Blended Source Imaging by Amplitude Encoding

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SUMMARY

The computational cost of conventional imaging is large for today's wide-azimuth seismic surveys. One strategy to reduce the overall cost of seismic imaging is to migrate with multiple shot-gathers at once, a technique which is known as blended source imaging. Blended source imaging trades the reduced cost of imaging with the presence of artifacts (cross-talk) in the image. We show that a theoretical framework using a matrix representation of the imaging process adequately describes conventional, and blended source imaging. Furthermore, the matrix representation predicts both the quantity and strength of cross-talk artifacts prior to imaging, thus allowing us to decide a priori the trade off between cross-talk and speed. By exploiting our theoretical framework, we are able to design an amplitude encoding scheme, referred to as Truncated Singular Vector (TSV), that trades a significantly reduced cost of imaging with spatial resolution and cross-talk noise. The TSV encoding allows us to reduce the cost of imaging by at least an order of magnitude relative to conventional shot-record migration. Overall, we provide a framework for finding blended source encoding schemes, that produce good quality images at lower computational cost.

INTRODUCTION

Today's seismic imaging challenges include imaging areas with increasingly complex geology, such as salt domes and overthrust regions. The major issues for imaging these areas are poor data quality and lack of seismic illumination, as the complex geology severely deforms seismic wavefields. One approach to resolving these issues is to obtain large amounts of redundant information from various acquisition directions via wide-azimuth or full-azimuth seismic surveys (Michell et al., 2006; Kapoor et al., 2007; Ting and Zhao, 2009). Subsequently, wide-azimuth surveys require significantly more time to acquire and even greater amounts of time to process and image the data.

However, recent technological advances may reduce the cost of imaging for large seismic surveys. One of these technologies is blended imaging, where multiple shot-gathers are combined together by applying an amplitude or phase-encoding prior to migration (Romero et al., 2000). This process reduces the number of migrations that are needed to produce a final image (Liu, 1999; Morton, 1999; Romero et al., 2000; Soubaras, 2006; Zhang et al., 2007; Berkhout et al., 2009; Perrone and Sava, 2009). The encoding process introduces additional noise into the image, called cross-talk (Romero et al., 2000). Previous work has shown that the cross-talk is related to the encoding scheme used, so many encodings have been developed including: planar (Liu, 1999), random (Romero et al., 2000), modulated (Soubaras, 2006), harmonic (Zhang et al., 2007), and planar with dithering (Perrone and Sava, 2009). The standard method to attenuate cross-talk in blended images is conventional stacking.

A special case of blended source imaging is simultaneous source imaging, i.e. linearly combining shot-gathers with zero phase- and time-delay. The major advantage of simultaneous source imaging compared to blended imaging is that simultaneous source data can be acquired using the same recording time length, whereas blended source acquisition requires long recording times due to time-delays between sources. Thus, simultaneous source imaging reduces both the cost of conventional acquisition and the data volume (Beasley, 2008; Hampson et al., 2008). However, the same problems with cross-talk in blended source imaging plague simultaneous source imaging as well (Romero et al., 2000). Presently, this issue is circumvented by deblending the simultaneous source shot-gathers to create separate shot-gathers for each source prior to imaging (Akerberg et al., 2008; Hampson et al., 2008; Spitz et al., 2008; Huo et al., 2009; Kim et al., 2009). Ultimately though, we want to image using simultaneous source data because this saves time in both imaging and acquisition.

This paper focuses on optimizing shot-encoding schemes used for blended source and simultaneous source imaging. We examine both amplitude and phase-encoding schemes that can be used to reduce the amount of cross-talk in blended images. Overall, the goal of this paper is to reduce the cost of imaging by an *order of magnitude* through the use of simultaneous source (amplitude) encodings.

BLENDED SOURCE ENCODINGS

The major issues affecting blended source imaging are cross-talk and the loss of spatial resolution for certain encodings. We find that we can address both issues by developing a theoretical framework of matrix operations that represent both conventional and blended imaging, of which simultaneous source imaging is a special case.

Wave-equation migration

In conventional shot-record migration, the source-and receiver-wavefields are reconstructed separately for each shot-gather and the wavefields are cross-correlated to form a partial image. All of the partial images are then stacked together to form the final image. Mathematically, each source and receiver-wavefield can be thought to be an element in a vector that holds all source- or receiver-wavefields respectively,

$$\mathbf{W}_S = \left[W_S^1, W_S^2, ..., W_S^i, ..., W_S^{N_s}\right], \quad (1)$$

$$\mathbf{W}_R = \left[W_R^1, W_R^2, ..., W_R^i, ..., W_R^{N_s}\right], \quad (2)$$

where N_s is the number of shot-gathers. The i^{th} elements of both W_S and W_R correspond to the same physical shot-gather. Thus, conventional seismic imaging is equivalent to the inner product of the two vectors,

$$R = W_S W_R^H, \quad (3)$$

where R is the constructed image, and the multiplication of two elements of the matrix is actually the application of the imaging condition between those wavefields. The H notation indicates the conjugate transpose, which reduces to the standard transpose T in the time-domain. The application of the imaging condition implies a summation over either time or frequency depending on which domain we use for wavefield reconstruction. The summation over elements implies stacking the partial images together.

In order to expand this notation to blended source imaging, we introduce an additional matrix, which we call the encoding matrix E. The encoding matrix is an $N_s \times N_e$ complex-valued matrix, where N_e is the number of experiments and N_s is the number of shots in the survey. Each column in the encoding matrix corresponds to a single blended source experiment, while each row contains an amplitude weight and a phase-shift to be applied to the wavefield. For example, a single element in the encoding matrix has the form $Ae^{-j\phi(\omega)}$ where A is the amplitude weight, and $\phi(\omega)$ is the frequency dependent phase-shift. The amplitude weights may be any real number, and the phase-shifts are complex valued, but are usually not frequency dependent. Thus, each column in the encoding matrix weighs the amplitudes of the wavefields and shifts the wavefields in time before linearly combining them together prior to imaging. We note that in our framework, phase-encodings from the literature are represented by encodings, where $A = 1$ for all elements. Conversely, an amplitude encoding refers to encodings where $\phi = 0$ for all elements. Since there are no time-delays, the amplitude encoding can be viewed as an encoding using simultaneous sources, hence we use the two terms interchangeably.

The encoding matrix reduces the effective number of reconstructed wavefields that are used for imaging as follows:

$$B = WE, then \begin{cases} B_S = W_S E \\ B_R = W_R E \end{cases} \quad (4)$$

where WE is the projection of the wavefield vector (i.e W_S or W_R) onto the encoding matrix E and B is the blended source or receiver wavefield, B_S and B_R respectively. Because the migration operator is linear, we can perform the combination of the source- and receiver-data prior to wavefield reconstruction, thus reducing the necessary number of migrations from N_s to N_e. The final simultaneous source image R_e is represented by

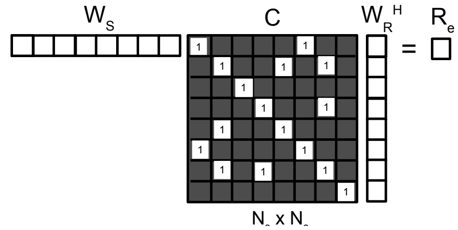


Figure 1: In blended source imaging, the R_e image is the original image plus additional artifacts from the cross-talk or the off-diagonal terms in C. When the cross-talk matrix C is the identity matrix I, then the product of W_S, C and W_R^H represents conventional seismic imaging and R_e becomes the conventional image R.

$$R_e = B_S B_R^H. \quad (5)$$

By substituting the expressions for B_S and B_R from equation 4 into equation 5, we obtain

$$R_e = W_S E E^H W_R^H, \quad (6)$$

which is similar to equation 3. We refer to the product EE^H as the cross-talk matrix C. Thus, equation 6 can be written as

$$R_e = W_S C W_R^H. \quad (7)$$

The cross-talk matrix C is similar to the identity I, but with additional off-diagonal terms as shown in Figure 1. This is a convenient description because equation 3, which describes conventional migration, can be rewritten to include the identity matrix I to represent the pairing of each source wavefield with its corresponding receiver wavefield as

$$R = W_S I W_R^H. \quad (8)$$

Thus, the cross-talk matrix C represents the formation of the conventional seismic image (i.e the diagonal terms) plus additional terms in the off-diagonals representing the pairing of wavefields that are not physically related to one another, i.e. the cross-talk.

By comparing equations 7 and 8, we find that the problem of designing optimal blended source encodings becomes the problem of finding an encoding matrix E, such that the resulting cross-talk matrix C is as close to the identity matrix I as possible. In fact, if we can find an encoding E such that $EE^T = I$, we can produce the same image as conventional seismic imaging but at a cost proportional to the number of blended experiments N_e instead of the number of shots N_s. Additionally, we can evaluate the quality of the images that an amplitude or phase-encoding scheme E produces by evaluating how close the cross-talk matrix C is to the identity matrix.

We note that the matrix representation in equation 7 unifies both the amplitude encoding scheme of Soubaras (2006) and the phase-encoding scheme of Romero et al. (2000). However, our framework does not restrict encodings to only these two cases. Indeed, we propose that there may be many hybrid, amplitude and phase, encodings that may be of interest for future research. Due to the limited space available, we only discuss amplitude or simultaneous source encodings.

Singular value decomposition

For simultaneous source encoding schemes, an alternative to finding E such that $EE^T \approx I$, is to design the cross-talk matrix C so that it is close to the identity matrix I, and then decompose C into E and E^T. The benefit is that we control how close the approximation C is to the I matrix, and the problem is reformulated as an inverse problem. To decompose C, we use Singular Value Decomposition (SVD) (Eckart and Young, 1936).

The SVD for the cross-talk matrix C, which is real-valued, square, and symmetric for amplitude (simultaneous source) encodings, is given by

$$C = U\Sigma U^T, \tag{9}$$

where U are the singular vectors, and Σ is a diagonal matrix of the singular values. This expression is similar to $C = EE^T$ in equation 9, except that U is a square matrix and E is rectangular. Thus, we can construct E by truncating the columns of U based on SVD theory, which indicates that we keep the first N_e singular vectors (U_{tc}) corresponding to the largest singular values. Additionally, we take the square root of the singular value matrix Σ and multiply it to both U_{tc} and U_{tc}^T. Therefore, E and E^T are respectively:

$$E = U_{tc}\sqrt{\Sigma_{tc}}, \tag{10}$$
$$E^T = \sqrt{\Sigma_{tc}}U_{tc}^T. \tag{11}$$

We refer to the constructed encoding matrix E as the Truncated Singular Vector (TSV) encoding. We note, that the product of EE^T is an *approximation* to C, as a result of truncating U, which in turn is an *approximation* of the identity matrix I, by design. Also, it should be noted that the E matrix from the SVD uses *all* sources, for each simultaneous source experiment.

CONSTRUCTING THE CROSS-TALK MATRIX

The construction of the C matrix to approximate I using SVD is of paramount importance to the quality of the final image R_e. We choose to use a Gaussian filter with a mean $\mu = 0$ and a standard deviation $\sigma = 3$ samples, to gradually taper the values away from the diagonal in the approximating matrix C. Regardless of choice, the additional components along the diagonal represent the construction of small plane-waves because this is equivalent to combining the images for spatially closest sources into the output image, which results in loss of spatial resolution. If the C matrix is close to the I matrix though, the loss in spatial resolution can be negligible.

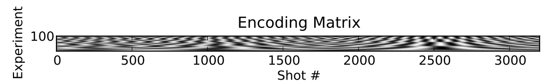


Figure 2: The transpose of the encoding matrix, E^T, for the Gaussian approximation to the identity matrix for 100 simultaneous source experiments. White values are (+1) and black values are (-1).

EXAMPLES

We conduct a series of examples to illustrate the ability of the SVD to construct encodings for simultaneous source imaging on the Sigsbee2A salt model. In this case, we create a new Sigsbee2A survey using 3200 shots to represent a shot at every possible shot location. Each shot is forward modelled using a finite-difference acoustic algorithm for the *full* aperture available in the Sigsbee model. We construct the encoding matrix E, Figure 2, by computing the singular values for the Gaussian approximation to the identity matrix, Figure 3, and truncating the singular vectors for the chosen number of experiments to perform (N_e). The shot-gathers and source-wavelets are blended according to the encoding matrix and then migrated using a standard downward continuation algorithm.

For reference, Figure 4(a) shows the target image obtained using conventional shot-record migration for all 3200 shots. Figure 4(b) shows the image for 10 experiments using the Gaussian approximation. Theoretically, the image in Figure 4(b) is created *320 times* faster than the conventional image, but there is additional overhead in terms of disk usage that slows down the process. For comparison, Figure 4(c) shows the result from the Gaussian approximation for 100 experiments. The 100 experiment image is less contaminated with artifacts in the salt body than the 10 experiment image, Figure 4(b). Additionally, the 100 experiment image is substantially clearer underneath the salt body ($x = 15.0km$, $4.5km < z < 8.0km$) and does not have as many cross-talk artifacts elsewhere. The 100 experiment image is created 32× faster than that of the conventional migration.

DISCUSSION

Our blended source framework adequately describes both amplitude and phase encodings and allows us to design encodings that minimize cross-talk in blended images. We have chosen to focus on amplitude encodings for this paper, but we speculate that it is possible to combine

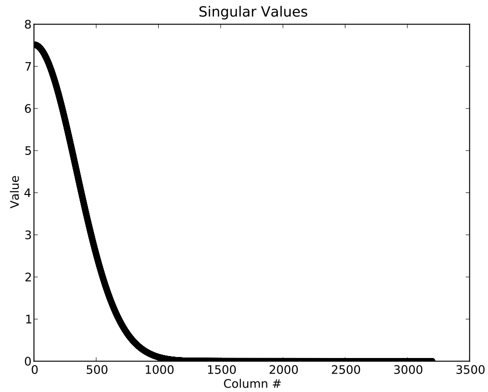
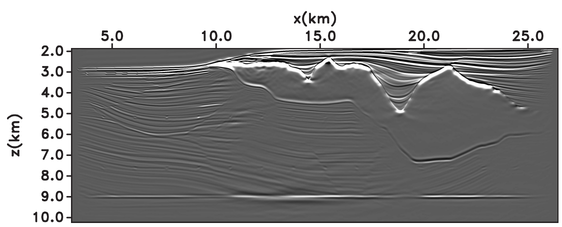
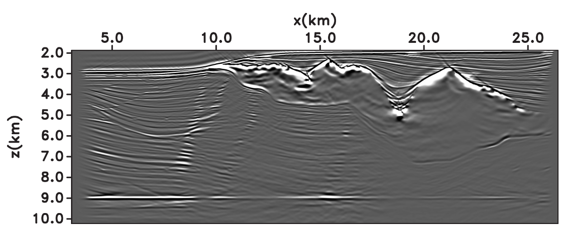


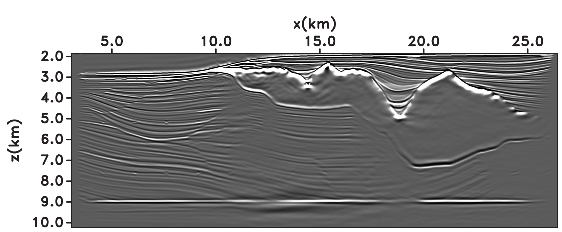
Figure 3: The singular values from the decomposition of the cross-talk matrix with the Gaussian approximation to the identity matrix. Since singular values close to zero do not contribute to the image, the best possible image for the Gaussian approximation is constructed for 1000 singular values (experiments).



(a)



(b)



(c)

Figure 4: The image of Sigsbee using the conventional shot-record migration (a), the Gaussian approximation to the identity matrix for 10 blended experiments (b), and 100 blended experiments (c).

both amplitude and phase encoding into a single encoding scheme, that retains the best aspects of each to produce superior images at much lower computational cost than previously discussed encodings.

Our theoretical model of seismic imaging has allowed the creation of an optimal simultaneous source (amplitude-only) encoding scheme that we refer to as the Truncated Singular Vector (TSV) encoding. For the Sigsbee data set, the cost of seismic imaging is reduced by at least an order of magnitude for 3200 shots. We note that 3200 shots for Sigsbee is excessive sampling. Regardless, the TSV encoding is equally valid for any number of shots, although the actual speed-up factor depends on the number of shots. The reduced computational cost comes at the expense of cross-talk noise in the image, a loss of spatial resolution, and a spatial variation of the amplitudes in the image, which is the result of the uneven weighting of shots in the encoding matrix. Presently, both the cross-talk noise and amplitude variation can be addressed by increasing the number of experiments used in the production of the final image. If all possible experiments are used, then the TSV scheme approximates conventional shot-record migration.

CONCLUSIONS

We develop a theoretical framework that adequately explains both conventional seismic imaging and blended source imaging. The framework allows us to reformulate the problem of blended source imaging in the context of matrix operations and leads to the use of singular value decomposition to construct optimal amplitude encodings. We examine the Truncated Singular Vector (TSV) encoding that trades reduced computational cost with increased noise in the image, spatial amplitude variation, and loss of spatial resolution. We demonstrate the validity of the TSV encoding through numerical experiments on the Sigsbee2A salt model. Overall, the TSV encoding scheme reduces the cost of imaging by at least an order of magnitude while restricting cross-talk noise and maintaining spatial resolution.

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