Wave-equation migration with dithered plane waves

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Motivation

- Reduce the cost of wave-equation migration
- Supply good quality images for CIP picking
- Explore the possibility for MVA
imaging with dithered plane waves
\[ f(x_n, \theta) = p_\theta(x_n - x_0) \]

\[ f(x_n, \theta) = t_{\text{max}} \mathcal{U}(\theta) \]
\[ f(x_n, \theta) = p_\theta(x_n - x_0) + \tau U(\theta) \]
 Migration by shot-encoding

**time-domain**

\[
I(x) = \sum_{\theta} \sum_{t} \sum_{m} s_m(x, t - f(x_m, \theta)) \sum_{n} r_n(x, t - f(x_n, \theta))
\]

**frequency-domain**

\[
I(x) = \sum_{\omega} \sum_{m} \sum_{n} \tilde{s}_m^*(x, \omega) \tilde{r}_n(x, \omega) \sum_{\theta} e^{i\omega(f(x_n,\theta)-f(x_m,\theta))}
\]
Crosstalk

**time-domain**

\[ l(x) = \sum_{\theta} \sum_{t} \sum_{m} s_m(x, t - f(x_m, \theta)) \sum_{n} r_n(x, t - f(x_n, \theta)) \]

**frequency-domain**

\[ l(x) = \sum_{\omega} \sum_{m} \sum_{n} \tilde{s}_m^*(x, \omega) \tilde{r}_n(x, \omega) \sum_{\theta} e^{i\omega(f(x_n, \theta) - f(x_m, \theta))} \]
Crosstalk matrix

$$\sum_{\theta} e^{i\omega[p_{\theta}(x_n-x_m)+(t_{\theta}^n-t_{\theta}^m)]}$$
Crosstalk matrix

\[ M \delta_{nm} + (1 - \delta_{nm}) \sum_{\theta} e^{i \omega [p_{\theta} \Delta x_{nm} + \Delta t_{nm}^\theta]} \]

\[ \Delta x_{nm} = x_n - x_m \]

\[ \Delta t_{nm}^\theta = t_n^\theta - t_m^\theta \]
Dithered linear delays

\[ p_\theta \Delta x_{nm} + \Delta t^\theta_{nm} \]
Dithered linear delays

\[
\begin{bmatrix}
 p_\theta + \frac{\Delta t^\theta_{nm}}{\Delta x_{nm}}
\end{bmatrix}
\Delta x_{nm}
\]
Equivalent ray parameter

\[ p_{eq} = p_\theta + \frac{\Delta t^\theta_{nm}}{\Delta x_{nm}} \]
\[ p_{eq} = p_\theta + \frac{\Delta t^\theta_{nm}}{\Delta x_{nm}} \]

\[ \Delta x_{nm} \text{ "small"} \]

\[ p_{eq} \rightarrow \frac{\Delta t^\theta_{nm}}{\Delta x_{nm}} \]
\[ p_{eq} = p_\theta + \frac{\Delta t_\theta}{\Delta x_{nm}} \]

\[ \Delta x_{nm} \text{ “large”} \]

\[ p_{eq} \longrightarrow p_\theta \]
velocity analysis with dithered plane waves
Objective function

\[
\min_{m} \left\{ \| d - Rm \|^2 + S(m) \right\}
\]

- \( d \): dithered PW gather
- \( R \): Radon transform
- \( m \): Radon-domain gather
- \( S(m) \): sparsity norm
Sparsity constraint

\[ S(m) = \sum_{k=1}^{N} \log \left( 1 + \frac{m_k m_k}{2\sigma^2} \right) \]

- \( \sigma \): scale parameter

(Ulrych et al., 2001)
Cauchy distribution

![Graph of the Cauchy distribution with different values of σ. The graph shows three curves, one for each value of σ: black for σ = 0.5, red for σ = 1, and blue for σ = 2. The x-axis represents the outcome, and the y-axis represents the probability density. The curves peak at 0 and spread out as σ increases.](image-url)
Cauchy process
Nonlinear inversion

\[(Q(m) + R^T R) m = R^T d\]

\[Q_{kk}(m) = \frac{2\sigma^2}{2\sigma^2 + m_k m_k}\]
Iterative linear inversion

\[
(Q(m^i) + R^T R) m^{i+1} = R^T d
\]

\[
Q_{kk}(m^i) = \frac{2\sigma^2}{2\sigma^2 + m_k^i m_k^i}
\]
Conclusion

- good quality image at lower cost
- CIGs equivalent to plane-wave migration
- suitable encoding for semblance-based MVA
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Questions?