Elastic wave mode separation for TTI media

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SUMMARY

The separation of wave modes for isotropic elastic wavefields is typically done using Helmholtz decomposition. However, Helmholtz decomposition using conventional divergence and curl operators is not satisfactory for anisotropic media and leaves the different wave modes only partially separated. The separation of anisotropic wavefields requires more sophisticated operators which depend on local material parameters. Wavefield separation operators for TI (transversely isotropic) models can be constructed based on the polarization vectors evaluated at each point of the medium by solving the Christoffel equation using local medium parameters. These polarization vectors can be represented in the space domain as localized filters, which resemble conventional derivative operators. The spatially-variable "pseudo" derivative operators perform well in 2D heterogeneous TI media even at places of rapid variation. Wave separation for 3D TI media can be performed in a similar way. In 3D TI media, P and SV waves are polarized only in symmetry planes, and SH waves are polarized orthogonal to symmetry planes. Using the mutual orthogonality property between these modes, we only need to solve for the P wave polarization vectors from the Christoffel equation, and SV and SH wave polarizations can be constructed using the relationship between these three modes. Synthetic results indicate that the operators can be used to separate wavefields for TI media with arbitrary strength of anisotropy.

INTRODUCTION

As the need for anisotropic imaging increases, processing and migration are more frequently performed based on anisotropic acoustic one-way wave equations (Alkhalifah, 1998, 2000; Shan, 2006; Shan and Biondi, 2005; Fletcher et al., 2008). However, less research has been done on anisotropic elastic migration based on two-way wave equations. One of the complexities that impedes anisotropic migration using elastic wave equations is the difficulty in separating anisotropic wavefields into different wave modes. Wave mode separation for isotropic media can be achieved by applying Helmholtz decomposition to the vector wavefields (Aki and Richards, 2002), which works for both homogeneous and heterogeneous isotropic media. Dellinger and Etgen (1990) extend the wave mode separation to homogeneous VTI (vertically transversely isotropic) media, where P and SV modes are obtained by projecting the vector wavefields onto the correct polarization vectors. Yan and Sava (2008) apply this technology to heterogeneous VTI media and show that the method is effective if locally varying filters are used, even for complex geology with high heterogeneity.

However, VTI models are only suitable for limited geological settings with horizontal layering. TTI (tilted transversely isotropic) models characterize more general geological settings like thrusts and fold belts. Many case studies have shown that TTI models are good representations of complex geology, e.g., in the Canadian Foothills (Godfrey, 1991). Using the VTI assumption to imaging structures characterized by TTI anisotropy introduces imaging errors both kinematically and dynamically (Zhang and Zhang, 2008; Behera and Tsvankin, 2009). For example, Isaac and Lawyer (1999) and Behera and Tsvankin (2009) show that seismic structures can be mispositioned if isotropy, or even VTI anisotropy, is assumed when the medium above the imaging targets is TTI. To make the separation work for more general TTI models, the wave mode separation algorithm needs to be adapted to TTI media. For sedimentary layers bent under geological forces, the model also needs to incorporate locally varying tilts that are consistent with the local beddings, under the assumption that the local symmetry axis of the model is orthogonal to the reflectors throughout the model. Because the symmetry axis varies from place to place, it is essential to use spatially-varying filters to separate the wave modes in complex TI models.

To date, separation of elastic wave modes based on computing polarization vectors has been applied only to 2D VTI models (Dellinger, 1991; Yan and Sava, 2008) and P wave mode separation for 3D anisotropic models (Dellinger, 1991). Conventionally, 3D elastic wavefields are usually first separated into in-plane (P and SV waves) and out-of-plane components (SH wave); then the in-plane wavefields can be separated into P and SV modes using 2D wave mode separation schemes. However, this procedure requires slicing the elastic wavefields along the planes of symmetry. This involves interpolation of the wavefields because 3D wavefields are usually modeled in Cartesian coordinates. Furthermore, it is difficult to determine the optimum number of azimuths for this procedure. In contrast to this procedure, we propose a new technique to separate elastic wavefields for TI media with 3D separators, which reduces the need for interpolation of 3D wavefields.

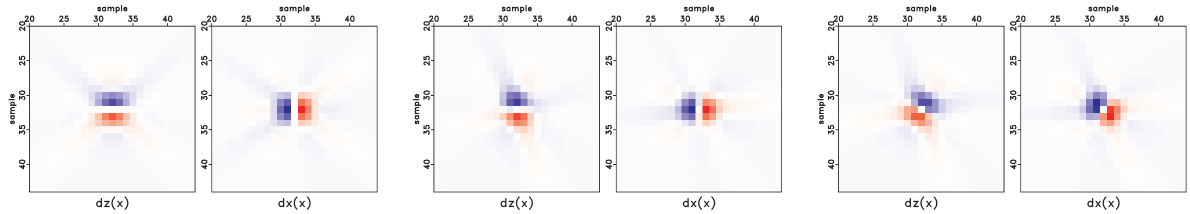
This paper first reviews the wave mode separation for 2D VTI media and then extends the algorithm to symmetry planes of TTI media. Then, we generalize this approach to wave mode separation in 3D TI media. We illustrate the separation with a 2D TTI synthetic example.

WAVE MODE SEPARATION FOR 2D TI MEDIA

Dellinger and Etgen (1990) suggest that wave mode separation of *quasi*-P and *quasi*-SV modes in 2D VTI media can be done by projecting the wavefields onto the directions in which the P and S modes are polarized. For example, we can project the wavefields onto the P wave polarization directions $\mathbf{U}_P$ to obtain *quasi*-P (qP) waves:

$$\widetilde{qP} = i\,\mathbf{U}_P(\mathbf{k}) \cdot \widetilde{\mathbf{W}} = iU_x\widetilde{W}_x + iU_z\widetilde{W}_z\,, \qquad (1)$$

where $\widetilde{qP}$ is the P wave mode in the wavenumber domain, $\mathbf{k} = \{k_x, k_z\}$ is the wavenumber vector, $\widetilde{\mathbf{W}}$ is the elastic wavefield in the wavenumber domain, and $\mathbf{U}_P(k_x, k_z)$ is the P wave



(a) (b) (c)

Figure 1: (a) Space domain wave mode separators for a VTI medium with parameters $V_{P0} = 3.0$ km/s, $V_{S0} = 1.5$ km/s, $\varepsilon = 0.25$, and $\delta = -0.29$. (b) Space domain separators for a TTI medium that has the same parameters as the VTI medium and a tilt angle $\nu = 30°$. Panel (c) is the projection of the separators shown in (b) on the tilt axis and the isotropy plane.

polarization vector as a function of $\mathbf{k}$. Generally, in anisotropic media, $\mathbf{U}_P(k_x, k_z)$ deviates from $\mathbf{k}$. Dellinger and Etgen (1990) demonstrate wave mode separation in the wave number domain using projection of the polarization vectors, as indicated by equation 1. However, for heterogeneous media, this procedure does not perform well because the polarization vectors are spatially varying. We can write an equivalent expression to equation 1 in the space domain at each grid point (Yan and Sava, 2008):

$$qP = \nabla_a \cdot \mathbf{W} = L_x[W_x] + L_z[W_z]\,, \tag{2}$$

where L_x and L_z represent the inverse Fourier transforms of iU_x and iU_z, and $L[\,]$ represents spatial filtering of the wavefield with anisotropic separators. L_x and L_z define pseudo-derivative operators in the x and z directions for a 2D VTI medium in the symmetry plane, and they can change from location to location according to the material parameters.

The separation of P and SV wavefields can be similarly accomplished for symmetry planes of VTI and TTI media. For a medium with arbitrary anisotropy, we obtain the polarization vectors $\mathbf{U}(\mathbf{k})$ by solving the Christoffel equation (Aki and Richards, 2002; Tsvankin, 2005):

$$\left[\mathbf{G} - \rho V^2 \mathbf{I}\right]\mathbf{U} = 0\,, \tag{3}$$

where $\mathbf{G}$ is the Christoffel matrix with $G_{ij} = c_{ijkl} n_j n_l$, in which c_{ijkl} is the stiffness tensor, n_j and n_l are the normalized wave vector components in the j and l directions with $i, j, k, l = 1, 2, 3$. The parameter V corresponds to the eigenvalues of the matrix $\mathbf{G}$ and represents the phase velocities of different wave modes as functions of the wave vector $\mathbf{k}$ (corresponding to n_j and n_l in the matrix $\mathbf{G}$). For plane waves propagating in a symmetry plane of a TTI medium, since qP and qSV modes are polarized in the symmetry planes and decoupled from the SH mode, we can set $k_y = 0$ and get

$$\begin{bmatrix} G_{11} - \rho V^2 & G_{12} \\ G_{12} & G_{22} - \rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_z \end{bmatrix} = 0\,, \tag{4}$$

where

$$G_{11} = c_{11}k_x^2 + 2c_{15}k_xk_z + c_{55}k_z^2\,, \tag{5}$$

$$G_{12} = c_{15}k_x^2 + (c_{13} + c_{55})\,k_xk_z + c_{35}k_x^2\,, \tag{6}$$

$$G_{22} = c_{55}k_x^2 + 2c_{35}k_xk_z + c_{33}k_z^2\,. \tag{7}$$

Equation 4 allows us to compute the polarization vectors $\mathbf{U}_P = \{U_x, U_z\}$ and $\mathbf{U}_{SV} = \{-U_z, U_x\}$ (the eigenvectors of the matrix) for P and SV wave modes given the stiffness tensor at every location of the medium. Here, the symmetry axis of the TTI medium is not aligned with vertical axis of the Cartesian coordinates, and the TTI Christoffel matrix takes a different form than the VTI form.

For TTI media, in order to maintain the continuity at the negative and positive Nyquist wavenumbers, $-\pi$ and π radians, we apply a rotational symmetric Gaussian taper

$$f(k) = Exp\left[-\frac{||\mathbf{k}||^2}{2\sigma^2}\right] \tag{8}$$

to both components of the TTI media polarization vectors. We choose a standard deviation of $\sigma = 1$. In this case, at $-\pi$ and π radians, the magnitude of this taper is about 3% of the peak value, and the components can be safely assumed to be continuous across the Nyquist wavenumbers. Compared to conventional finite difference operators which are 1D stencils, the derivatives constructed after the application of the Gaussian taper are represented by 2D stencils.

Figures 1(a) and 1(b) are the space domain representations of the polarization vector components for a VTI medium and a TTI medium, respectively. Here, the TTI medium has the same medium parameters as the VTI medium and the tilt angle of the TTI medium is 30°. Figure 1(c) shows the separators projected onto the symmetry axis and isotropy plane of the TTI medium.

We can apply the procedure described here to heterogeneous media by computing two different operators, namely U_x and U_z, at every grid point. For each point, we pre-compute the polarization vectors as a function of the local medium parameters and transform them to the space domain to obtain the wave mode separators. We assume that the medium parameters vary smoothly (locally homogeneous), but even for complex media, the localized operators work similarly as long finite difference operators would work for locations where medium parameters change rapidly. If we represent the stiffness coefficients using Thomsen parameters (Thomsen, 1986), then the pseudo-derivative operators L_x and L_z depend on ε, δ, V_{P0}/V_{S0} ratio and title angle ν, all of which can be spatially varying. We can compute and store the operators for each grid point in the medium and then use these operators to separate P and S modes from reconstructed elastic wavefields at different time steps. Thus, wavefield separation in TI media can be achieved simply by non-stationary filtering with operators L_x and L_z.

WAVE MODE SEPARATION FOR 3D TI MEDIA

Since SV and SH waves have the same velocity along the symmetry axis in 3D TI media, it is not possible to obtain the shear mode polarization vectors in this particular direction by solving the Christoffel equation. This point is known by the name “kiss singularity” (Tsvankin, 2005). For elastic wavefields excited by a point source in 3D TI media, S wave modes near the singularity directions have complicated nonlinear polarization that cannot be characterized by a plane wave solution. Consequently, we cannot construct 3D global separators for both S waves based on the 3D Christoffel solution to the TI elastic wave equation. However, since the P wave mode is always well-behaved and does not have the problem of singularity, we can always construct a P wave separator represented by the polarization vector $\mathbf{U}_P = \{U_x, U_y, U_z\}$. We obtain the P mode by projecting the 3D elastic wavefields onto the vector $\mathbf{U}_P$. The P wave polarization $\{U_x, U_y, U_z\}$ is obtained by solving the 3D Christoffel matrix (Tsvankin, 2005):

$$\begin{bmatrix} G_{11}-\rho V^2 & G_{12} & G_{13} \\ G_{12} & G_{22}-\rho V^2 & G_{23} \\ G_{13} & G_{23} & G_{33}-\rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = 0, \quad (9)$$

where $G_{ij} = c_{ijkl} n_k n_l$.

We can remove the S wave mode singularity by using the relative P-SV-SH mode polarization orthogonality in cylindrical system. In every symmetry plane of the 3D TTI model, SV and SH mode polarization are defined to be perpendicular to the P mode in and out of the symmetry plane, respectively. For this 3D TTI medium, without losing generality, the symmetry axis ($\mathbf{n}$) can be expressed by $\mathbf{n} = \{n_x, n_y, n_z\} = \{\sin\nu\cos\alpha, \sin\nu\sin\alpha, \cos\nu\}$, where ν and α are the symmetry axis tilt and azimuth angles, respectively. A symmetry plane is the plane that contains the symmetry axis and the line connecting the source ($\mathbf{S}$) and an arbitrary point in space. Due to the rotational symmetry of TI media, P and SV waves are only polarized in symmetry planes, and the SH wave is polarized in isotropy planes. We can first calculate the SH wave polarization $\mathbf{U}_{SH}$ by

$$\begin{aligned} \mathbf{U}_{SH} &= \mathbf{n} \times \mathbf{U}_P \\ &= \{n_x, n_y, n_z\} \times \{U_x, U_y, U_z\} \\ &= \{-n_z U_y + n_y U_z, \\ &\quad n_z U_x - n_x U_z, \\ &\quad -n_y U_x + n_x U_y\}. \end{aligned} \quad (10)$$

Then we can calculate the SV polarization $\mathbf{U}_{SV}$ by

$$\begin{aligned} \mathbf{U}_{SV} &= \mathbf{U}_P \times \mathbf{U}_{SH}, \\ &= \{-n_y U_x U_y + n_x U_y^2 - n_z U_x U_z + n_x U_z^2, \\ &\quad n_y U_x^2 - n_x U_x U_y - n_z U_y U_z + n_y U_z^2, \\ &\quad n_z U_x^2 + n_z U_y^2 - n_x U_x U_z - n_y U_y U_z\}. \end{aligned} \quad (11)$$

Here, the P wave polarization vector is normalized with

$$U_x^2 + U_y^2 + U_z^2 = 1. \quad (12)$$

Suppose each component of the vector wavefields is composed of P, SV, and SH modes projected onto their respective normalized polarization directions, we can express the vector wavefields $\mathbf{W}$ as:

$$\widetilde{W}_x = PU_x + SV\frac{U_{SVx}}{|U_{SV}|} + SH\frac{U_{SVx}}{|U_{SV}|}, \quad (13)$$

$$\widetilde{W}_y = PU_y + SV\frac{U_{SVy}}{|U_{SV}|} + SH\frac{U_{SVy}}{|U_{SV}|}, \quad (14)$$

$$\widetilde{W}_z = PU_z + SV\frac{U_{SVz}}{|U_{SV}|} + SH\frac{U_{SVz}}{|U_{SV}|}. \quad (15)$$

Here, P, SV, and SH are the magnitudes of the P, SV, and SH mode in the k domain, respectively. Note that all the wave modes are projected onto their normalized polarization vectors. We can verify that, in the direction of the symmetry axis, the SV and SH wave polarizations are not well-defined, which corresponds to the singularity mentioned earlier.

However, because we use un-normalized polarization vectors in Equations 10 and 11 to filter the wavefields, the polarization vectors in the symmetry axis propagation direction become

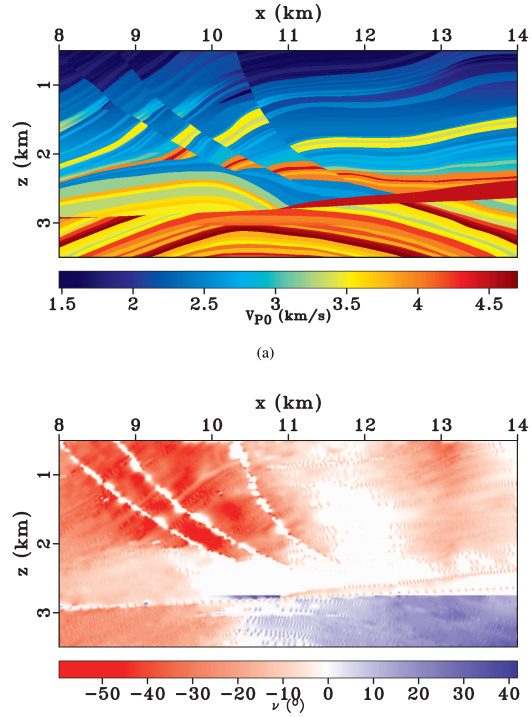
$$\begin{aligned} \mathbf{U}_P &= \{n_x, n_y, n_z\}, \\ \mathbf{U}_{SV} &= \{0, 0, 0\}, \\ \mathbf{U}_{SH} &= \{0, 0, 0\}. \end{aligned} \quad (16)$$

The magnitude of both SV and SH waves becomes zero in the singularity direction. This procedure naturally avoids the singularity problem which impedes us from constructing 3D separators for S modes. The zero amplitude of S wave modes in the vertical direction is not abrupt but a continuous change over nearby propagation angles. The amplitudes of separated S modes are zero in the symmetry axis direction and increase towards the horizontal propagation directions. However, since the symmetry axis direction usually corresponds to normal incidence, it is important to obtain more accurate S wave amplitudes in this direction. One possibility is to approximate the anomalous polarization with the major axes of the quasi-ellipses of the S-wave polarization, which can be obtained by incorporating the first-order term in the ray tracing method (Kieslev and Tsvankin, 1989; Tsvankin, 2005).

The wave polarization vectors for P, SV, and SH waves can be brought to space domain to construct spatial filters for 3D heterogeneous TI media. Therefore, wave mode separation would work for models with complex structures and arbitrary tilts.

EXAMPLES

We illustrate the anisotropic wave mode separation with an elastic model based on the elastic Marmousi II model (Bourgeois et al., 1991). In our modified model, the P wave velocity (Figure 2(a)) is taken from the original model, the $V_{P0}/V_{S0} = 2$, the parameter ε ranges from 0.13 to 0.36, and parameter δ ranges from 0.11 to 0.24. Figure 2(b) represents the local dips of the model obtained from the density model using plane wave decomposition. The dip model is used to simulate the TTI wavefields and also used to construct wavefield separators. A displacement source oriented at 45° to the vertical direction is located at coordinates $x = 11$ km and $z = 1$ km to simulate the elastic anisotropic wavefield. Figure 3(a) shows



(b)

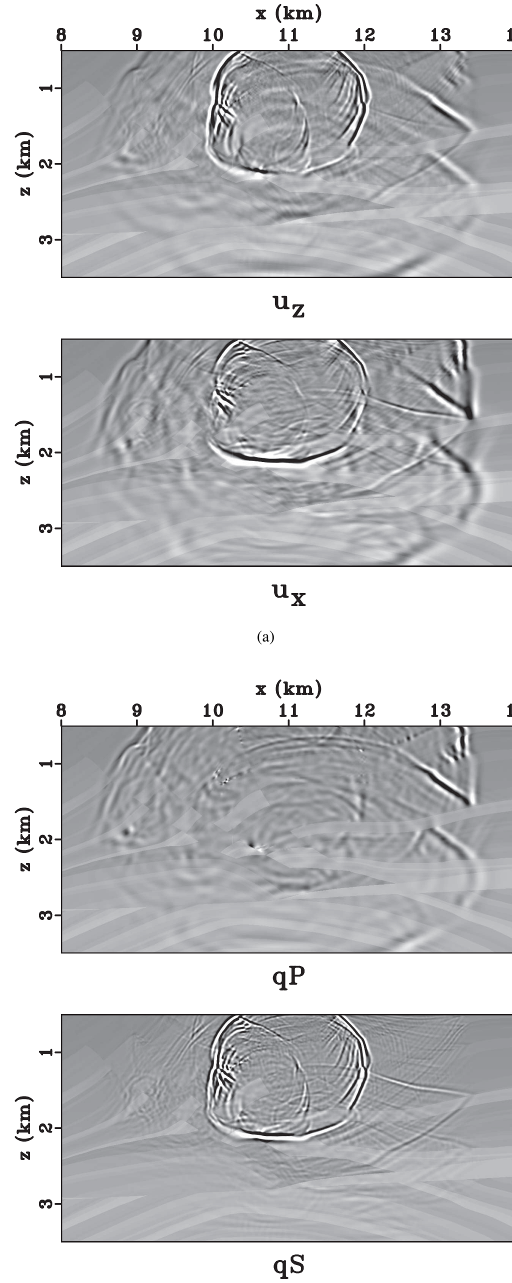
Figure 2: Anisotropic elastic Marmousi II model with (a) P wave velocity along the local symmetry axis and (b) local tilt angle ν.

one snapshot of the simulated elastic anisotropic wavefields using the model shown in Figure 2. Figure 3(b) shows the P and SV mode separation using the correct TTI filters. The TTI filters are constructed using the dips used for modeling. As expected, separation using TTI separators is effective even at locations where medium parameters change rapidly.

CONCLUSIONS

We present a method of obtaining spatially-varying derivative operators for TI models, which can be used to separate elastic wave modes in complex media. The main idea is to utilize polarization vectors constructed in the wavenumber domain using the local medium parameters and then transform these vectors back to the space domain. The advantage of applying the derivative operators in the space domain is that they are suitable for heterogeneous media. In order for the operators to work for TTI models with non-zero tilt angles, we incorporate a parameter, local tilt angle ν, in addition to other parameters needed for the VTI operators.

Wave mode separation is also applicable to 3D TI models. The P, SV, and SH wavefield separators can all be constructed by solving the Christoffel equation for the P wave eigenvectors with local medium parameters. Constructing 3D separation operators saves us the processing step of decomposing the wavefields in azimuthally-dependent slices. The wave mode separators obtained using this method are spatially-variable filters and can be used to separate wavefields in TI media with arbitrary strength of anisotropy.



(b)

Figure 3: (a) A snapshot of the vertical and horizontal displacement wavefield simulated for model shown in Figure 2. Panels (b) is the P and SV wave separation using TTI separators using correct model parameters.

ACKNOWLEDGMENT

We acknowledge the support of the sponsors of the Center for Wave Phenomena at Colorado School of Mines.