Elastic wave-mode separation for VTI media

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Elastic reverse time migration

- wavefield reconstruction

- imaging condition
Elastic reverse time migration

- wavefield reconstruction
  separate wave-modes before/after

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Elastic reverse time migration

- wavefield reconstruction
  separate wave-modes before/after

- imaging condition
  displacement/potential-based
Elastic reverse time migration

- wavefield reconstruction
  separate wave-modes before/after

- imaging condition
  displacement/potential-based
Imaging condition

- displacement-based
  cross-correlate Cartesian components

- potential-based
  cross-correlate wave-modes
Isotropic wavefield decomposition

\[ W = \nabla \phi + \nabla \times \psi \]

- \( \phi \): scalar potential
- \( \psi \): vector potential
Isotropic wavefield decomposition

\[ W = \nabla \phi + \nabla \times \psi \]

- \( \phi \): scalar potential
- \( \psi \): vector potential

\[ P = \nabla \cdot W \]
\[ S = \nabla \times W \]

- \( P \): compressional mode
- \( S \): shear mode
Isotropic media

\[ P = \nabla \cdot \mathbf{W} = D_x \ast W_x + D_y \ast W_y + D_z \ast W_z \]
Isotropic media

\[ P = \nabla \cdot \mathbf{W} = D_x \ast W_x + D_y \ast W_y + D_z \ast W_z \]

\[ P = i \mathbf{k} \cdot \mathbf{\tilde{W}} = i k_x \mathbf{\tilde{W}}_x + i k_y \mathbf{\tilde{W}}_y + i k_z \mathbf{\tilde{W}}_z \]
Polarization vectors: isotropic media
Anisotropic media

\[ qP = i \mathbf{U}(\mathbf{k}) \cdot \mathbf{\tilde{W}} = i \ U_x \mathbf{\tilde{W}}_x + i \ U_y \mathbf{\tilde{W}}_y + i \ U_z \mathbf{\tilde{W}}_z \]
Anisotropic media

\[ qP = i \mathbf{U}(\mathbf{k}) \cdot \mathbf{\tilde{W}} = i U_x \mathbf{\tilde{W}}_x + i U_y \mathbf{\tilde{W}}_y + i U_z \mathbf{\tilde{W}}_z \]

\[ qP = \nabla_a \cdot \mathbf{W} = L_x * W_x + L_y * W_y + L_z * W_z \]
Christoffel equation

\[
\left[ G - \rho V^2 I \right] \mathbf{U} = 0
\]
Christoffel equation

\[
\begin{bmatrix}
G - \rho V^2 \mathbf{I}
\end{bmatrix} \mathbf{U} = 0
\]

VTI media

\[
\begin{bmatrix}
\left( c_{11}k_x^2 + c_{55}k_z^2 - \rho V^2 \right) & (c_{13} + c_{55})k_xk_z \\
(c_{13} + c_{55})k_xk_z & c_{55}k_x^2 + c_{33}k_z^2 - \rho V^2
\end{bmatrix}
\begin{bmatrix}
U_x \\
U_z
\end{bmatrix}
= 0
\]
Polarization vectors: VTI media

$\epsilon = 0.25$, $\delta = -0.29$
\[ P = \nabla \cdot W \]
\[ = D_x \ast W_x \]
\[ + D_y \ast W_y \]
\[ + D_z \ast W_z \]

ISO

stationary

\[ qP = \nabla_a \cdot W \]
\[ = L_x \ast W_x \]
\[ + L_y \ast W_y \]
\[ + L_z \ast W_z \]

ANI

nonstationary
“Derivative” operators

ISO

\[ \frac{\partial}{\partial x} \]

\[ \frac{\partial}{\partial z} \]

ANI

\[ \frac{\partial}{\partial x} \]

\[ \frac{\partial}{\partial z} \]
Model 1: constant velocities

\[ V_{P0} = 3.0 \, \text{km/s}, \quad V_{S0} = 1.5 \, \text{km/s} \]
\[ \epsilon = 0.25, \quad \delta = -0.29 \]

\[ \rho = 1.0 \, \text{g/cm}^3 \]

\[ \rho = 2.0 \, \text{g/cm}^3 \]
Isotropic displacement
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla_a \cdot$ and $\nabla_a \times$
Anisotropic displacement

Position

Depth

$u_z$

$u_x$
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla_a \cdot$ and $\nabla_a \times$
Model 2: heterogeneous

- \( V_P \) Depth Position
- \( V_S \) Depth Position
- \( \rho \) Depth Position
- \( \epsilon \) Depth Position
- \( \delta \) Depth Position
Anisotropic stencils: nonstationary
Anisotropic displacement
Anisotropic displacement

Position

Depth

$u_z$

Position

$u_x$
Anisotropic displacement

Depth

Position

$u_z$

Position

$u_x$
Anisotropic displacement

Depth

\[ u_z \]

\[ u_x \]
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla_a \cdot$ and $\nabla_a \times$
Model 3: elastic Sigsbee 2A
Anisotropic displacement
Anisotropic displacement
Anisotropic displacement

Position

Depth

$u_z$

Position

$u_x$
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla \cdot$ and $\nabla \times$
Separation with $\nabla_a \cdot$ and $\nabla_a \times$
Conclusions

- anisotropic (VTI) wave-mode separation
- non-stationary filters
- complex media
Future work

- sensitivity analysis to anisotropy
- TTI extension
- 3D wave-mode separation
- anisotropic elastic RTM