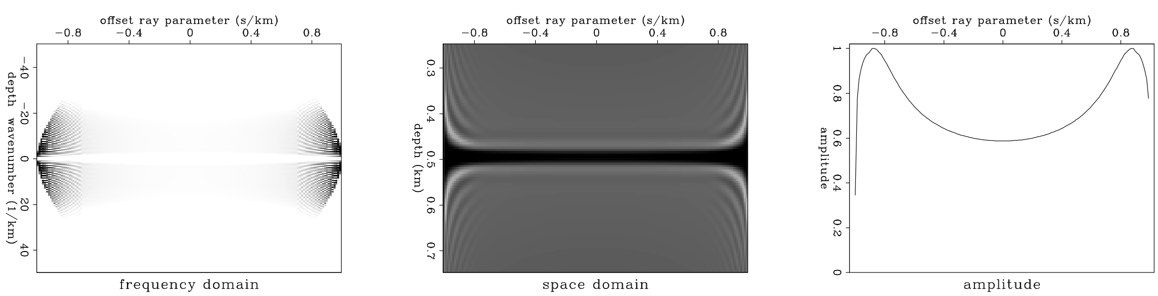

Figure 6: Angle gather computed in the data space. The amplitudes are distorted by the non-unitary nature of the migration operator.

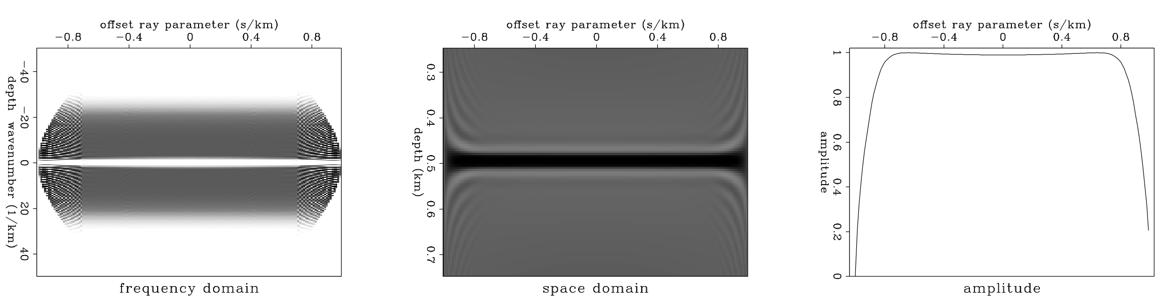

Figure 7: Angle gather computed in the data space. The weighting factors restore correct amplitudes.

the Jacobian of the transformation from the temporal frequency ω to the vertical wavenumber k_z, that is $d\omega/dk_z$.

The Jacobian is different for the two methods of computing ADCIGs because the imaging is performed at constant offset ray parameter p_h in the first case, and at constant offset wavenumber k_h in the other case. In one case the weighting function W_p is equal to

$$W_p^{-1} = \left.\frac{dk_z}{d\omega}\right|_{p_h=const} = \frac{k_{zs}\left[\omega s^2 - \frac{(k_m+p_h\omega)p_h}{4}\right] + k_{zr}\left[\omega s^2 + \frac{(k_m-p_h\omega)p_h}{4}\right]}{k_{zs}k_{zr}} \quad (7)$$

and in the other case W_h is equal to

$$W_h^{-1} = \left.\frac{dk_z}{d\omega}\right|_{k_h=const} = \frac{\omega s^2\,(k_{zs}+k_{zr})}{k_{zs}k_{zr}}, \quad (8)$$

where s is the local slowness and k_{zs} and k_{zr} are respectively the vertical wavenumber for the source component and the receiver component, and k_m is the midpoint wavenumber. As discussed above, in case of variable velocity media, these quantities are evaluated at the reflector location. For both ADCIG methods we have that $W\mathbf{L}' \approx \mathbf{L}^{-1}$. Figures 5 and 7 show the restored amplitudes for the case of our simple synthetic example.

"True amplitude" migration weights can be easily derived from equations (7) and (8) by spliting the weighting factor of modeling. If we adopt the definitions introduced by Stolt and Benson (1986), we can multiply W_p and W_h by $4k_{zs}k_{zr}/(i\omega s^2)$. For example, W_h becomes

$$\widehat{W_h}^{-1} = \frac{4}{i}\,(k_{zs}+k_{zr}). \quad (9)$$

For exemplification, Figure 9 shows an image gather obtained for a real dataset. The left panel is the gather computed without the compensation factor, while the right panel shows the gather after amplitude compensation. The amplitude weighting attenuates most of the energy at high incidence angles, and enhances the amplitude of the reflections at lower incidence angles.

CONCLUSIONS

This paper demonstrates that angle-domain common image gathers generated by wave-equation migration can be used for AVA analysis. Angle gathers can be computed both in the data space and in the image space. The two most important factors on the accuracy of the amplitude response are the finite sampling of the offset axis and the non-unitary nature of the migration operator. While limited sampling cannot be easily repaired, the migration algorithm can be modified to incorporate correction factors.

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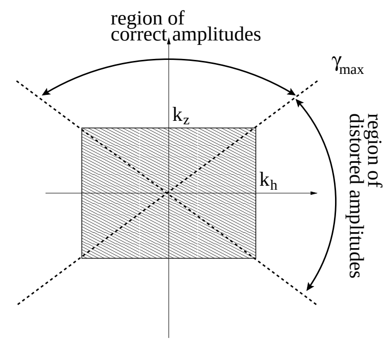

Figure 8: The finite spatial bandwidth limits the range of the angles for which we can reliably reconstruct the reflection amplitudes.

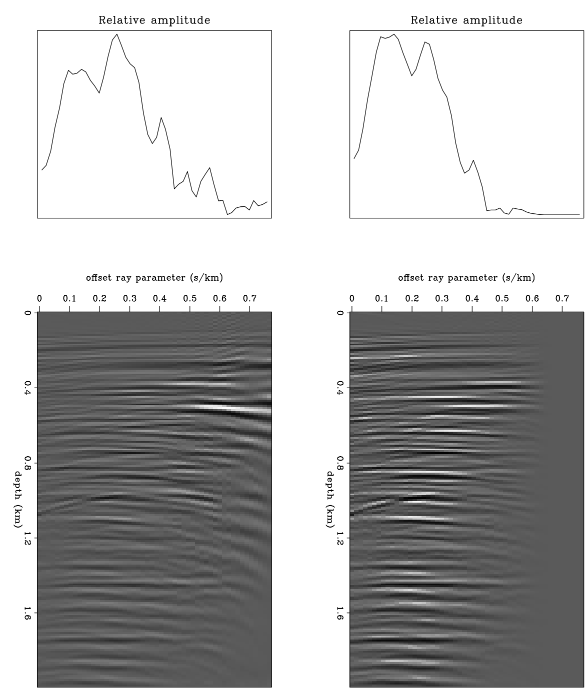

Figure 9: Real data example. The bottom panels are image gathers computed in the data space with (right) and without (left) amplitude compensation. The top panels show the amplitude variation with offset ray parameter for an event at about 1.75 km depth.