Seismic tomography with extended images

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the objective

an affordable wavefield-based technique for velocity model building
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an affordable wavefield-based technique for velocity model building

ignore elasticity
ignore anisotropy
ignore density
ignore attenuation

...
geometry of a seismic experiment
the core idea

update the velocity by exploiting differences between recorded and simulated wavefields
the core idea

update the velocity by exploiting differences between recorded and simulated wavefields

waveform inversion

velocity analysis

data domain

image domain
We (x_r, t), De (x_r, t)

\[ W^e_s(x, t) = D^e_s(x_s, +t) \]

run time forward

compare the wavefields at the receivers
\[ D_s^e (x_s, t), \ W_r^e (x_s, t) \]

\[ \mathcal{L} (v) \left[ W_r^e (x, t) \right] = D_r^e (x_r, -t) \]

run time backward

compare the wavefields at the sources
**objective functions**

\[
J_s(v) = \sum_{e} \| D_s^e(x_s, t) - W_r^e(x_s, t) \|^2
\]

\[
J_r(v) = \sum_{e} \| W_s^e(x_r, t) - D_r^e(x_r, t) \|^2
\]

minimize wavefield differences
waveform inversion

pro

known analysis locations
use kinematic and dynamic info
waveform inversion

**pro**
- known analysis locations
- use kinematic and dynamic info

**con**
- incorrect assumptions
- cycle skipping
the core idea

update the velocity by exploiting differences between recorded and simulated wavefields

waveform inversion

data domain

velocity analysis

image domain
run time both forward and backward

compare the wavefields everywhere
a fundamental ambiguity

the wavefields should match only at the unknown reflector positions
a fundamental ambiguity

the wavefields should match only at the unknown reflector positions

The semblance principle:

*if the velocity is correct, independent experiments image the same structure*
wave-equation imaging

wavefield reconstruction

imaging condition
4D objects

$t$-domain

\[ W_s^e (x, t) \quad W_r^e (x, t) \]

$\omega$-domain

\[ W_s^e (x, \omega) \quad W_r^e (x, \omega) \]
conventional imaging condition

**t-domain**

\[ R (x) = \sum_{e} \sum_{t} W_s^e (x, t) W_r^e (x, t) \]

**ω-domain**

\[ R (x) = \sum_{e} \sum_{\omega} \overline{W_s^e (x, \omega)} W_r^e (x, \omega) \]
extended imaging condition

$t$ -domain

\[ R(x, \lambda, \tau) = \sum_{e} \sum_{t} W_s(x - \lambda, t - \tau) W_r(x + \lambda, t + \tau) \]

\[ e \]

\[ t \]

$\omega$ -domain

\[ R(x, \lambda, \tau) = \sum_{e} \sum_{\omega} \overline{W_s(x - \lambda, \omega)} W_r(x + \lambda, \omega) e^{2i\omega\tau} \]

\[ e \]

\[ \omega \]
extended images

\[ R(x, \lambda, \tau) \]

too big to compute

too difficult to analyze
$R \left( x, \lambda, \tau \right)$
ideal CIP moveout

one shot

$$\delta \left( (q \cdot \lambda) \sin \theta - \nu \tau \right)$$

all shots

$$\delta (q \cdot \lambda) \delta (\nu \tau)$$
one shot
one shot
one shot
one shot
one shot
one shot
one shot
one shot
one shot
form extended images: \( R(x, \lambda, \tau) \)
objective function

\[ J(v) = \| P(\lambda, \tau) [R(x, \lambda, \tau)] \|^2 \]

minimize defocusing
penalty operator

\[ P(\lambda, \tau)[\cdot] = \sqrt{(q \cdot \lambda)^2 + (v \tau)^2} \]

- \(q\): reflector orientation
- \(v(x)\): local velocity
velocity analysis

pro

no cycle skipping
better assumptions
velocity analysis

pro

no cycle skipping

better assumptions

con

unknown analysis locations

use kinematic info only
conclusions

WI and VA use the same reconstructed seismic wavefields

VA uses sparse extended CIPs

VA provides robustness; WI provides resolution
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