Wavefield reconstruction using the wavelet transform

Iga Pawelec\textsuperscript{1}, Paul Sava\textsuperscript{1} and Michael Wakin\textsuperscript{2}

\textsuperscript{1} Center for Wave Phenomena, CSM
\textsuperscript{2} SINE Center for Research in Signals and Networks, CSM
CSM Geophysics Field Camp

- dense nodal acquisition (1.25m)
- non-linear sweep (4-140Hz)
large dynamic range
sampling: 1.25m
sampling: 2.5m
data aliasing

dynamic range

gaps pattern

land seismic data
constant amplitude
Fourier domain alternatives

curvelet transform
+ optimally sparse
+/- highly redundant
- limited to 3D
- slow computation

wavelet transform
- limited directionality
+ no redundancy
+ implemented in nD
+ fast computation
Fourier domain alternatives

curvelet transform
+ optimally sparse
+- highly redundant
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wavelet transform: multi-resolution

A: low frequency
H: horizontal details
V: vertical details
D: diagonal details
wavelet transform: multi-resolution

A: low frequency
H: horizontal details
V: vertical details
D: diagonal details

\[ A_1 = A_2 \oplus H_2 \oplus V_2 \oplus D_2 \]
coiflet 6
## Data Reconstruction Approaches

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sparse recovery

\[ \hat{\alpha} = \arg \min_{\alpha} \| \alpha \|_1 \quad \text{s. t.} \quad d = T\Phi^T\alpha \]

\( \alpha \): signal in the wavelet domain

\( T \): sampling matrix

\( \Phi \): sparsifying transform

\( d \): data in time domain
projection onto convex sets

$$\min_{f \in \mathcal{P}} J = \|f - g\| \quad \text{s. t.} \quad Pf = g$$

$\mathcal{P}_a$ - subspace for data with gaps
$\mathcal{P}_b$ - subspace for fully sampled data
projection onto convex sets

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\( \mathcal{P}_a \) - subspace for data with gaps
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field data examples

• large dynamic range
• complex wave patterns
• slow surface waves
full data
data with gaps
sparse recovery
data with gaps
data with gaps
sparse recovery
data with gaps
POCS
## Summary

### Sparsity Promotion
- + accurate kinematics
- + relative amplitudes
- - geometry sensitive
- - missing details
- + no parametrization

### POCS
- + accurate kinematics
- + relative amplitudes
- + robust geometry
- - missing details
- - non-intuitive thresholding
future research directions

• expansion to 5D
• increasing sensor spacing
• higher reconstruction accuracy
Wavelet domain reconstruction can overcome the large dynamic range problem.

**take home message**
acknowledgements

• Geophysical Technology Inc.
• Dawson Geophysical
• CSM Geophysics Field Camp crew