Elastic wavefield tomography with physical constraints

Yuting Duan and Paul Sava

Center for Wave Phenomena
Colorado School of Mines
fitting

single model parameter

\[ 2J(m) = \left\| W_D \left( G(m) - \bar{d} \right) \right\|^2 \]

fit the data
fitting
multiple model parameters

$$2J(\alpha, \beta) = \| W_D (G(\alpha, \beta) - \bar{d}) \|^2$$

fit the data
fitting + sensitivity

inconsistent model parameters

\[ 2J(\alpha, \beta) = \|W_D (G(\alpha, \beta) - \bar{d}) \|^2 \]

fit the data

maximize sensitivity
fitting + sensitivity + shaping
regularize model parameters

\[ 2J(\alpha, \beta) = \| W_D (G(\alpha, \beta) - \bar{d}) \|^2 + \| W_\alpha (\alpha - \bar{\alpha}) \|^2 + \| W_\beta (\beta - \bar{\beta}) \|^2 \]

fit the data
maximize sensitivity
impose model shape
The resulting model may not be physical/geological, i.e. no plausible lithology may be associated with the recovered models.

Must impose petrophysical constraints to link model parameters.
fitting + sensitivity + shaping + constraints

constrain model parameters

\[ 2J(\alpha) = \|W_D (G(\alpha, \beta(\alpha)) - \bar{d}) \|^2 + \|W_\alpha (\alpha - \bar{\alpha}) \|^2 \]

fit the data

maximize sensitivity

impose model shape

explicit constraints
fitting + sensitivity + shaping + constraints

constrain model parameters

\[ 2J(\alpha, \beta) = \| W_D (G(\alpha, \beta) - \bar{d}) \|^2 \quad \text{fit the data} \]
\[ + \| W_\alpha (\alpha - \bar{\alpha}) \|^2 \quad \text{maximize sensitivity} \]
\[ + \| W_\beta (\beta - \bar{\beta}) \|^2 \quad \text{impose model shape} \]
\[ + p(\alpha, \beta) \quad \text{implicit constraints} \]
Objective function \( J = J_D \)

\[
2J_D = \| G(\alpha, \beta) - \bar{d} \|^2
\]

- \( G(\alpha, \beta) \): predicted data
- \( \bar{d} \): observed data
model parameters

$$\ddot{u} = \alpha \nabla (\nabla \cdot u) - \beta \nabla \times (\nabla \times u)$$

- $\alpha = \frac{\lambda + 2\mu}{\rho}$
- $\beta = \frac{\mu}{\rho}$
ASM gradient

\[
\begin{bmatrix}
\partial_\alpha J_D \\
\partial_\beta J_D
\end{bmatrix} = \sum_e \begin{bmatrix}
- [\nabla (\nabla \cdot u)]^T \ast a \\
[\nabla \times (\nabla \times u)]^T \ast a
\end{bmatrix}
\]

- \textbf{u}: state variable
- \textbf{a}: adjoint variable
The objective function is given by:

\[ J = J_D + J_M \]

Furthermore, the term \( J_M \) is defined as:

\[ 2J_M = \| W_\alpha (\alpha - \bar{\alpha}) \|^2 + \| W_\beta (\beta - \bar{\beta}) \|^2 \]

- \( W_\alpha, W_\beta \): shaping operators
- \( \bar{\alpha}, \bar{\beta} \): reference models
petrophysical relation
physical constraint

\[ h_u = -\alpha + c_u \beta + b_u = 0 \]

\[ h_l = +\alpha - c_l \beta + b_l = 0 \]
physical constraint

\[ h_u = -\alpha + c_u \beta + b_u > 0 \]

\[ h_l = +\alpha - c_l \beta + b_l < 0 \]
objective function $J = J_D + J_M + J_C$

$$J_C = -\eta \sum_x [\log (-h_u) + \log (h_l)]$$

$\eta$: weighting parameter
cross-well example
Marmousi example
starting models
unconstrained inversion
constrained inversion
loose constraints
inaccurate constraints
conclusions

- multi-parameter inversion suffers from
  - differences in illumination between wave modes
  - ambiguity between parameters

- petrophysical constraints enforce
  - consistent updates for different parameters
  - plausible ranges for model parameters
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