interferometric imaging of sparsely-sampled data

Paul Sava

Center for Wave Phenomena
Colorado School of Mines
psava@mines.edu
notation

- $x$: receiver coordinates
- $y$: image coordinates
- $t$: time
conventional imaging

\[ V(x, y, t) = D(x, t) *_{t} G(x, y, t) \]
conventional imaging

\[ V(x, y, t) = D(x, t) \ast_t G(x, y, t) \]

\[ U(y, t) = \int dx \ V(x, y, t) \]
conventional imaging

\[ V(x, y, t) = D(x, t) \ast_t G(x, y, t) \]

\[ U(y, t) = \int dx \ V(x, y, t) \]

\[ R(y) = U(y, t = 0) \]
WE imaging assumptions

- **velocity model**: known
- **data sampling**: dense
- **array aperture**: wide
WE imaging assumptions

- velocity model: known
- data sampling: dense
- array aperture: wide
WE imaging assumptions

- **velocity model**: known
- **data sampling**: dense
- **array aperture**: wide
WE imaging assumptions

- velocity model: known
- data sampling: dense
- array aperture: wide
WE imaging assumptions

- **velocity model**: known
- **data sampling**: dense
- **array aperture**: wide
Wigner distribution functions

Definition
time-frequency transformations of complex signals

\[ U(x, t) \leftrightarrow W(x, t, k, \omega) \]
Wigner distribution functions

- Wigner (1932): quantum physics
- Ville (1958): signal processing
- Cohen (1995): image processing
$W(x, t, k, \omega) = \int d\xi \int d\tau \, U\left(x - \frac{\xi}{2}, t - \frac{\tau}{2}\right) U\left(x + \frac{\xi}{2}, t + \frac{\tau}{2}\right) e^{-i(k \cdot \xi + \omega \tau)}$

- $U(x, t)$: wavefield
- $W(x, t, k, \omega)$: WDF
pseudo-WDF

\[ W(x, t) = \int d\xi \int d\tau U\left( x - \frac{\xi}{2}, t - \frac{\tau}{2} \right) U\left( x + \frac{\xi}{2}, t + \frac{\tau}{2} \right) \]

- \( U(x, t) \): wavefield
- \( W(x, t) \): pseudo-WDF
conventional imaging

\[ V(x, y, t) = D(x, t) \ast_t G(x, y, t) \]

\[ U(y, t) = \int dx \ V(x, y, t) \]

\[ R(y) = U(y, t = 0) \]
WDF imaging

\[ V(x, y, t) = D(x, t) \ast_t G(x, y, t) \]

\[ U(y, t) = \int dx \ V(x, y, t) \]

\[ W_y(y, t) = \int d\tau \int d\eta \ U \left( y - \frac{\eta}{2}, t - \frac{\tau}{2} \right) U \left( y + \frac{\eta}{2}, t + \frac{\tau}{2} \right) \]

\[ R_y(y) = W_y(y, t = 0) \]
WDF imaging

conventional imaging
summary

WDFs attenuate wavefield fluctuations

- random velocity variation
- sparse data sampling