Wavefield Tomography
with extended images

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WT definition

A procedure to estimate the model by exploiting discrepancies between seismic wavefields.
WT implementation

1. What is the model?
2. What are the seismic wavefields?
3. How do we measure discrepancies?
source seismic wavefield

\[ D_s(e, x_s, t) \quad W_s(e, x, t) \]
$D_s(e, x_s, t)$

$W_s(e, x, t)$

$L(v)[W_s(e, x, t)] = D_s(e, x_s, +t)$

*forward time propagation*
Receiver seismic wavefield

\[ D_r(e, x_r, t) \quad W_r(e, x, t) \]
\[ D_r(e, x_r, t) \]

\[ W_r(e, x, t) \]

\[ L(v)[W_r(e, x, t)] = D_r(e, x_r, -t) \]

\textit{backward time propagation}
model

velocity

v (x)
model \quad \text{wavefields}

velocity \quad \text{source} \quad \text{receiver}

\nu (x) \quad W_s (e, x, t) \quad W_r (e, x, t)
WT implementation

1. What is the model?
2. What are the seismic wavefields?
3. How do we measure discrepancies?
\[ D(e, x, t) = W_s(e, x, t) - W_r(e, x, t) \]

*subtract the wavefields*
\( K_r(e, x) D(e, x, t) \)
formulate a data domain objective function

\[ J_d(v) = \sum_e \| K_r(e, x) D(e, x, t) \|^2 \]
sensitive to cycle skipping...
Why compare the wavefields only by differencing?
\[ C(e, x, \tau) = \sum_t W_s(e, x, t - \tau) W_r(e, x, t + \tau) \]

*cross-correlate the wavefields*
\[ K_r(e, x) C(e, x, \tau) \]
formulate a data domain objective function

\[ J_d(v) = \sum_{e} \| P(\tau) K_r(e, x) C(e, x, \tau) \|^2 \]
robust to cycle skipping...
<table>
<thead>
<tr>
<th>difference</th>
<th>correlation</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(e, x, t)$</td>
<td>$C(e, x, \tau)$</td>
<td>?</td>
</tr>
</tbody>
</table>
Why compare the wavefields only at the surface?
cross-correlate the wavefields
$R(x, \tau) = \sum_{e} C(e, x, \tau)$

construct a time-lag extended image
restrict the time-lag extended image
formulate an image domain objective function

\[ J_i(v) = \| P(\tau) K_i(x) R(x, \tau) \|^2 \]
WT options

Why cross-correlate the wavefields only in time?
$$C(e, x, \tau) = \sum_t W_s(e, x, t - \tau) W_r(e, x, t + \tau)$$

*ignore wavefield directionality*
\[ C (e, x, \lambda, \tau) = \sum_{t} W_s (e, x - \lambda, t - \tau) W_r (e, x + \lambda, t + \tau) \]

exploit wavefield directionality
\[ R(x, \lambda, \tau) = \sum_e C(e, x, \lambda, \tau) \]

*construct an extended image*
restrict the extended image

\[ K_i(x)R(x, \lambda, \tau) \]
formulate an image domain objective function

\[ J_i(v) = \| P(\lambda, \tau) K_i(x) R(x, \lambda, \tau) \|^2 \]
The objective function is defined as:

\[ J_d(v) = \sum_e \| K_r(e, x) D(e, x, t) \|^2 \]

For the data domain:

\[ J_d(v) = \sum_e \| P(\tau) K_r(e, x) C(e, x, \tau) \|^2 \]

For the image domain:

\[ J_i(v) = \| P(\tau) K_i(x) \sum_e C(e, x, \tau) \|^2 \]

\[ J_i(v) = \| P(\lambda, \tau) K_i(x) \sum_e C(e, x, \lambda, \tau) \|^2 \]
objective function gradient
adjoint state method

1. state variables
2. adjoint source
3. adjoint variables
4. gradient

..., Plessix (2006), Symes (2009), ...
example
velocity

$v(x)$

$x$(km)

$z$(km)

$correct$
velocity

\[ v(x) \]

\[ x(\text{km}) \]

\[ z(\text{km}) \]

background
conventional image

$R(x)$

$x(km)$

$z(km)$
correct
conventional image
extended image
gradient

\[ \nabla J(x) \]
gradient

\( \nabla J(x) \)
conventional image
conventional image
extended image
velocity

$v(x)$

Updated

$x(km)$

$z(km)$
summary

waveform inversion

data domain

velocity analysis

image domain
summary

waveform inversion

velocity analysis

data domain

image domain

same seismic wavefields

different objective functions

same optimization procedure
food for thought

Why do we really need low frequencies?
food for thought

Why do we really need low frequencies?

What is an optimal objective function?
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