Efficient elastic wave-mode separation in TTI media

Jia Yan and Paul Sava

Center for Wave Phenomena
Colorado School of Mines
elastic imaging
anisotropic
heterogeneous
3D
Helmholtz decomposition

\[ \mathbf{W} = \nabla \theta + \nabla \times \psi \]
Helmholtz decomposition

\[ \mathbf{W} = \nabla \theta + \nabla \times \psi \]

\[ P = \nabla \cdot \mathbf{W} \]

\[ \mathbf{S} = \nabla \times \mathbf{W} \]
Isotropic wave-mode separation

x-domain: stationary filtering

\[ P = \nabla \cdot \mathbf{W} = \sum_{c} D_c [W_c] \]

c = \{x, y, z\}: Cartesian coordinates
Isotropic wave-mode separation

k-domain: vector projection

\[ \tilde{P} = i \mathbf{k} \cdot \tilde{\mathbf{W}} = \sum_c i k_c \tilde{W}_c \]

\( c = \{x, y, z\} \): Cartesian coordinates
Anisotropic wave-mode separation
k-domain: vector projection

\[ \tilde{q} \tilde{P} = i \mathbf{u} \cdot \tilde{\mathbf{W}} = \sum_c i u_c \tilde{W}_c \]

Dellinger and Etgen (1990)
Anisotropic wave-mode separation

\[ qP = \nabla a \cdot W = \sum_c \mathcal{L}_c [W_c] \]

Yan and Sava (2009)
Wave-mode separation

x-domain
- heterogeneous model
- expensive

k-domain
- homogeneous model
- cheap
Efficient wave-mode separation

step 1: separate in k-domain

\[ P^r (x) = \mathcal{F}^{-1} \left\{ i u^r (k) \cdot \tilde{W} (k) \right\} \]

\( r: \) reference models
Efficient wave-mode separation

step 2: interpolate in x-domain

\[ P(x) = \sum_{r} w^{r}(x) P^{r}(x) \]

\( r \): reference models
Multidimensional interpolation

\[ w^r = \frac{1}{\|m - m^r\|} \]

\[ m = \{\epsilon, \delta, \nu, \alpha\} \]
Conclusions

- wave-mode separation is applicable in 3D
- k-domain separation is cheap
- accuracy controlled by reference selection
Acknowledgments

the sponsors of the Center for Wave Phenomena at Colorado School of Mines