3D elastic wave mode separation for TTI media

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SUMMARY

The separation of wave-modes for isotropic elastic wavefields is typically done using Helmholtz decomposition. However, Helmholtz decomposition using conventional divergence and curl operators in anisotropic media only partially separates the elastic wave-modes. The separation of anisotropic wavefields requires operators which depend on local material parameters. Wavefield separation operators for TI (transverse isotropic) models can be constructed based on the polarization vectors evaluated at each point of the medium by solving the Christoffel equation. These polarization vectors can be represented in the space domain as localized filters, which resemble conventional derivative operators. The spatially-variable pseudo-derivative operators perform well in heterogeneous media even at places of rapid variation. In 3D TTI media, P and SV waves are polarized only in symmetry planes, and SH waves are polarized orthogonal to symmetry planes. Using the mutual orthogonality property between these modes, we need to solve only for the P wave polarization vectors from the Christoffel equation, and construct SV and SH wave polarizations using the relationship between these three modes. Synthetic results indicate that the operators can be used to separate wavefields for TI media with arbitrary strength of anisotropy.

INTRODUCTION

As the need for anisotropic imaging increases, processing and migration are more frequently performed based on anisotropic acoustic one-way wave-equations (Alkhalifah, 2000; Shan and Biondi, 2005; Fletcher et al., 2008). One of the complexities that impedes anisotropic migration using elastic wave equations is the difficulty in separating anisotropic wavefields into different wave-modes. Wave-mode separation for isotropic media can be achieved by applying Helmholtz decomposition to the elastic vector wavefields (Aki and Richards, 2002), which works for both homogeneous and heterogeneous isotropic media. Dellinger and Etgen (1990) extend the wave-mode separation to homogeneous VTI (vertically transversely isotropic) media, where P and SV modes are obtained by projecting the vector wavefields onto the correct polarization vectors. Yan and Sava (2008) apply this technology to heterogeneous VTI media and show that the method is effective if locally varying filters are used, even for complex geology with large heterogeneity.

However, VTI models are only suitable for limited geological settings with horizontal layering. TTI (tilted transversely isotropic) models characterize more general geological settings like thrusts and fold belts, e.g., in the Canadian Foothills (Godfrey, 1991). Using the VTI assumption to imaging structures characterized by TTI anisotropy introduces imaging errors both kinematically and dynamically. For sedimentary layers bent under geological forces, the model also needs to incorporate locally varying tilts that are consistent with layering, under the assumption that the local symmetry axis of the model is orthogonal to the reflectors throughout the model. Because the symmetry axis varies from place to place, it is essential to use spatially-varying filters to separate the wave-modes.

To date, separation of elastic wave modes based on computing polarization vectors has been applied only to 2D VTI models (Dellinger, 1991; Yan and Sava, 2008) and P wave mode separation for 3D anisotropic models (Dellinger, 1991). Conventionally, 3D elastic wavefields are usually first separated into in-plane (P and SV waves) and out-of-plane components (SH wave); then the in-plane wavefields can be separated into P and SV modes using 2D wave-mode separation schemes. However, this procedure requires slicing the elastic wavefields along the planes of symmetry. This involves interpolation because 3D wavefields are usually modeled in Cartesian coordinates. Furthermore, it is difficult to determine the optimum number of azimuths for this procedure. In this paper, we propose an alternative method for wave-mode separation which eliminates the need for wavefield interpolation along radial directions. The method is applicable to complex models with arbitrary heterogeneity and TI anisotropy.

WAVE-MODE SEPARATION FOR 2D TI MEDIA

Dellinger and Etgen (1990) suggest that wave-mode separation of *quasi*-P and *quasi*-SV modes in 2D VTI media can be done by projecting the wavefields onto the directions in which the P and S modes are polarized. For example, we can project the wavefields onto the P wave polarization directions $\mathbf{U}_P$ to obtain *quasi*-P (qP) waves:

$$\widetilde{qP} = i\,\mathbf{U}_P(\mathbf{k})\cdot\widetilde{\mathbf{W}} = iU_x\widetilde{W}_x + iU_z\widetilde{W}_z\,, \qquad (1)$$

where $\widetilde{qP}$ is the P wave-mode in the wavenumber domain, $\mathbf{k}=\{k_x,k_z\}$ is the wavenumber vector, $\widetilde{\mathbf{W}}$ is the elastic wavefield in the wavenumber domain, and $\mathbf{U}_P(k_x,k_z)$ is the P wave polarization vector as a function of $\mathbf{k}$. Generally, in anisotropic media, $\mathbf{U}_P(k_x,k_z)$ deviates from $\mathbf{k}$.

Dellinger and Etgen (1990) demonstrate wave-mode separation in the wave number domain using projection of the polarization vectors, as indicated by equation 1. However, for heterogeneous media, this procedure does not perform well because the polarization vectors are spatially varying. We can write an equivalent expression to equation 1 in the space domain at each grid point (Yan and Sava, 2008):

$$qP = \nabla_a\cdot\mathbf{W} = L_x[W_x] + L_z[W_z]\,, \qquad (2)$$

where L_x and L_z represent the inverse Fourier transforms of iU_x and iU_z, and $L[\,]$ represents spatial filtering of the wavefield with anisotropic separators. L_x and L_z define pseudo-derivative operators in the x and z directions for a 2D VTI medium in the symmetry plane, and they can change from location to location according to the material parameters.

The separation of P and SV wavefields can be similarly accomplished for symmetry planes of VTI and TTI media. For a medium with arbitrary anisotropy, we obtain the polarization vectors $\mathbf{U}(\mathbf{k})$ by solving the Christoffel equation (Aki and Richards, 2002):

$$\left[\mathbf{G} - \rho V^2 \mathbf{I}\right] \mathbf{U} = 0 \,, \tag{3}$$

where $\mathbf{G}$ is the Christoffel matrix with $G_{ij} = c_{ijkl} n_j n_l$, c_{ijkl} is the stiffness tensor, n_j and n_l are the normalized wave vector components in the j and l directions with $i,j,k,l = 1,2,3$. The parameter V corresponds to the eigenvalues of the matrix $\mathbf{G}$ and represents the phase velocities of different wave-modes as functions of the wave vector $\mathbf{k}$ (corresponding to n_j and n_l in the matrix $\mathbf{G}$). For plane waves propagating in a symmetry plane of a TTI medium, since qP and qSV modes are polarized in the symmetry planes and decoupled from the SH mode, we can set $k_y = 0$ and get

$$\begin{bmatrix} G_{11} - \rho V^2 & G_{12} \\ G_{12} & G_{22} - \rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_z \end{bmatrix} = 0 \,. \tag{4}$$

Equation 4 allows us to compute the polarization vectors $\mathbf{U}_P = \{U_x, U_z\}$ and $\mathbf{U}_{SV} = \{-U_z, U_x\}$ (the eigenvectors of the matrix) for P and SV wave-modes given the stiffness tensor at every location of the medium. Here, the symmetry axis of the TTI medium is not aligned with vertical axis of the Cartesian coordinates, and the TTI Christoffel matrix is different from the VTI equivalent.

For TTI media, in order to maintain the continuity at the negative and positive Nyquist wavenumbers, $-\pi$ and π radians, we apply a rotational symmetric Gaussian taper

$$f(\mathbf{k}) = Exp\left[-\frac{||\mathbf{k}||^2}{2\sigma^2}\right] \tag{5}$$

to the components of the TTI media polarization vectors. We choose a standard deviation of $\sigma = 1$. In this case, at $-\pi$ and π radians, the magnitude of this taper is about 3% of the peak value, and the components can be safely assumed to be continuous across the Nyquist wavenumbers. We can apply the procedure described here to heterogeneous media by computing two different operators, corresponding to U_x and U_z, at every grid point. In any symmetry plane of a TTI medium, the operators are 2D and depend on the local values of the stiffness coefficients. For each point, we pre-compute the polarization vectors as a function of the local medium parameters and transform them to the space domain to obtain the wave-mode separators. We assume that the medium parameters vary smoothly (locally homogeneous), but even for complex media, the localized operators act similarly to long finite difference operators used for modeling at locations where medium parameters change rapidly.

If we represent the stiffness coefficients using Thomsen parameters (Thomsen, 1986), then the pseudo-derivative operators L_x and L_z depend on ε, δ, V_P/V_S ratio along the symmetry axis and title angle ν, all of which can be spatially varying. We can compute and store the operators for each grid point in the medium and then use these operators to separate P and S modes from reconstructed elastic wavefields at different time steps. Thus, wavefield separation in TI media can be achieved simply by non-stationary filtering with operators L_x and L_z.

WAVE-MODE SEPARATION FOR 3D TI MEDIA

Since SV and SH waves have the same velocity along the symmetry axis in 3D TI media, it is not possible to obtain the shear mode polarization vectors in this particular direction by solving the Christoffel equation. This point is known by the name "kiss singularity" (Tsvankin, 2005). For elastic wavefields excited by a point source in 3D TI media, S wave-modes near the singularity directions have complicated nonlinear polarization that cannot be characterized by a plane wave solution. Consequently, we cannot construct 3D global separators for both S waves based on the 3D Christoffel solution to the TI elastic wave equation. However, since the P wave-mode is always well-behaved and does not have the problem of singularity, we can always construct a P wave separator represented by the polarization vector $\mathbf{U}_P = \{U_x, U_y, U_z\}$ by solving the 3D Christoffel equation 3. Then, we obtain the P mode by projecting the 3D elastic wavefields onto the vector $\mathbf{U}_P$.

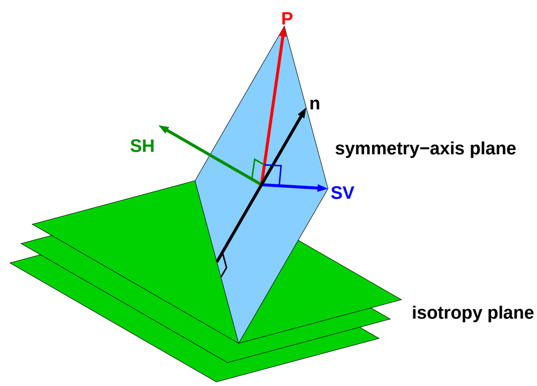

Figure 1: A schematic showing the elastic wave-modes polarization in a 3D VTI medium. Vector $\mathbf{n}$ indicates the symmetry axis of the TI medium. The P, SV, and SH modes are polarized orthogonal to each other.

We can remove the S wave-mode singularity by using the relative P-SV-SH mode polarization orthogonality. In every symmetry plane of the 3D TTI model, SV and SH mode polarization are defined to be perpendicular to the P mode in and out of the symmetry plane, respectively. For this 3D TTI medium, without losing generality, the symmetry axis ($\mathbf{n}$) can be expressed by $\mathbf{n} = \{n_x, n_y, n_z\} = \{\sin\nu\cos\alpha, \sin\nu\sin\alpha, \cos\nu\}$, where ν and α are the symmetry axis tilt and azimuth angles, respectively. A symmetry plane is the plane that contains the symmetry axis ($\mathbf{n}$) and the line connecting the source and an arbitrary point in space. Due to the rotational symmetry of TI media, P and SV waves are only polarized in symmetry planes, and the SH wave is polarized in isotropy planes (Figure 1). We can construct the SH wave polarization vector by

$$\mathbf{U}_{SH} = \mathbf{n} \times \mathbf{U}_P \,, \tag{6}$$

and the SV polarization vector by

$$\mathbf{U}_{SV} = \mathbf{U}_P \times \mathbf{U}_{SH}\,. \tag{7}$$

The P wave polarization vector is normalized by $U_x^2 + U_y^2 + U_z^2 = 1$. The wave polarization vectors for P, SV, and SH waves can be transformed to space domain to construct spatial filters for 3D heterogeneous TI media. Therefore, wave-mode separation works for models with complex structures and arbitrary tilts.

Suppose that each component of the vector wavefields is composed of P, SV, and SH modes projected onto their respective normalized polarization directions, we can construct the vector wavefields $\mathbf{W}$ as:

$$\widetilde{W}_x = PU_x + SV\frac{U_{SVx}}{|U_{SV}|} + SH\frac{U_{SVx}}{|U_{SV}|}\,, \tag{8}$$

$$\widetilde{W}_y = PU_y + SV\frac{U_{SVy}}{|U_{SV}|} + SH\frac{U_{SVy}}{|U_{SV}|}\,, \tag{9}$$

$$\widetilde{W}_z = PU_z + SV\frac{U_{SVz}}{|U_{SV}|} + SH\frac{U_{SVz}}{|U_{SV}|}\,. \tag{10}$$

Here, P, SV, and SH are the magnitudes of the P, SV, and SH mode in the k domain, respectively. Note that all the wave-modes are projected onto their normalized polarization vectors. We can verify that, in the direction of the symmetry axis, the SV and SH wave polarizations are not well-defined, which corresponds to the singularity mentioned earlier.

However, we use un-normalized polarization vectors from equations 6 and 7, as opposed to the normalized ones represented in equation 8 to filter the wavefields. In this case, the polarization vectors in the symmetry axis propagation direction are $\mathbf{U}_P = \{n_x, n_y, n_z\}$, $\mathbf{U}_{SV} = \{0,0,0\}$ and $\mathbf{U}_{SH} = \{0,0,0\}$. Our procedure naturally avoids the singularity problem which impedes us from constructing 3D separators for S modes. The zero amplitude of S wave-modes in the symmetry axis direction is not abrupt but a continuous change over nearby propagation angles. The amplitudes of separated S modes are zero in the symmetry axis direction and increase towards the isotropy plane propagation directions. The amplitudes change in S wave-modes can be seen from Figure 2. However, since the symmetry axis direction usually corresponds to normal incidence, it is important to obtain more accurate S wave amplitudes in this direction. One possibility is to approximate the anomalous polarization with the major axes of the quasi-ellipses of the S-wave polarization, which can be obtained using the first-order term in the ray tracing method (Tsvankin, 2005).

We illustrate the anisotropic wave-mode separation with a 3D VTI homogeneous model. The model has the parameters $V_{P0} = 3.5$ km/s, $V_{S0} = 1.75$ km/s, $\rho = 2.0$ g/cm^3, $\varepsilon = 0.4$, $\delta = 0.1$, and $\gamma = 0.0$. Figure 3 shows a snapshot of the elastic wavefields in the z, x and y directions, respectively. A displacement source located at the center of the model oriented at vector direction $\frac{1}{\sqrt{3}}\{1,1,1\}$ is used to simulate the wavefield. Figure 4 shows the separation into P, SV, and SH modes using the algorithm described in this paper. We can observe that the wave-modes are correctly separated even at locations where the wavefields triplicate due to the strong anisotropy.

CONCLUSIONS

We present a method for obtaining spatially-varying wave-mode separation operators for TI models, which can be used for elastic wave-modes in complex media. The main idea is to utilize polarization vectors constructed in the wavenumber domain using the local medium parameters and then transform these vectors to the space domain. The main advantage of applying the derivative operators in the space domain constructed in this way is that they are suitable for heterogeneous media. The construction uses the local tilt and azimuth angles, in addition to the other anisotropy parameters needed for the VTI operators.

For 3D TI models, the P, SV, and SH wavefield separators are constructed by solving the Christoffel equation for the P wave eigenvectors with local medium parameters. SV and SH polarization vectors are constructed using the mutual orthogonality of the wave-modes and their relations to the local symmetry planes of the TI media. This construction avoids wave-mode singularities, although the procedure is only kinematically accurate. The wave-mode separators obtained using this method are spatially-variable filters and can be used to separate wavefields in TI media with arbitrary strength of anisotropy.

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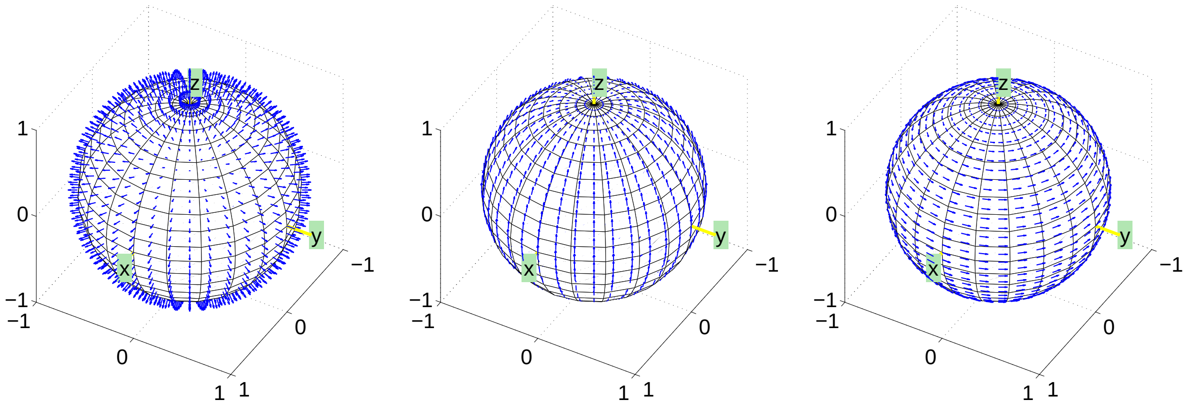
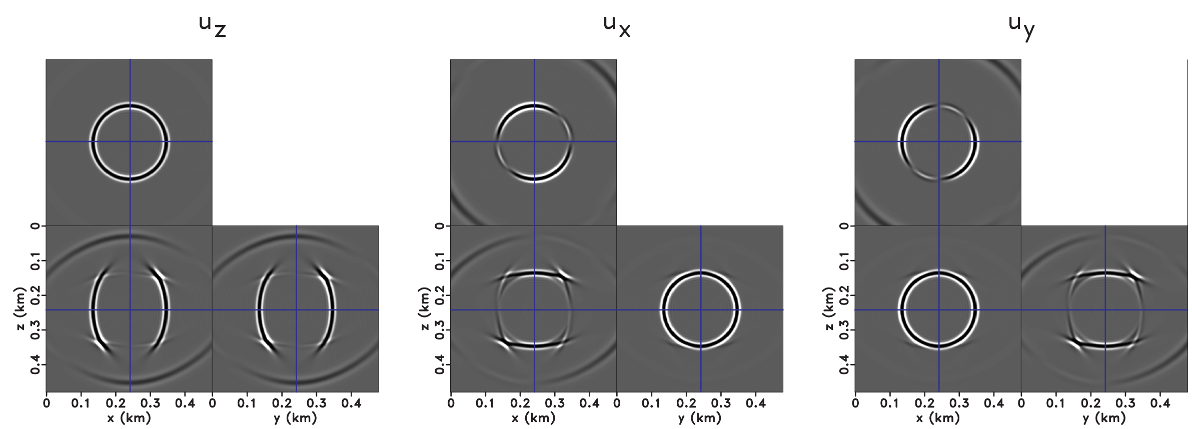


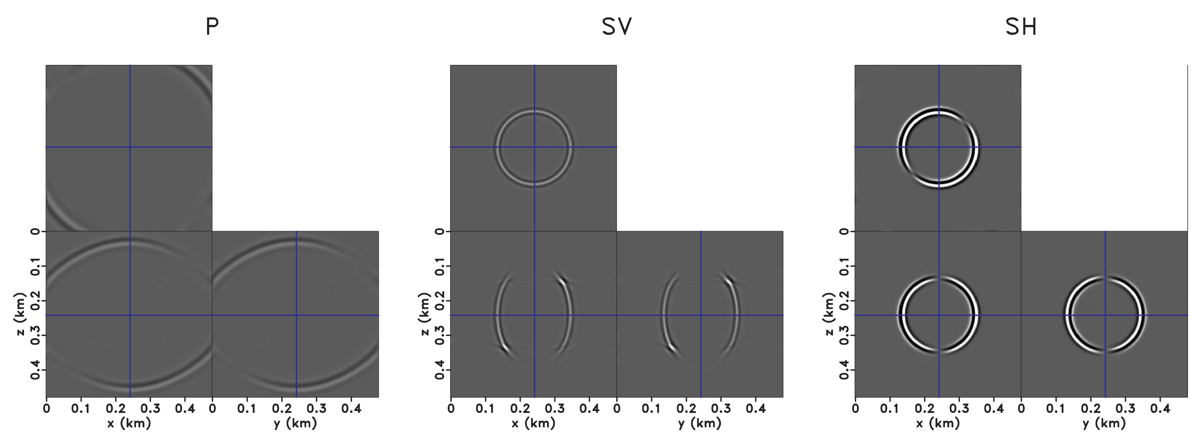
(a) (b) (c)

Figure 2: Panels (a) to (c) correspond to the wave-mode polarization for P, SV, and SH mode for a VTI medium obtained using Equations 6 and 7. Note that the SV and SH wave polarization vectors have zero amplitude in the symmetry axis direction.



(a) (b) (c)

Figure 3: A snapshot of the elastic wavefield in the z, x and y directions for a 3D VTI model. The model has parameters $V_{P0} =$ 3.5 km/s, $V_{S0} = 1.75$ km/s, $\rho = 2.0$ g/cm^3, $\varepsilon = 0.4$, $\delta = 0.1$, and $\gamma = 0.0$. A displacement source located at the center of the model oriented at vector direction $\{1,1,1\}$ is used to simulate the wavefield.



(a) (b) (c)

Figure 4: Separated P, SV and SH wave modes for the elastic wavefields shown in Figures 3(a)-3(c). P, SV, and SH are well separated from each other.