Multiple attenuation in the image space

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SUMMARY

Multiples can be suppressed in the angle-domain image space, after migration. For a given velocity model, primaries and multiples have different angle-domain moveouts and, therefore, can be separated using techniques similar to the ones employed in the data space, prior to migration. We use Radon transforms in the image space to discriminate between primaries and multiples. This method has the advantage of working with 3-D data and complex geology. Therefore, it offers an alternative to the more expensive Delft approach.

INTRODUCTION

The current most robust multiple attenuation techniques exploit moveout discrepancies that exist between primaries and multiples (Weglein, 1999). For instance, for relatively simple geology, our well-trusted Normal Move Out (NMO) correction efficiently flattens the primaries and leaves the multiples curved. Then the primaries and multiples can be separated in the Radon domain. However, it has been recognized that NMO and Radon transforms are not optimal when complex wavefield propagation occurs in the subsurface. The main reason is that the moveout of primaries and multiples cannot be describe with simple functions (parabolic or hyperbolic) anymore (Bishop et al., 2001). Therefore, more sophisticated methods are needed to perform the multiple attenuation.

One method that takes propagation effects into account is the Delft approach (Verschuur et al., 1992). This technique has the advantage of working with the surface data only and for any type of geology. Thus, it is often the method of choice for multiple attenuation in complex geology (Miley et al., 2001). To be accurate, the Delft method requires a very dense coverage of sources and receivers. If this condition is relatively easy to meet in 2-D, it becomes much more difficult to fulfill with 3-D surveys. New multiple prediction techniques are therefore developed to circumvent this limitation (van Dedem and Verschuur, 2001; Levin and Johnston, 2001).

A powerful multiple attenuation technique would be one that first takes the wavefield propagation into account, with whatever data we have, and then uses moveout discrepancies to remove multiples. To achieve this goal, we first propose using prestack depth migration as our imaging operator. Assuming that we have the correct velocity and an accurate migration scheme, we can then handle/image any type of complex geology very accurately. In this process, *both* primaries and multiples are migrated, after which they are transformed to angle gathers using standard techniques. In the angle domain, primaries are flat and multiples are curved, mimicking the situation we have after NMO for simple geology. Finally, we propose mapping the angle gathers into a Radon domain where the signal/noise separation can be achieved. This method has the potential to work with 2-D or 3-D data as long as angle gathers can be estimated. It is also much cheaper than the Delft approach, but it can still handle complicated geologic media.

ANGLE TRANSFORM

Angle-domain common image gathers (ADCIGs) are decompositions of seismic images in components proportional to the reflection magnitude for various incidence angles at the reflector. Given correct velocities and migration algorithms, primaries map into flat gathers and multiples map into events with moveouts.

Angle-gathers can be constructed by two classes of methods: data-space methods (de Bruin et al., 1990; Prucha et al., 1999; Mosher and Foster, 2000), with reflectivity described function of offset ray parameter, and image-space methods (Weglein and Stolt, 1999; Sava and Fomel, 2003; Rickett and Sava, 2001), with reflectivity described function of scattering angle. The moveout behavior of primaries and multiples are similar, irrespective of the method used to construct them.

Figure 1 shows on the left a CMP gather for a model with flat reflectors and $v(z)$ velocity. Most of the events in the gather are multiples, and just a few of the top-most events are primaries. On the right, Figure 1 shows an angle-domain CIG for the data in the left panel. The primary events are imaged correctly and are flat, but the multiples are not imaged correctly and have strong moveouts.

Angle-domain common image gathers are useful for multiple suppression for several reasons. First, events imaged with the wrong velocity show substantial moveouts, which allows us to discriminate between primaries imaged with correct velocity, and multiples, imaged with incorrect velocity. Second, angle-domain common image gathers describe the reflectivity at the reflection point, independent, in principle, from the actual structure for which they are computed, so they capture all 3-D propagation effects at every individual CIG.

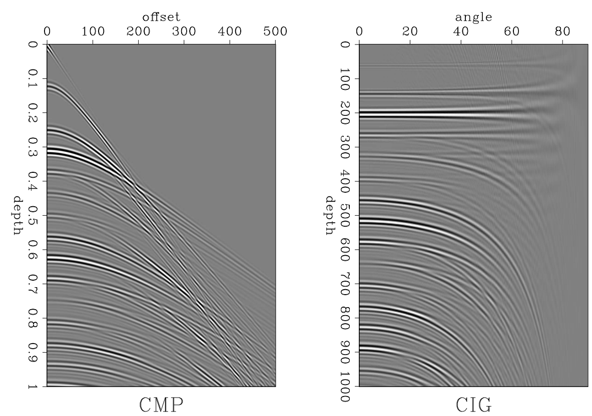

Figure 1: Simple synthetic model. A data-space CMP (left) and a corresponding image-space CIG (right).

MULTIPLE SUPPRESSION

Multiple attenuation with Radon transforms (RT) are popular and robust methods (Foster and Mosher, 1992). These techniques use the moveout discrepancy between primaries and multiples in order to separate them. Usually, the multiple attenuation is carried out with Common Mid Point (CMP) gathers after Normal Move Out (NMO) correction (Kabir and Marfurt, 1999). Then, the NMOed data are mapped with a parabolic Radon transform (PRT) in a domain where primaries and multiples are separable.

One desirable property of a Radon transform is that events in the Radon domain are well focused. This property makes the signal/noise separation much easier and decreases the transformation artifacts. These artifacts come essentially from the null-space associated with the RTs (Thorson and Claerbout, 1985). The RTs can be made sparse in the Fourier domain (Hugonnet et al., 2001) or in the time domain (Sacchi and Ulrych, 1995). The Fourier domain approach has the advantage of allowing fast computation of

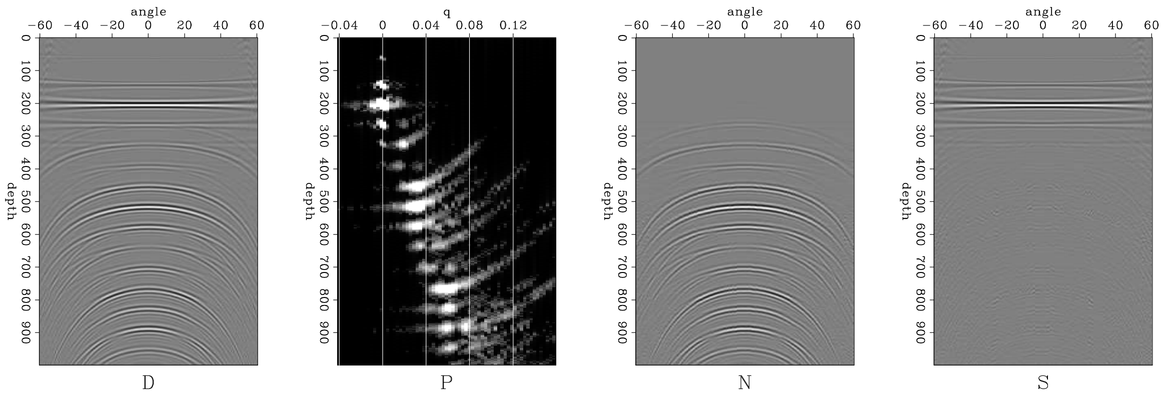

Figure 2: Synthetic example for S/N separation in the image space: (D) data in the image domain; (P) data in the Parabolic Radon domain; (N) multiples (noise); (S) primaries (signal).

the Radon panel (Kostov, 1990). However, the sparse condition developed so far (Hugonnet et al., 2001) does not focus the energy in the time axis. Therefore, in our implementation of the RTs, we opted for a time domain formulation with a Cauchy regularization in order to enforce sparseness.

A generic equation for a Radon transform is

$$z(q,\gamma) = z_0 + q\ g(\gamma), \tag{1}$$

where z_0 is the zero-angle depth, γ the angle, q is a curvature parameter, and $g(\gamma)$ is a function that represents the moveout in the CIGs. The modeling equation from the Radon domain to the image domain and its adjoint are

$$\begin{aligned} d(z,\gamma) &= \sum_{z_0}\sum_{q} m(z_0,q)\delta\left[z_0 - (z - q\ g(\gamma))\right], && (2)\\ m(z_0,q) &= \sum_{z}\sum_{\gamma} d(z,\gamma)\delta\left[z - (z_0 + q\ g(\gamma))\right]. && (3) \end{aligned}$$

At first order, we can assume that $g(\gamma) = \gamma^2$, which shows that Equation (1) corresponds to the definition of a parabola. However, for angle-domain common image gathers, Biondi and Symes (2003) demonstrate that a better approximation for g is $g(\gamma) = tan(\gamma)^2$. This definition of $g(\gamma)$ gives better focusing in the Radon domain (Figure 3), since it encapsulates a better physical model.

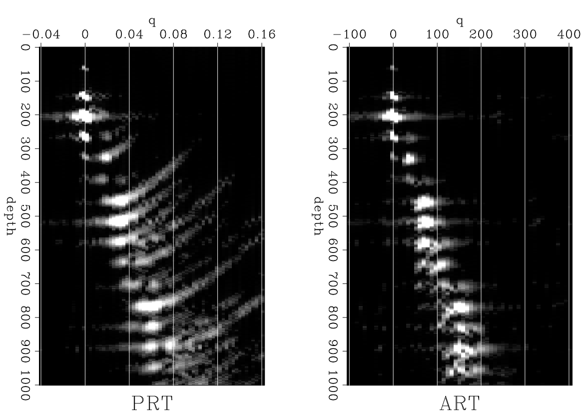

Figure 3: Radon transform of angle-domain CIGs using the parabolic equation (left) and the tangent equation (right).

Equation (2) can be rewritten as

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{4}$$

where **d** is the image in the angle domain, **m** is the image in the Radon domain, **L** is the forward RT operator. Our goal now is to find the vector **m** that best synthesizes, in a least-squares sense, the data **d** *via* the operator **L**. We, therefore, want to minimize the objective function:

$$f(\mathbf{m}) = \|\mathbf{L}\mathbf{m} - \mathbf{d}\|^2. \tag{5}$$

We also add a regularization term that enforces sparseness in the model space **m**. High resolution can be obtained by imposing a Cauchy distribution in the model space (Sacchi and Ulrych, 1995):

$$f(\mathbf{m}) = \|\mathbf{L}\mathbf{m} - \mathbf{d}\|^2 + \epsilon^2 \sum_{i=1}^{n} ln(b + m_i^2), \tag{6}$$

where n is the size of the model space, ϵ and b two constants chosen a-priori: ϵ controls the amount of sparseness in the model space and b relates to the minimum value below which everything in the Radon domain should be zeroed. The least-squares inverse of **m** is

$$\hat{\mathbf{m}} = \left[\mathbf{L}'\mathbf{L} + \epsilon^2 \mathbf{diag}(1/(b + m_i^2))\right]^{-1} \mathbf{L}'\mathbf{d}, \tag{7}$$

where **diag** defines a diagonal operator. Because the model or data space can be large, we estimate **m** iteratively. The objective function in Equation (6) is non-linear because the model appears in the definition of the regularization term. Therefore, we use a limited-memory quasi-Newton method (Guitton and Symes, 1999) to find the minimum of $f(\mathbf{m})$.

From the estimated model **m**, we separate the multiples from primaries in the Radon domain, using their distinct q values. We transform back the multiples to the image domain by applying **L**, and subtract them from the input data to obtain multiple-free gathers.

EXAMPLES

Our first example corresponds to a synthetic model with flat reflectors and $v(z)$ velocity. The left panel in Figure 2 is a representative CMP for a survey shot over this method. Most of the energy in the CMP is represented by multiples. The right panel in Figure 2 depicts a corresponding CIG. Again, most of the energy in the gather is represented by multiples, described by non-flat moveouts which distinguish them from the flat primaries.

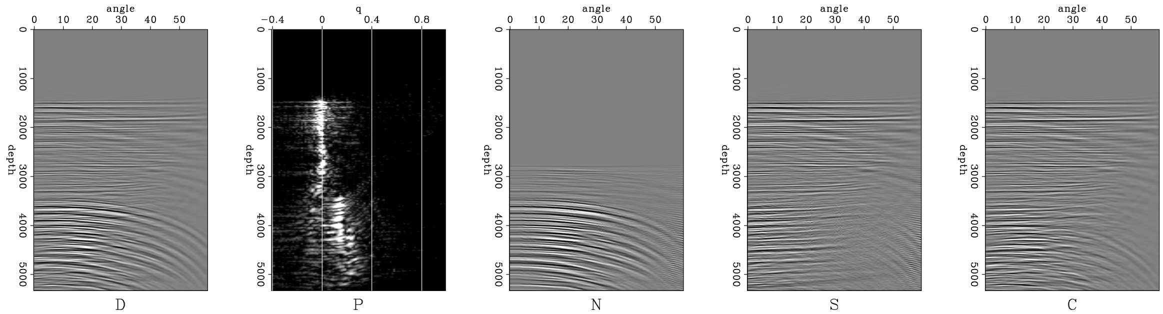

Figure 4: Gulf of Mexico example. S/N separation in the image space: (D) signal + multiples (data); (P) data the Parabolic Radon domain; (N) multiples (noise); (S) primaries (signal) separated in the image space; (C) primaries (signal) separated in the data space.

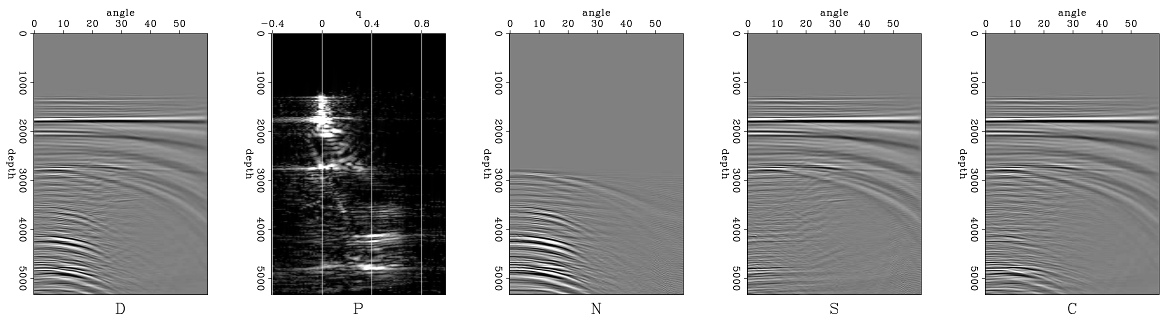

Figure 5: Gulf of Mexico example. S/N separation in the image space: (D) signal + multiples (data); (P) data the Parabolic Radon domain; (N) multiples (noise); (S) primaries (signal) separated in the image space; (C) primaries (signal) separated in the data space.

Figure 2 shows from left to right: (D) the data = primaries + multiples, in the image space; (P) the data transformed to the Radon domain, where the flat primaries are represented in the vicinity of $q = 0$, in contrast to the multiples at non-zero q; (N) the multiples isolated in the Radon domain and transformed back to the image domain; (S) the primaries left after subtraction of the multiples (N) from the data (D).

Figure 3 shows a comparison between RT using the parabolic equation $g(\gamma) = \gamma^2$ (left), and the more accurate tangent equation $g(\gamma) = tan(\gamma)^2$ (right). Not surprisingly, we observe better focusing using the tangent equation, which makes it easier to isolate the multiples.

We also apply our technique to a Gulf of Mexico dataset from a salt dome environment. This is a more complicated example, since it illustrates many of the difficulties encountered by multiple suppression in complicated areas, around salt bodies and in the presence of notable 3-D effects. Following the pattern used in the preceding example, Figures 4 and 5 show our multiple analysis at two different locations in the data. The first figure, corresponds to an area more or less away from the salt body, while the second one corresponds to a region right under the salt. From left to right, we present the data (D), the Radon domain (P), the noise (multiples) (N), and the signal (primaries) (S).

In both cases, primaries and multiples separate remarkably well in the Radon domain. We obtain the noise model after mute in the Radon domain and inverse RT, and the signal model by subtracting the noise from the data.

For comparison, in both Figures 4 and 5 we include one more panel (C) which represents the same image gather obtained by migration of the signal obtained by multiple suppression in the data space using a high resolution HRT with Cauchy regularization. The image space multiple suppression creates cleaner CIGs, compared with the data space method, although some of the inherent noise associated with RT can be still observed in the image.

Figure 6 shows the stacks of the images obtained without multiple suppression (D), with multiple suppression in the image space (S), and with multiple suppression in the data space (C). Here, too, we observed that our method removes a lot of the multiple energy, better than the data space method.

DISCUSSION

Primaries and multiples can have shapes which are neither parabolic, nor hyperbolic in the data space. All multiple suppression strategies based on PRT, HRT or similar methods approximate the data moveouts: it may fail in complex areas. Furthermore, primaries and multiples often have comparable shapes which are hard to discriminate. Similarly, primaries and multiples have different shapes in the image space: primaries are mostly flat and multiples are non-flat, which allows, in principle, for robust signal/noise separation strategies.

For complex geology, the multiples are better attenuated if the propagation effects are taken into account. This is why the Delft approach performs so well (Verschuur et al., 1992). For 3-D data, the latter can be rather difficult to utilize because interpolating the sources and receivers on a regular grid is very expensive. Alternatively, we can migrate the data and do the separation in the image space with Radon transforms, as we demonstrated in this paper.

Potential pitfalls for the multiple suppression strategy in the image space include situations where our velocity model is far from the truth. We encounter the theoretical possibility that some multiples are flat and some primaries are not flat. However, even in such situations, we can still discriminate primaries from multiples, given enough separation in the Radon domain.

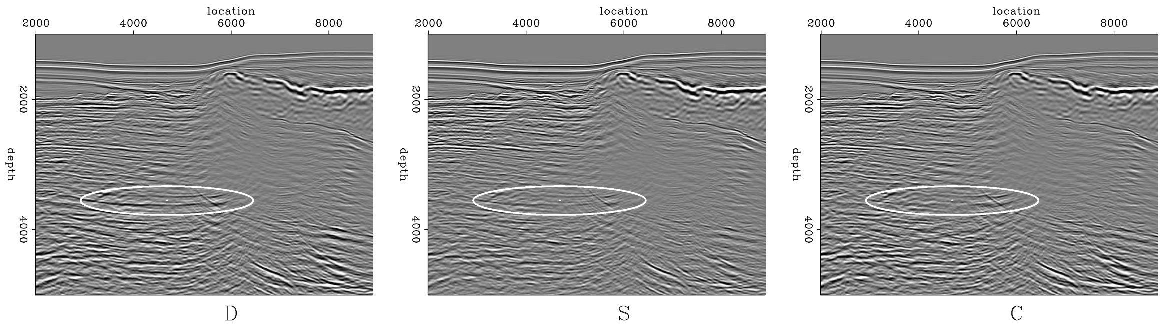

Figure 6: Gulf of Mexico example. Images obtained by angle-domain stacks: (D) signal + multiples (data); (S) primaries (signal) after separation in the image space; (C) primaries (signal) after separation in the data space.

CONCLUSIONS

Multiples can be suppressed in the angle-domain, after migration. For a given velocity model, primaries and multiples have different moveouts in the image space, and therefore they can be separated using similar techniques as the ones employed in the data space, prior to migration. We use Radon transforms, although these methods are neither unique, nor ideal.

Because we are using prestack depth migration, this method takes into account the effects of complex wavefield propagation in the same way that the Delft approach does. However, our proposed scheme has the potential to be affordable with 3-D data and cheap to apply. Therefore, for complex geology, this method stands between multiple attenuation in the data space with Radon transforms and the Delft approach where multiples are first predicted and then subtracted.

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