Data-domain wavefield tomography using local correlations

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Global correlation

\[ c(\tau) = \int f(t' - \frac{\tau}{2})g(t' + \frac{\tau}{2}) dt' \]

\[ c = Cf \]
Local correlation

\[ c(t, \tau) = \int w(t - t') f(t' - \frac{\tau}{2}) g(t' + \frac{\tau}{2}) dt' \]

\[ w(t - t') \equiv e^{-\tau^2/4\sigma^2} e^{-(t-t')^2/\sigma^2} \]
Local correlation

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Local correlation

\[ c(t, \tau) = \int w(t - t') f(t' - \frac{\tau}{2}) g(t' + \frac{\tau}{2}) \, dt' \]

\[ c = Cf \]
Local Correlation $C = GS$

Local Convolution $C^\top = S^\top G$

$G$: Gaussian filter matrix

$S$: matrix with shifted versions of $g$
the value of local correlations
The graph shows a function $f(t)$ plotted against time $t$ in seconds. The function appears to have a periodic pattern with sharp peaks at certain intervals. The x-axis represents time in seconds, ranging from 0 to 1, while the y-axis represents the value of the function, ranging from -0.04 to 0.08.
$g(t)$
$f \star g, \sigma = 0.04s$
\( f \star g, \sigma \to \infty \)
\[ f^* = \mathbf{C}^\top \mathbf{c}, \quad \sigma \to \infty \]
\[ f^* = C^\top c, \sigma = 0.04s \]
replacing the adjoint operator
\[
\min_c J = \left\| f - C^\top c \right\|_2^2 + \epsilon \left\| c \right\|_2^2
\]

\textbf{c}: time-variant matched filter
\( c = C_f, \sigma = 0.04s \)
$c_{inv}, \sigma = 0.04s$
$C_{inv}, \sigma \rightarrow \infty$
\[ f^* = C^\top c, \quad \sigma = 0.04s \]
\[ f = C^\top \mathbf{c}_{inv}, \quad \sigma = 0.04 \text{s} \]
\[ f = C^\top c_{\text{inv}}, \sigma \rightarrow \infty \]
\[ f = C^\top c_{inv}, \sigma \to \infty \]
\[ f = C^T c_{inv}, \sigma = 0.04s \]
tomography with local correlations
The objective function is given by:

$$J = \frac{||PCd^m||^2_2}{||Cd^m||^2_2}$$

**P**: penalty matrix  
**d^m**: modeled data  
**C ← d^{obs}**: correlation matrix
Penalty matrix

\[ P(\tau) = |\tau| \]
Penalty matrix

\[ P(\tau) = \frac{\tau}{\text{Env}(s^s(\tau)) + \epsilon} \]
Gradient calculation
augmented functional

\[ H(m, u_s, a_s) = J(m) - \langle a_s, Lu_s - s \rangle \]

\[ u_s: \text{ source wavefield (state variable) } \]
\[ a_s: \text{ adjoint variable } \]
state equation

$$\partial_{a_s} H(m, u_s, a_s) = 0$$

$$\langle \delta a_s, Lu_s \rangle = \langle \delta a_s, s \rangle$$

$$Lu_s = s$$
state equation

\[ \partial a_s H(m, u_s, a_s) = 0 \]

\[ \langle \delta a_s, Lu_s \rangle = \langle \delta a_s, s \rangle \]

\[ Lu_s = s \]
adjoint equation

$$\partial u_s H(m, u_s, a_s) = 0$$

$$\langle \delta u_s, g_s \rangle = \langle \delta u_s, L^\top a_s \rangle$$

$$L^\top a_s = g_s$$
adjoint equation

\[ \partial u_s H(m, u_s, a_s) = 0 \]

\[ \langle \delta u_s, g_s \rangle = \langle \delta u_s, L^\top a_s \rangle \]

\[ L^\top a_s = g_s \]
adjoint source

\[ g_s = \frac{\left( C^\top P^\top PCu_s - JC^\top Cu_s \right)}{\langle Cu_s, Cu_s \rangle} \]
gradient calculation

\[ \partial_m H(m, u_s, a_s) = 0 \]

\[ \partial_m J(x) = \sum_t \ddot{u}_s(x, t) a_s(x, t) \]
examples
correct
\( \sigma = 0.1 \text{s} \)
\[ \sigma = 0.1\, \text{s} \]

fast
$\sigma = 0.1 \text{s}$

slow
a cross-well transmission example
local correlation

\[ \sigma = 0.1 \text{s} \]
global correlation

$\sigma \rightarrow \infty$
global correlation
local correlation

\[ j \]

\text{iteration}
observed data
local correlation
Marmousi transmission example
true model
\( \sigma = 0.1 \text{s} \)
conclusions

better event discrimination

reduced cross-talk

improved convergence