Do we really need low frequencies in waveform inversion?

Paul Sava

Center for Wave Phenomena
Colorado School of Mines
psava@mines.edu
Waveform inversion

\[ \min_{m} J = \left\| d_{obs}(x_r, t) - d_{pre}(x_r, t, m) \right\|^2 \]

- **d**: data
- **m**: model
- **x_r**: receiver coordinates
- **t**: observation time
in phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
off phase
in phase
We need low frequencies.
the question

Do we really need low frequencies?
wavefield tomography: definition

A procedure for estimating the earth model using seismic wavefield discrepancies.
\[ u_s(e, x_s, t) \]

\[ L(m) [u_s(e, x, t)] = u_s(e, x_s, +t) \]

*forward time propagation*
\( L(m)[u_r(e, x, t)] = u_r(e, x_r, -t) \)

*backward time propagation*
data domain WT
\[ D(e, x, t) = u_s(e, x, t) - u_r(e, x, t) \]
\[ J_d(v) = \sum_e \| K_r(e, x)D(e, x, t) \|^2 \]

\[ D(e, x, t) = u_s(e, x, t) - u_r(e, x, t) \]
\[
C(e, x, \tau) = \sum_t u_s(e, x, t - \tau) u_r(e, x, t + \tau)
\]
\[ J_d(v) = \sum_e \| P(\tau) K_r(e, x) C(e, x, \tau) \|^2 \]

\[ C(e, x, \tau) = \sum_t u_s(e, x, t - \tau) u_r(e, x, t + \tau) \]
image domain WT
\[ C(e, x, \lambda, \tau) = \sum_t u_s(e, x - \lambda, t - \tau) u_r(e, x + \lambda, t + \tau) \]
\[ R(x, \lambda, \tau) = \sum_e C(e, x, \lambda, \tau) \]
The diagram consists of several plots and images. The top left plot shows the function $\tau(s)$ with a contour indicating variations with depth $z$ (in km). The top middle plot depicts $\lambda_x$ (in km) as a function of $z$, where $\lambda_x$ represents the characteristic length scale. The top right plot continues to show $\lambda_x$ with a different orientation.

Below these plots is a large horizontal image labeled $R(x)$, which likely represents a cross-sectional view of a medium or system. The vertical green line in the $R(x)$ image might indicate a specific condition or measurement point.

The term "correct" is also present, suggesting that the diagram or its interpretation is accurate.
\[ J_i(v) = \| P(\lambda, \tau) K_i(x) R(x, \lambda, \tau) \|^2 \]
WT objective functions

\[ J_d(v) = \sum_{e} \| K_r(e, x) D(e, x, t) \|^2 \]

\[ J_i(v) = \| P(\lambda, \tau) K_i(x) \sum_{e} C(e, x, \lambda, \tau) \|^2 \]
difference

local minima
correlation

global minimum
difference

high resolution
correlation

low resolution
Marmousi example
correct model
shot gather
data spectrum
starting velocity
objective functions
starting velocity
image-domain WT
image-domain WT + data-domain WT
We don’t really need low frequencies!
acknowledgments