waveform inversion in the image domain

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waveform inversion

estimate the model by exploiting discrepancies between seismic wavefields
source wavefield – propagation forward in time

\[
L(v) [W_s(e, x, t)] = D_s(e, x_s, +t)
\]
receiver wavefield – propagation backward in time

\[ L(v) [W_r(e, x, t)] = D_r(e, x_r, -t) \]
source wavefield

$W_s(e, x, t)$

receiver wavefield

$W_r(e, x, t)$

- e: experiment index
- x: position
- t: time
\[ D(e, x, t) = W_s(e, x, t) - W_r(e, x, t) \]
$J_d(v) = \sum_{e} \| K_r(e, x) D(e, x, t) \|^2$

$D(e, x, t) = W_s(e, x, t) - W_r(e, x, t)$
\[ J_d(v) = \sum_e \| K_r(e, x) D(e, x, t) \|^2 \]

\[ D(e, x, t) = W_s(e, x, t) - W_r(e, x, t) \]

wavefield phase and amplitude

sensitive to cycle skipping
\[ \text{C} (e, x, \tau) = \sum_{t} W_s (e, x, t - \tau) W_r (e, x, t + \tau) \]
\[ J_d (v) = \sum_{e} \| K_r (e, x) P (\tau) C (e, x, \tau) \|^2 \]

\[ C (e, x, \tau) = \sum_{t} W_s (e, x, t - \tau) W_r (e, x, t + \tau) \]
\[ J_d (v) = \sum_{e} \| K_r (e, x) P (\tau) C (e, x, \tau) \|^2 \]

\[ C (e, x, \tau) = \sum_{t} W_s (e, x, t - \tau) W_r (e, x, t + \tau) \]

wavefield phase

robust to cycle skipping
implementation domain

waveform inversion

data domain

velocity analysis

image domain
$C(e, x, \tau)$

wavefield cross-correlation
\[ R(x, \tau) = \sum_{e} C(e, x, \tau) \]
\[ R(x_c, \tau) \]
$R(x_c, \tau)$  

$R(x, \tau = 0)$
\[ J_i(v) = \| K_i(x) P(\tau) R(x, \tau) \|^2 \]
\[ J_d (v) = \sum_{e} \| K_r (e, x) P (\tau) C (e, x, \tau) \|^2 \]

*(data domain objective function)*
adjoint state method

1. state variables
2. adjoint source
3. adjoint variables
4. gradient

Plessix (2006), Symes (2009)
example
velocity

\[ v(x) \]
velocity

\[ v(x) \]

[Graph showing a blank area labeled 'background']
conventional image

$R(x, \tau = 0)$

$x$(km)

$z$(km)

background
$R(x_c, \tau)$
extended image

\[ R(x_c, \tau) \]
Gradient

$\nabla J(x)$
gradient

$\nabla J(x)$
gradient

\[ \nabla J(x) \]
conventional image
extended image

$R(x_c, \tau)$

background
$R(x_c, \tau)$
velocity

\[ v(x) \]
velocity

$v(x)$

$x(km)$

$z(km)$

updated
waveform inversion
data domain

velocity analysis
image domain
Waveform inversion in the data domain yields the same seismic wavefields as velocity analysis in the image domain, with different objective functions.
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