Acoustic wavefield imaging using the energy norm

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ABSTRACT

Wavefield energy can be measured by the so-called *energy norm*. We extend the concept of norm to obtain the energy inner-product between two related wavefields. Considering an imaging condition as an inner product between source and receiver wavefields at each spatial location, we propose a new imaging condition that represents the total reflection energy. Investigating this imaging condition further, we find that it accounts for wavefield directionality in space-time. Based on the directionality discrimination provided by this imaging condition, we apply it to attenuate backscattering artifacts in reverse-time migration (RTM). This imaging condition can be designed not only to attenuate backscattering artifacts, but also to attenuate any selected reflection angle. By exploiting the flexibility of this imaging condition for attenuating certain angles, we develop a procedure to preserve the type of events that are near to 90° reflection angle, i.e., backscattered, diving and head waves, leading to a suitable application for full waveform inversion (FWI). This application involves filtering the FWI gradient to preserve the tomographic term (waves propagating in the same path) and attenuate the migration term (reflections) of the gradient. We illustrate the energy imaging condition applications for RTM and FWI using numerical experiments in simple (horizontal reflector) and complex models (Sigsbee).

Key words: imaging condition, conservation of energy, backscattering, reverse-time migration, waveform inversion

1 INTRODUCTION

Wavefield extrapolation is commonly known as a mathematical technique for numerically generating a wavefield in space and time, using a known portion of the wavefield itself (Margrave and Ferguson, 1999). One can use wavefield extrapolation to form an image of the subsurface, which is related to the Earth's reflectivity, in the case where the velocity is known and sufficiently accurate or to obtain subsurface velocity information by solving an inverse problem. In particular, when the two-way wave-equation is used, the former method is called reverse-time migration (Baysal et al., 1983; McMechan, 1983; Levin, 1984), and the latter is called full waveform inversion (Lailly, 1983; Tarantola, 1984; Gauthier et al., 1986; Plessix et al., 2013).

Reverse-time migration (RTM), as well as other wave-equation imaging procedures, obtains an image of a reflector in the subsurface in two steps. The first step is to extrapolate two wavefields from surface seismic data, one originating at the source location and the other at receiver locations. The second step is to crosscorrelate these extrapolated wavefields in time and form an image as the zero-lag correlation (Claerbout, 1985). This procedure is based on the assumption that, for the single scattering approximation, the reflector is at the location where source and receiver wavefields coexist in time and space (Cohen and Bleistein, 1979; Oristaglio, 1989; Perrone and Sava, 2014). At the source location, the real injected pressure function is often unknown and an estimated source function is used, i.e., the source wavefield is synthetic for the majority of RTM implementations.

Full waveform inversion (FWI) using two-way wave-equation obtains a velocity update by crosscorrelation between the state wavefield, which is similar to the source wavefield in RTM, and the adjoint wavefield (Tarantola, 1984; Hindlet and Kolb, 1988; Plessix, 2006). The adjoint wavefield is extrapolated from a data residual, which is often defined as the difference between observed and synthetic data. The velocity model is updated by minimizing the residual in a least-square sense. Mathematically, this velocity update is associated with the gradient of an objective function, which represents a norm of the residual vector.

Both RTM and FWI methods rely on the crosscorrelation of wavefields, which causes some problems. On one hand, besides imaging reflectors, RTM also outputs spurious arti-

facts caused by the crosscorrelation of wavefields propagating in the same direction: backscattering, head and diving waves (Diaz and Sava, 2015). For FWI, on the other hand, the crosscorrelation of wavefields propagating in the same direction forms low-wavenumber events that translate into useful velocity update information (tomographic term); and the imaged reflectors (migration term) represent the source of non-linearity that prevents the optimization from converging (Mora, 1989). Given these problems, alternatives to crosscorrelation have been investigated for both RTM and FWI (Chang and McMechan, 1986; Fletcher et al., 2005; Gao et al., 2012).

For RTM, the simplest and most common approach for overcoming the low-wavenumber events is to apply a high-pass filter. The Laplacian filter serves this purpose (Youn and Zhou, 2001) and is also convenient regarding processing cost since it involves post-imaging filtering. Other post-imaging filtering techniques are shown in the literature (Guitton et al., 2007; Xu et al., 2014), but such techniques do not account for multipathing. This filtering can also be applied prior to imaging by using the propagation directions of wavefields obtained by Poynting vectors (Yoon and Marfurt, 2006; Costa et al., 2009) or by wavefield decomposition (Suh and Cai, 2009; Liu et al., 2011) to attenuate wavefields traveling in the same direction. However, the imaging condition using Poynting vectors requires the design of a weighting function; and wavefield decomposition only separates wavefields into their up-going and down-going parts, resulting in unaltered backscattering artifacts from nearly horizontal reflected waves.

For FWI, in order to preserve the tomographic component and attenuate the migration component in the velocity update, some authors have used filtering on the FWI gradient (Almomin and Biondi, 2012; Albertin et al., 2013; Tang et al., 2013; Alkhalifah, 2015). All of these approaches use an extension of the gradient, similar to extended images in wave-equation migration (Sava and Fomel, 2006; Sava and Vasconcelos, 2011), to separate tomographic and migration components by exploiting the angle between the wave vectors and to mute the reflections from the gradient. With extended images, it is also possible to implement image-domain wavefield tomography, using an objective function that penalizes energy away from zero-lag (i.e., reflections) and preserves the backscattering energy, which is mostly present at zero-lag (Diaz and Sava, 2015).

Considering these recurrent problems of crosscorrelation in RTM and FWI, we are motivated to find an imaging condition that is implemented by a simple operation between vectors that represent wavefield directionality. These vectors are defined in the 4-D space-time and can be made orthogonal for undesired events. Then, a simple dot product eliminates these events and preserves the reflectors (i.e., the image). This operation is simple to apply since it does not involve identifying specific propagation directions, as in the case for Poynting vector methods (Yoon and Marfurt, 2006).

The proposed imaging condition is also discussed by Tarantola (1984) in the context of waveform inversion using impedance as the model parameter. This imaging condition attenuates the backscattering and is related to the Laplacian filter, as shown by other authors (Douma et al., 2010; Whitmore and Crawley, 2012; Pestana et al., 2013; Brandsberg-Dahl et al., 2013; Sun and Wang, 2013). Although more general in nature, the imaging condition is ideally suited for backscattering filtering, as discussed later in this paper.

We start by reviewing the theory of energy conservation in 4-D space-time in order to develop the physical explanation of the proposed imaging condition. Then, we show how this imaging condition can be applied to RTM for attenuating the backscattering artifacts and to FWI for obtaining the tomographic term, free of reflection artifacts.

2 REVERSE-TIME MIGRATION

In this section, we develop the theoretical aspects of the proposed imaging condition. We describe the conventional imaging condition as an inner product between source and receiver wavefields at each spatial location. Then, we formulate an alternative imaging condition based on the mechanical energy of the extrapolated wavefields. Appropriately modifying the inner product enables us to use it as an imaging condition to attenuate certain reflection angles, and consequently, to attenuate backscattering artifacts from RTM images. Conversely, for transmission FWI, we attenuate the contribution of waves propagating at arbitrary angles and preserve the waves which propagate along the same paths.

2.1 The L_2 norm and the conventional imaging condition

For a given seismic experiment with index e, consider a wavefield $W\left(e,\mathbf{x},t\right)$ as a solution to the acoustic wave-equation

$$\frac{\partial^2 W}{\partial t^2} - v^2\nabla^2 W = f\ , \tag{1}$$

where $v\left(\mathbf{x}\right)$ is the medium velocity and $f\left(e,\mathbf{x},t\right)$ is the source function. If we, for example, try to evaluate how close to one another two wavefields are, a global measure of wavefield strength is of interest in wavefield imaging. We can characterize the strength of the wavefield by the L_2 norm

$$\|W\|^2 = \sum_{e,\mathbf{x},t} W^2\ . \tag{2}$$

An application of this norm in wavefield imaging is present in data-domain wavefield tomography, where it is used to minimize the wavefield difference at the recorded data locations. Another application is in image-domain wavefield tomography, where the norm is used to minimize the energy away from zero lag in extended images (Yang and Sava, 2015). In linear algebra, a norm is also defined as the inner product of a vector with itself (Jain et al., 2004). Hence, one can define an L_2 inner product between the two wavefields $U\left(e,\mathbf{x},t\right)$ and $V\left(e,\mathbf{x},t\right)$ as

$$\langle U,V\rangle = \sum_{e,\mathbf{x},t} UV\ . \tag{3}$$

A common application of this L_2 inner product in wavefield imaging is to compare source and receiver wavefields $U(\mathbf{x},t)$ and $V(\mathbf{x},t)$, respectively, and form an image $I(\mathbf{x})$ of the subsurface:

$$I = \sum_{e,t} UV \; . \tag{4}$$

Note that equation (4) is commonly known as the conventional imaging condition (CIC), forming an image at every location where the wavefields coexist in space and time. However, the propagation directions of the different wavefields are not taken into account when forming the image. Consequently, the resulting image contains backscattering artifacts, which occur wherever source and receiver wavefields propagate along the same path. We illustrate this idea with the simple 2-D theoretical experiment depicted in Figure 1. The rays from source and receiver wavefields are represented in the space-time continuum. When they backscatter at the reflector depth ($z = 2.5$ km) (Figure 1(a) and Figure 1(b)), the CIC (dashed lines in Figure 1(c)) forms an image at all locations above the reflector where the wavefields coexist.

This imaging failure stems from the fact that the imaging condition does not account for the wavefield propagation direction. As indicated earlier, alternative imaging conditions attempt to address this challenge. In the following section, we use energy conservation laws and alternative wavefield norms to account for the wavefield propagation direction and derive a new imaging condition that exploits not only the wavefield position and timing, but also its orientation.

2.2 The energy norm and the proposed imaging condition

One can define a measure of energy for a solution to the acoustic homogeneous wave-equation within a spatial domain Ω (Evans, 1997; McOwen, 2003):

$$E(t) = \frac{1}{2}\int\limits_{\Omega}\left[\frac{1}{v^2}\left(\frac{\partial W}{\partial t}\right)^2 + |\nabla W|^2\right] d\mathbf{x} \; . \tag{5}$$

The time derivative term corresponds to the kinetic energy of the wavefield, and the spatial gradient term corresponds to its potential energy. The wavefield energy is conserved over time as seen by taking the derivative of equation (5) with respect to time:

$$\frac{1}{2}\frac{d}{dt}\int\limits_{\Omega}\left[\frac{1}{v^2}\left(\frac{\partial W}{\partial t}\right)^2 + |\nabla W|^2\right] d\mathbf{x} = \int\limits_{\Omega}\left[\frac{1}{v^2}\frac{\partial W}{\partial t}\frac{\partial^2 W}{\partial t^2} + \nabla W\cdot\nabla\frac{\partial W}{\partial t}\right] d\mathbf{x} \; . \tag{6}$$

Then, the second term on the right-hand side of equation (6) is integrated by parts:

$$\begin{aligned}\int\limits_{\Omega}\left[\nabla W\cdot\nabla\frac{\partial W}{\partial t}\right] d\mathbf{x} &= \int\limits_{\Omega}\nabla\cdot\left(\frac{\partial W}{\partial t}\nabla W\right) d\mathbf{x} \\ &- \int\limits_{\Omega}\left(\frac{\partial W}{\partial t}\nabla^2 W\right) d\mathbf{x} \; .\end{aligned} \tag{7}$$

The first integral on the right-hand side can be turned into a surface integral by the Divergence Theorem. Assuming homogeneous boundary conditions, this integral goes to zero since the wavefield and its derivatives vanish on the boundary. Substituting the remaining term in (6) yields

$$\dot{E}(t) = \frac{1}{2}\int\limits_{\Omega}\frac{\partial W}{\partial t}\left(\frac{1}{v^2}\frac{\partial^2 W}{\partial t^2} - \nabla^2 W\right) d\mathbf{x} = 0 \; , \tag{8}$$

for a wavefield that satisfies the homogeneous wave-equation in (1), thus indicating that the total energy of the wavefield is conserved.

A norm and an inner product in a vector space can be defined if they result in a scalar quantity and have linear properties. The energy norm is defined in this context as (Zeidler, 1999; Tanushev et al., 2009)

$$\|W\|_E^2 = \sum_{e,\mathbf{x},t}\left[\frac{1}{v^2}\left(\frac{\partial W}{\partial t}\right)^2 + |\nabla W|^2\right] \; , \tag{9}$$

and the respective inner-product for two wavefields $U(e,\mathbf{x},t)$ and $V(e,\mathbf{x},t)$ is

$$\langle U, V\rangle_E = \sum_{e,\mathbf{x},t}\left(\frac{1}{v}\frac{\partial U}{\partial t}\frac{1}{v}\frac{\partial V}{\partial t} + \nabla U\cdot\nabla V\right) \; . \tag{10}$$

Note that the inner-product in equation (10) is composed of the dot product between the gradient vectors of the wavefields plus the product of their time derivatives scaled by the slowness ($1/v$). The time dimension provides an additional derivative component beyond the components of the spatial gradient. Thus, a wavefield $W(\mathbf{x},t)$ can be represented in a Euclidean space with spatial coordinates $\mathbf{x} = \{x,y,z\}$ and an additional coordinate vt, assuming a locally constant velocity. The gradient of W in this four-dimensional space can be defined as follows:

$$\left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}, \frac{\partial W}{\partial z}, \frac{1}{v}\frac{\partial W}{\partial t}\right) \; . \tag{11}$$

Notice that the first 3 components of this vector form the conventional spatial gradient of the wavefield. The fourth component contains a time derivative, which is related to the temporal forward or backward character of the wavefield and is scaled by the slowness ($1/v$). Therefore, this vector indicates the space and time directionality of the wavefield at a particular position and time. This notion of a four-dimensional gradient is used in other research areas, such as special relativity and wave theory, with the following compact notation (Feyn-

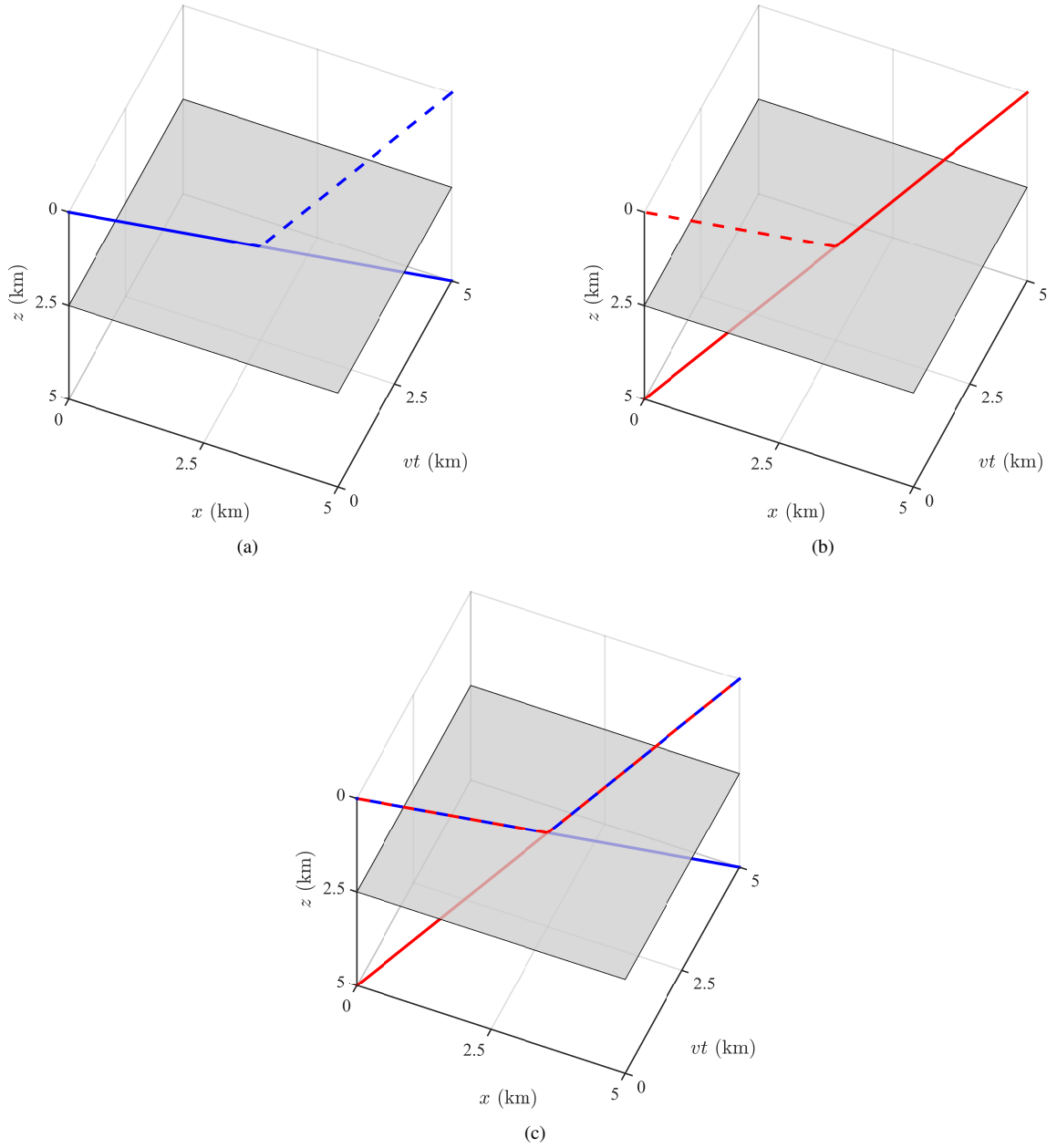


Figure 1. Schematic representation of (a) the source wavefield, (b) the receiver wavefield, and (c) conventional imaging, for a horizontal reflector at z=2.5 km in a 2-D experiment. The backscattered waves from source and receiver wavefields coexist in space and time and, consequently, overlap in the conventional image.

man et al., 1964; Chappel et al., 2010):

$$\Box W = \left(\nabla W, \frac{1}{v} \frac{\partial W}{\partial t} \right) . \tag{12}$$

We can rewrite equations (9) and (10) using the dot product between such four-dimensional vectors:

$$\|W\|_E^2 = \|\Box W\|^2 \tag{13}$$

$$\langle U, V \rangle_E = \langle \Box U, \Box V \rangle . \tag{14}$$

If the inner product in equation (10) is evaluated at each spatial location, and the time-reversal aspect of the receiver wavefield is considered, which means this wavefield is oriented backward in time ($V(e, \mathbf{x}, T - t)$), we can define an

imaging condition as

$$I_E = \sum_{e,t} \Box U \cdot \Box V \ , \\ = \sum_{e,t} \left(\frac{1}{v}\frac{\partial U}{\partial t} \frac{1}{v}\frac{\partial V}{\partial t} + \nabla U \cdot \nabla V \right) \ , \quad (15)$$

The energy imaging condition forms an image at every location where the dot product between the vectors $\Box U$ and $\Box V$ is nonzero. Notice that the time derivatives should be evaluated in reverse time, considering V is generated by an adjoint wave-equation operator. Figure 2 shows $\Box U$ and $\Box V$ for the theoretical experiment depicted in Figure 1. These figures show that these vectors describe the wavefield propagation direction in this space-time continuum. Notice that these vectors are not orthogonal for the backscattering events using the imaging condition in (15), and we need to slightly change the imaging condition for the purpose of preserving only the reflection information. In order to remove certain undesired events, such as backscattering artifacts, one can properly use this imaging condition to make their respective vectors $\Box U$ and $\Box V$ orthogonal to each other. In fact, this idea can be implemented to attenuate any event with a certain bisection angle.

2.3 Wavefield orientation in the imaging condition

Wavefields are often composed of plane waves near a reflector, which leads us to the reflection angle definition: the angle between the wavenumber vector of the plane wave and the normal to the reflector. For a given experiment, the Fourier transform of the wavefield is a function of the wavenumber vector and frequency: $\tilde{W}(e, \mathbf{k}, \omega)$. Using U and V transformed to the Fourier domain, one can apply the conventional imaging condition

$$I(\mathbf{k}) = \sum_{e,\omega} \tilde{U}\tilde{V}^* \ , \quad (16)$$

where $*$ is the complex conjugate operator in frequency. The energy imaging condition uses the vectors $\Box U$ and $\Box V$ in the time domain. In the Fourier domain, these vectors can be defined in the same way as shown in equation (12), but using the derivative operators with respect to frequency and wavenumber components:

$$\widetilde{\Box U} = \left(ik_x\tilde{U}, ik_y\tilde{U}, ik_z\tilde{U}, -i\tfrac{\omega}{v}\tilde{U} \right) \\ = i\left(\mathbf{k}_s, -\tfrac{\omega}{v}\right)\tilde{V} \ , \quad (17)$$

$$\widetilde{\Box V} = \left(ik_x\tilde{V}, ik_y\tilde{V}, ik_z\tilde{V}, +i\tfrac{\omega}{v}\tilde{V} \right) \\ = i\left(\mathbf{k}_r, +\tfrac{\omega}{v}\right)\tilde{V} \ , \quad (18)$$

considering that the adjoint derivative for V is the opposite to the forward derivative in the Fourier domain. Therefore, the dot product between $\widetilde{\Box U}$ and $\widetilde{\Box V}$ is

$$\widetilde{\Box U} \cdot \widetilde{\Box V} = \left[-\mathbf{k}_s \cdot \mathbf{k}_r + \frac{\omega^2}{v^2} \right] \tilde{U}\tilde{V}^* \\ = \left[-||\mathbf{k}_s||||\mathbf{k}_r|| \cos(2\theta) + \frac{\omega^2}{v^2} \right] \tilde{U}\tilde{V}^*, \quad (19)$$

where 2θ is the reflection aperture angle and also the angle between the spatial gradient vectors. Using the dispersion relation

$$||\mathbf{k}_s|| = ||\mathbf{k}_r|| = \frac{\omega}{v} \quad (20)$$

in equation (19), leads to

$$\widetilde{\Box U} \cdot \widetilde{\Box V} = \left[-\frac{\omega^2}{v^2}\cos(2\theta) + \frac{\omega^2}{v^2} \right] \tilde{U}\tilde{V}^* \ . \quad (21)$$

In equation (21), in order to attenuate a certain reflection of angle θ_c, we need to apply a scaling factor to the second term (product of the time derivatives of the wavefields), thus nullifying the term in the brackets:

$$\widetilde{\Box U} \cdot \widetilde{\Box V} = \left[-\frac{\omega^2}{v^2}\cos(2\theta) + \frac{\omega^2}{v^2}\cos(2\theta_c) \right] \tilde{U}\tilde{V}^* \\ = 0 \ , \text{if } \theta = \theta_c \ . \quad (22)$$

Therefore, in the Fourier domain, the imaging condition with this angle factor is

$$I_E(\mathbf{k}) = \sum_{e,\omega} \widetilde{\Box U} \cdot \widetilde{\Box V} \\ = \sum_{e,\omega} \left[\frac{\omega^2}{v^2}\cos(2\theta_c) - \mathbf{k}_s \cdot \mathbf{k}_r \right] \tilde{U}\tilde{V}^* \ . \quad (23)$$

In the space-time domain, we can modify the imaging condition (15) by including the angle factor:

$$I_E = \sum_{e,t} \left(\cos(2\theta_c)\frac{1}{v}\frac{\partial U}{\partial t}\frac{1}{v}\frac{\partial V}{\partial t} + \nabla U \cdot \nabla V \right) \ . \quad (24)$$

Figure 3 illustrates how the imaging condition in equation (24) attenuates a reflection with a certain angle. The dot-product between the spatial gradient vectors leads to the orthogonal projection of one of the gradient vectors onto the other (Figure 3(a)). Considering the time derivative components of $\Box U$ and $\Box V$ (Figures 3(b) and 3(c)), in order to attenuate this reflection, we need to scale the product of the time derivatives by $\cos(2\theta_c)$ (Figures 3(c) and 3(d)).

2.4 Backscattering attenuation

The backscattering artifact, in particular, is not a reflection. However, its wavenumber vectors $\mathbf{k_s}$ and $\mathbf{k_r}$ are collinear and opposite, with an angle of 180° between them. The artifact can be treated as having a 90° reflection angle, which implies that $\cos(2\theta_c) = -1$. Therefore, we can rewrite equation (24) to form an imaging condition specifically designed to attenuate

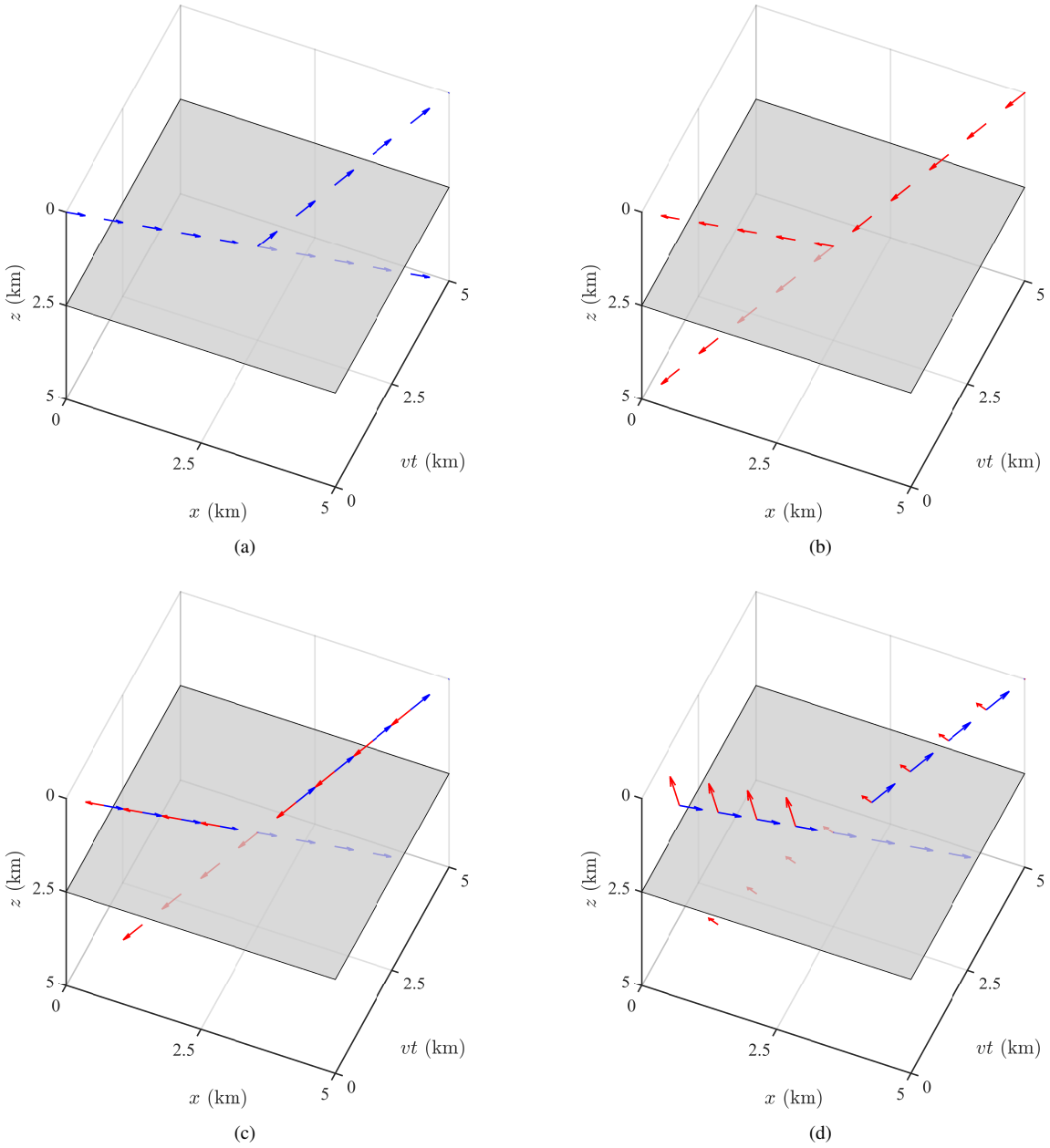


Figure 2. Schematic representation of the vector fields (a) $\Box U$, (b) $\Box V$, (c) the vector fields plotted together and (d) the imaging condition in equation (25). Notice that in (d), the backscattering events are characterized by orthogonal vectors $\Box U$ and $\Box V$, while the reflections are characterized by non-orthogonal vectors $\Box U$ and $\Box V$.

the backscattering energy:

$$I_E = \sum_{e,t} \left(-\frac{1}{v}\frac{\partial U}{\partial t}\frac{1}{v}\frac{\partial V}{\partial t} + \nabla U \cdot \nabla V \right) . \qquad (25)$$

Figure 2(d) shows the vectors $\Box U$ and $\Box V$ for the imaging condition in equation (25). Notice that, for the backscattering artifacts, these vectors are orthogonal, which makes their dot-product zero. This means that the backscattered energy is attenuated and reflectors are preserved if we apply the imaging condition in equation (25).

The attenuation of the backscattering artifacts is one practical benefit of the energy imaging condition over its conventional L_2 counterpart. The Laplacian operator applied on the conventional image is the most common form of backscattering filtering. The proposed imaging condition is related to the

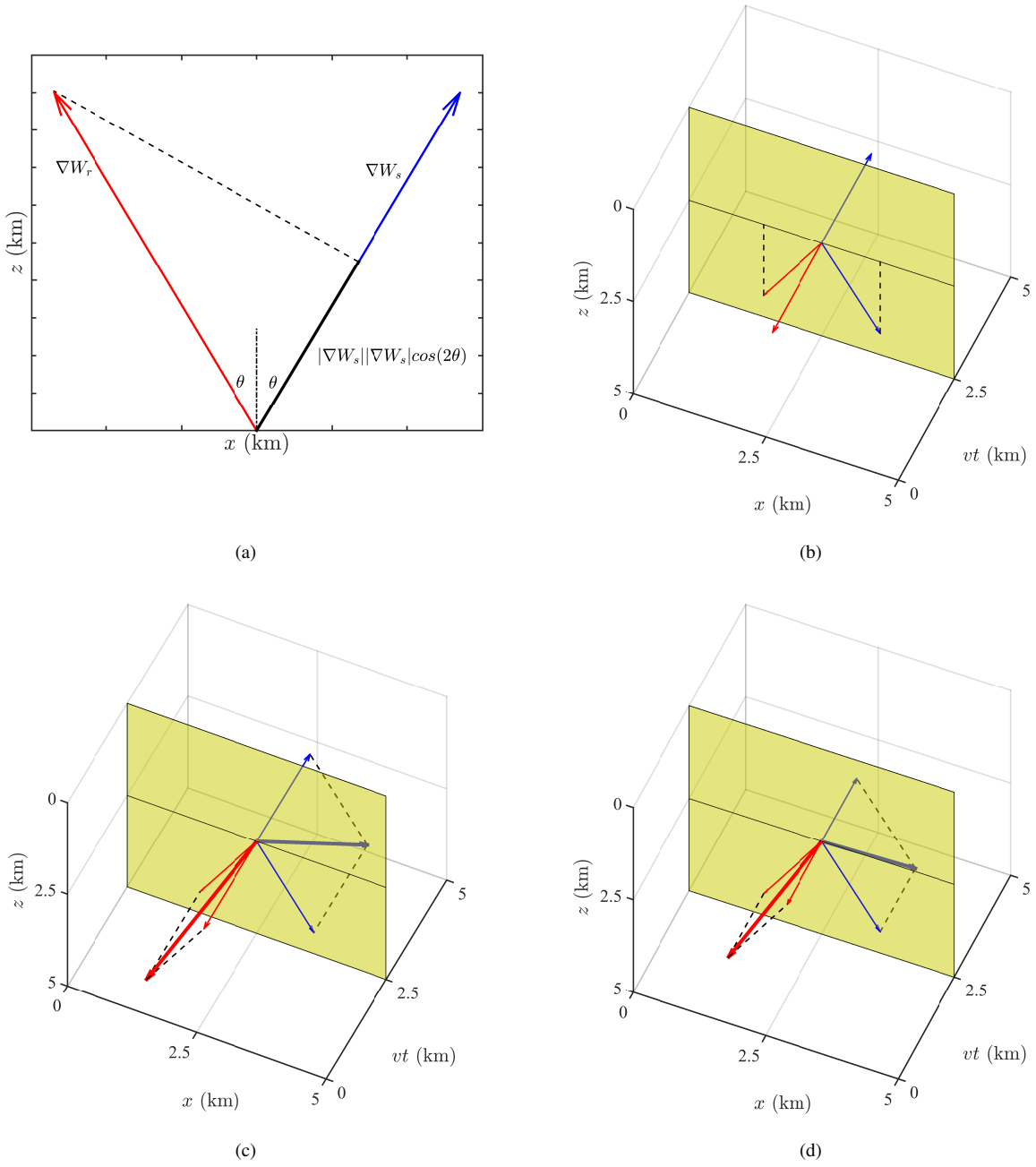


Figure 3. Schematic representation of (a) the gradients for source and receiver wavefields (blue and red, respectively) and the projection of ∇U onto ∇V (bold black); (b) Same schematic representation in (a), but with the additional time dimension and the time-derivatives components of $\Box U$ and $\Box V$; (c) Plot of $\Box U$ and $\Box V$ (bold blue and red vectors) and their respective components; (d) Time-derivative components scaled properly to make $\Box U$ and $\Box V$ orthogonal.

Laplacian filter by (Appendix A)

$$I_E = \frac{1}{2}\nabla^2 I_{L_2} \ . \tag{26}$$

Therefore, the proposed imaging condition is the general form of the filtering using the Laplacian operator for any reflection angle θ_c.

2.5 Amplitude and phase of the energy imaging condition

Compared to the conventional imaging condition (CIC), the angle between $\Box U$ and $\Box V$ influences the amplitude of the energy image, whereas in CIC the amplitude of the image only depends on the strength of U and V. Using I_E from equation (23) for the special case when $\cos(2\theta_c) = -1$, we have

$$I_E = -\sum_{e,\omega} \left[1 + \cos(2\theta)\right] \frac{\omega^2}{v^2} \tilde{U}\tilde{V}^* \ . \tag{27}$$

Following the strategy from Zhang and Sun (2009), in order to preserve the phase compared to the CIC image, one can integrate the source function and receiver data before wavefield extrapolation, which means scaling I_E by $\frac{1}{\omega^2}$ in the frequency domain; after applying the imaging condition, one can multiply the image by $v^2(\mathbf{x})$ at each space location. The amplitude of the resulting image is not exactly equal to the amplitude in CIC due to the term depending on the reflection angle $1 + \cos(2\theta)$. Only if the reflection angle is known for each time and space location of the correlated wavefields, one can multiply I_E by $\frac{-1}{1+\cos(2\theta)}$ along with v^2/ω^2 to get the exact amplitude and phase of CIC. This is also true for the Laplacian filter, and we can obtain the same expression from Zhang and Sun (2009) starting from (26):

$$-\sum_{\omega} 2\left[1 + \cos(2\theta)\right] \frac{\omega^2}{v^2} \tilde{U}\tilde{V}^* = -||\mathbf{k}||^2 \sum_{\omega} \tilde{U}\tilde{V}^* \ . \tag{28}$$

Using the trigonometric identity

$$1 + \cos(2\theta) = 2\cos^2(\theta) \ , \tag{29}$$

we obtain from equation (28) the following relation:

$$||\mathbf{k}||^2 = \frac{4\omega^2 \cos^2(\theta)}{v^2} \ , \tag{30}$$

which means the Laplacian filter also has a term that depends on the reflection angle θ.

Another particular case is when the energy imaging condition is used to attenuate normal-incidence reflections ($\cos(2\theta_c) = 1$). Using equation (23) and the following trigonometric identity

$$1 - \cos(2\theta) = 2\sin^2(\theta) \ , \tag{31}$$

we obtain the following pair of imaging conditions:

$$I_E(\mathbf{k}, \theta_c = 0^\circ) \quad = +2\sum_{e,\omega} \sin^2(\theta) \frac{\omega^2}{v^2} \tilde{U}\tilde{V}^* \ , \tag{32}$$

$$I_E(\mathbf{k}, \theta_c = 90^\circ) \quad = -2\sum_{e,\omega} \cos^2(\theta) \frac{\omega^2}{v^2} \tilde{U}\tilde{V}^* \ , \tag{33}$$

whose difference multiplied by $\frac{v^2}{\omega^2}$ is equivalent to the conventional image. Later in this paper, we exploit the complementary behavior of these two imaging conditions.

2.6 Examples

With simple synthetic examples, we illustrate the application of the proposed imaging condition. Then, we show its effectiveness with a complex model (Sigsbee).

Figure 4 shows a simple constant velocity experiment with a flat reflector at $z = 2$ km, and with one source and one receiver on the surface at $x = 2.5$ km and $x = 7.5$ km, respectively. As expected, the conventional image (Figure 4(a)) shows backscattering artifacts, which are very strong along the path at the 45° reflection angle. The Laplacian filter applied on the conventional image, Figure 4(b), attenuates backscattering artifacts. The energy image in Figure 4(c), which uses the imaging condition in equation (25), shows a similar result compared to Figure 4(b), proving the equivalency of these images for the far-field.

Figure 5 shows the application of the imaging condition in equation (24). In these images, every reflection of a certain angle is attenuated by scaling the time derivative product by $\cos(2\theta_c)$. This proves that our new imaging condition can be used to attenuate reflections for arbitrary angles. Later in this paper, we describe a specific application exploiting this generalization.

Figures 6 show imaging results from selected shots using these different imaging conditions for the Sigsbee model. Stacking all individual migrated images, we obtain the migrated images in Figure 7. Figure 7(a) shows the stratigraphic velocity model that was used to generate the synthetic data for the experiment. The conventional image in Figure 7(b) is masked by the low-frequency backscattered energy. The application of the energy norm imaging condition removes the low-frequency artifacts in Figure 7(c).

3 FULL WAVEFORM INVERSION

The same methodology applied in the preceding section for migration can also be applied to waveform inversion. The gradient of the objective function (required to update the velocity) is equivalent to an RTM image of the data difference, instead of the data themselves, if the adjoint-state method is implemented (Plessix et al., 1999). We begin this section by reviewing the adjoint-state method. Then, we use the energy norm to obtain the inner product between state and adjoint variables, which is required in the calculation of the objective function gradient. Modifying the inner product properly, we are able to filter the gradient and preserve the tomographic term while attenuating the migration term.

3.1 Adjoint-state method

Data-domain wavefield tomography consists of comparing simulated and observed wavefields at specific data locations, improving their similarity at each iteration, and then generating an optimized velocity. The difference between these two wavefields, called the data residual, is the most common form

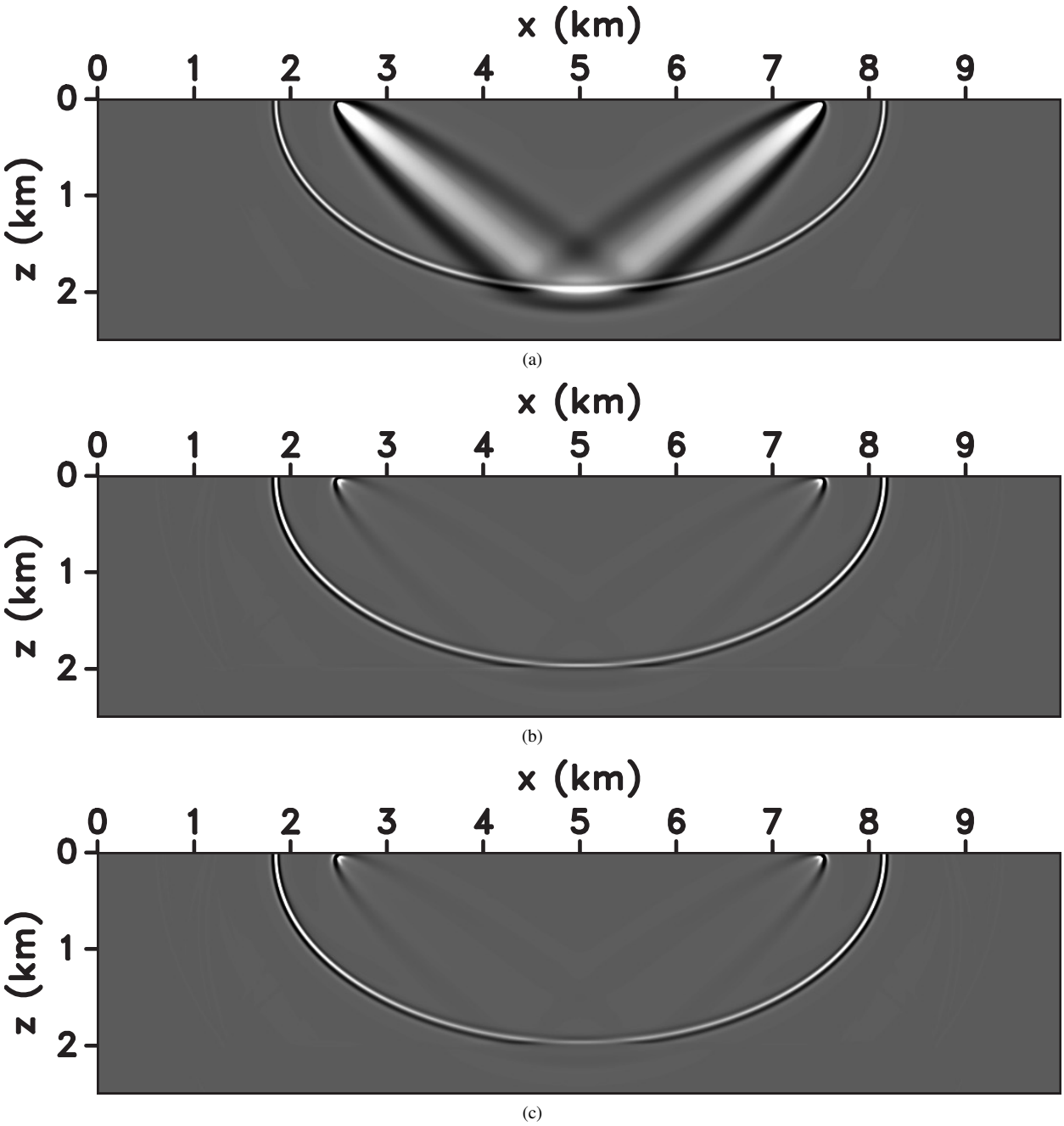


Figure 4. Experiment with a flat reflector, one source and one receiver on the surface at the locations $x = 2.5$ km and $x = 7.5$ km, respectively: (a) conventional imaging condition and (b) with Laplacian filter applied, and (c) energy norm imaging condition.

of comparison:

$$r_d = \mathbf{W}_u(u_s - u_r) \ , \tag{34}$$

where $\mathbf{W}_u\ (e, \mathbf{x}, t)$ is an operator that restricts the wavefields at data locations, and $u_s\ (e, \mathbf{x}, t)$ and $u_r\ (e, \mathbf{x}, t)$ are the source and receiver wavefields, respectively. The source and receiver wavefields restricted at the receiver locations are equivalent to the synthetic and observed data, respectively. The data residual is also a function of space, time and experiment index: $r_d\ (e, \mathbf{x}, t)$. These wavefields are obtained using a proper wave equation, which can be represented by the differential operator $\mathcal{L}$ that admits the adjoint operator $\mathcal{L}^{\mathrm{T}}$ for back propagation. The equations governing the wave propagation for each one of the wavefields are the physical constraints for the inverse

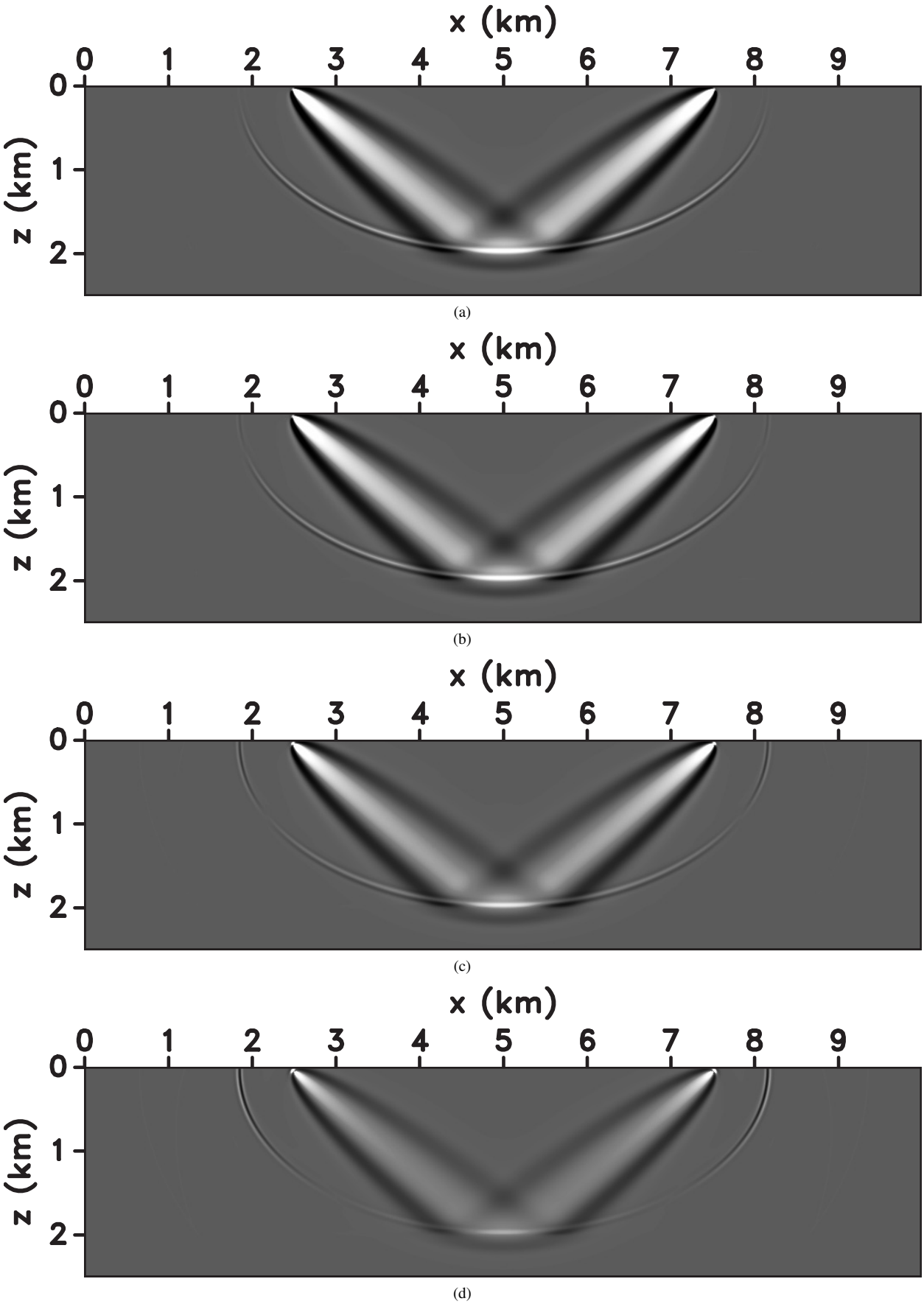


Figure 5. Same experiment geometry and model from Figure 4, but using the energy norm imaging condition to attenuate certain reflection angles: (a) normal-incidence reflections, (b) 15° reflections, (c) 30° reflections, and (d) 45° reflections.

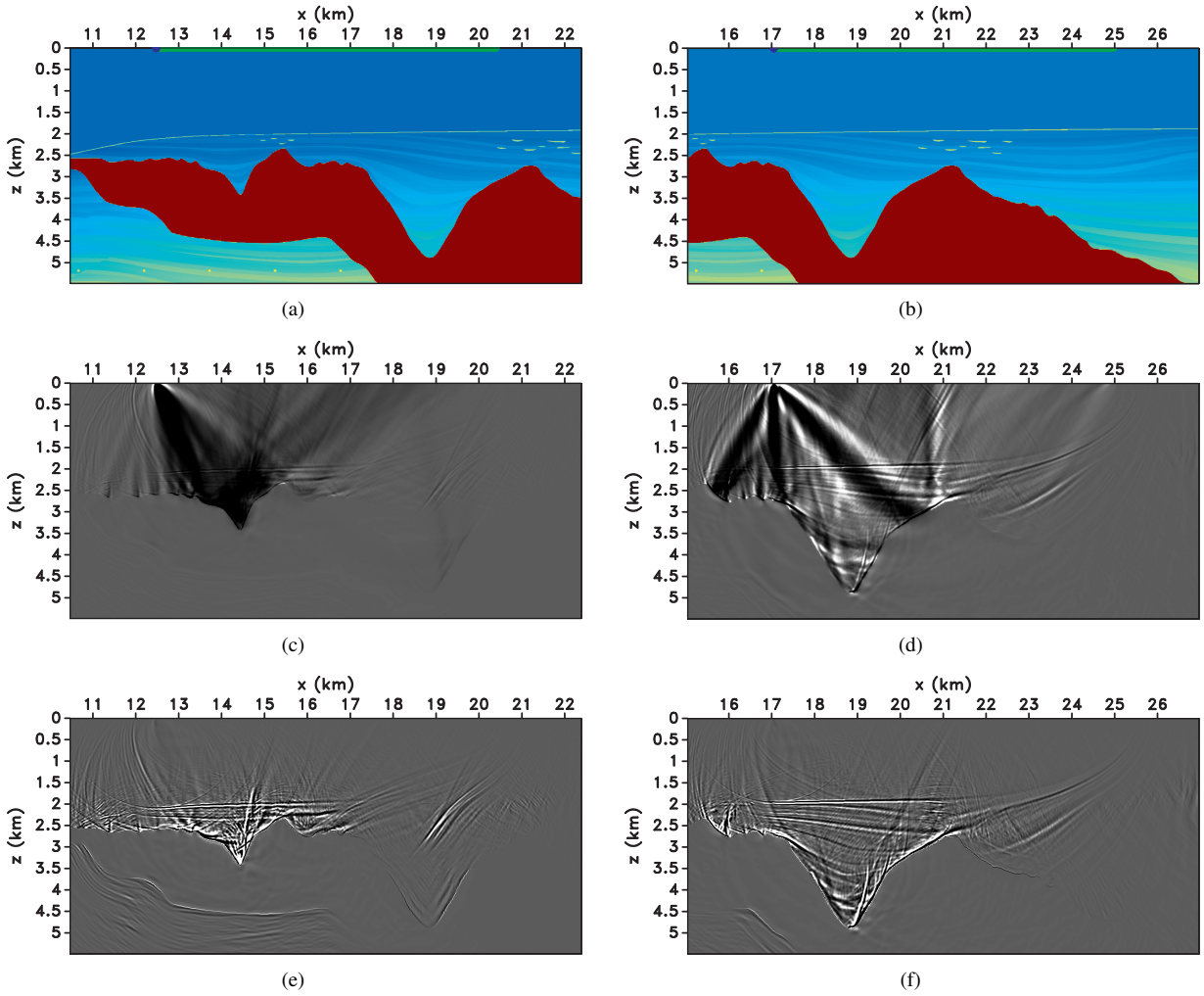


Figure 6. Single-shot Sigsbee migrations. (a) and (b): sections of the stratigraphic velocity model overlaid by source and receiver positions (blue and green, respectively); (c) and (d): conventional images; (e) and (f) energy images.

problem (Sava, 2014):

$$\begin{bmatrix} \mathcal{F}_s \\ \mathcal{F}_r \end{bmatrix} = \begin{bmatrix} \mathcal{L} & 0 \\ 0 & \mathcal{L}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} u_s \\ u_r \end{bmatrix} - \begin{bmatrix} d_s \\ d_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} , \tag{35}$$

where $d_s\,(e,\mathbf{x},t)$ and $d_r\,(e,\mathbf{x},t)$ are the source functions of the wave equations using $\mathcal{L}$ and $\mathcal{L}^{\mathrm{T}}$, respectively.

With a residual, we can define an objective function:

$$\mathcal{J}_{L_2} = \sum_e \frac{1}{2} ||r_d||^2_{L_2} . \tag{36}$$

In order to minimize the defined objective function, we need to compute its gradient with respect to model parameters. The gradient is robustly obtained by the adjoint-state method:

$$\frac{\partial \mathcal{J}}{\partial m} = -\left\langle a, \frac{\partial \mathcal{F}}{\partial m} \right\rangle , \tag{37}$$

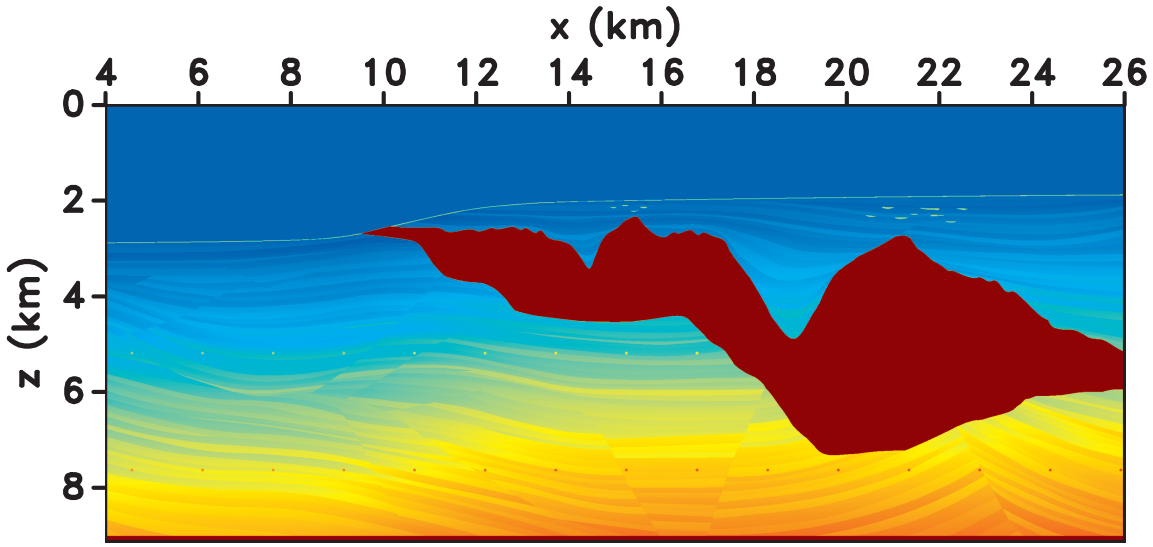
where $m\,(\mathbf{x})$ is the model parameter; $\mathcal{F}$ is the physical realization equation, which can be either $\mathcal{F}_s$ or $\mathcal{F}_r$ in equation (35); $a\,(e,\mathbf{x},t)$ is the adjoint-state variable; and $\langle , \rangle$ is the inner product between the adjoint-state variable and the derivative of $\mathcal{F}$ with respect to the model parameters (Plessix, 2006).

An inner product must output a scalar quantity from two vectors and must have linear properties (Jain et al., 2004). Both inner products in L_2 or L_E norms satisfy these requirements. Therefore,

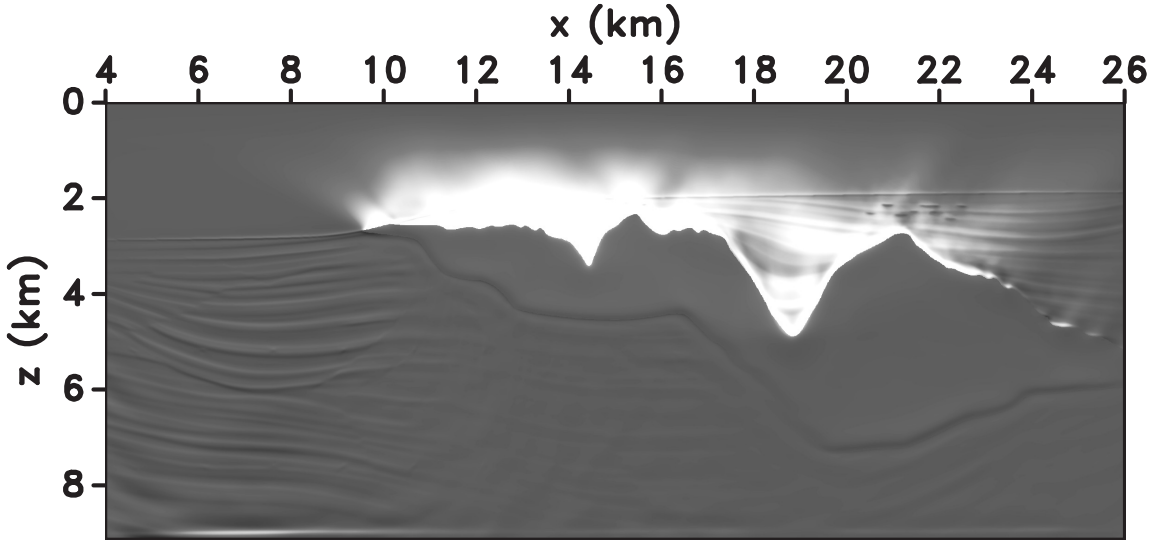
$$\frac{\partial \mathcal{J}}{\partial m} = -\left\langle a, \frac{\partial \mathcal{F}}{\partial m} \right\rangle_{L_2} = -\left(\frac{\partial \mathcal{F}}{\partial m} \right)^{\mathrm{T}} a , \tag{38}$$

$$\frac{\partial \mathcal{J}}{\partial m} = -\left\langle a, \frac{\partial \mathcal{F}}{\partial m} \right\rangle_{E} = -\left\langle \Box a, \Box \frac{\partial \mathcal{F}}{\partial m} \right\rangle . \tag{39}$$

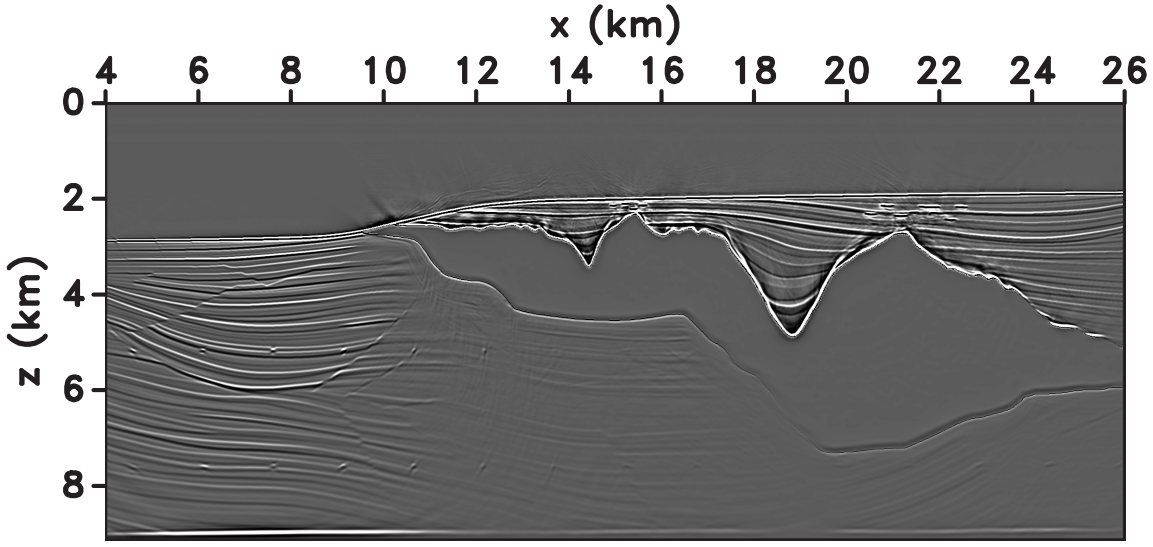
The adjoint-state variable $a\,(e,\mathbf{x},t)$ is computed using the adjoint of the wave-equation operator in either one of the equations in (35). The adjoint-state variable can be defined as a_s or a_r depending on which physical constraint is used from equa-



(a)



(b)



(c)

Figure 7. Mutiple-shot Sigsbee migration: (a) stratigraphic velocity model, (b) conventional image, and (c) energy image.

tion (35):

$$\begin{bmatrix} \mathcal{L}^{\mathrm{T}} & 0 \\ 0 & \mathcal{L} \end{bmatrix} \begin{bmatrix} a_s \\ a_r \end{bmatrix} = \begin{bmatrix} g_s \\ g_r \end{bmatrix} , \tag{40}$$

where $g_s\,(e, \mathbf{x}, t)$ and $g_r\,(e, \mathbf{x}, t)$ are the adjoint sources, and they are defined as the derivative of the objective function with respect to state variables. We can use equation (36) to obtain the adjoint sources:

$$\begin{bmatrix} g_s \\ g_r \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathcal{J}}{\partial u_s} \\ \frac{\partial \mathcal{J}}{\partial u_r} \end{bmatrix} = \begin{bmatrix} +\mathbf{W}_u^{\mathrm{T}} \\ -\mathbf{W}_u^{\mathrm{T}} \end{bmatrix} r_d \; . \tag{41}$$

In summary, our goal is to compute the gradient by using equation (39) in a form that favors the tomographic term and attenuates the migration term.

3.2 Application of the energy norm imaging condition to preserve the tomographic term

In the previous section, we discuss the application of the energy norm imaging condition for attenuating a certain reflection angle, leading to a pair of complementary imaging conditions in equations (32) and (33). As these two images are complementary with respect to angle, the maximum value of one imaging condition is the minimum value of the other imaging condition, and vice-versa. We are interested in selecting a narrow range of reflection angles near 90°, i.e., from backscattering, diving and head waves. However, the image $I_E(\mathbf{k}, \theta_c = 0^\circ)$, which has maximum amplitude at 90°, selects a broad range of reflection angles. A suitable option to make this range narrower is to apply an exponential function that has the greatest decay for angles far from 90°. The complementary image $I_E(\mathbf{k}, \theta_c = 90^\circ)$ indicates how far a certain angle is from 90°. Considering this, we can define the filtered image by an exponential function applied to $I_E(\mathbf{k}, \theta_c = 0^\circ)$ as

$$I_E^{tomo} = I_E(\mathbf{k}, \theta_c = 0^\circ) e^{-a I_E(\mathbf{k}, \theta_c = 90^\circ)} \; , \tag{42}$$

where I_E^{tomo} stands for the tomographic component of the image. In equation (42), by inserting the complementary image $I_E(\mathbf{k}, \theta_c = 90^\circ)$ into the exponential function with a proper factor $a > 1$, we are able to select a narrower range of reflection angles from $I_E(\mathbf{k}, \theta_c = 0^\circ)$, without directly computing these angles. The image inside the exponential should be normalized, such that only the $\sin^2(\theta)$ factor is accounted for in the exponential. In equations (32) and (33), the term that depends on the reflection angle for each imaging condition is $\sin^2(\theta)$ and $\cos^2(\theta)$, respectively. Figure 8 presents a plot of these terms and a plot of $\sin^2(\theta)$ with the exponential function applied. The application of the exponential function to $\sin^2(\theta)$ resembles the exponential filter application in equation (42).

The complementary relation found in equations (32) and (33) is also applicable for the source and receiver wavefields at each time step. The multipathing character of the wavefields causes different reflection angles to overlap in the image. Consequently, a more ideal procedure would involve applying exponential filtering in the wavefield domain, before stacking over time.

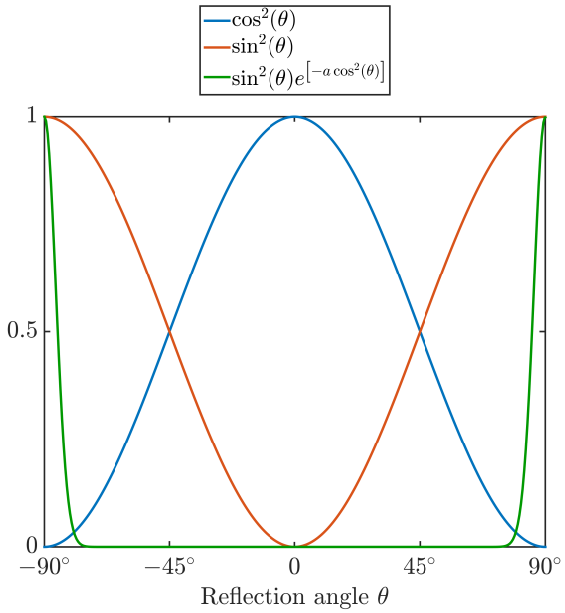


Figure 8. Plots of the angle-dependent functions of the imaging conditions in equation (32) (red line) and (33) (blue line). An exponential in function of $\cos^2(\theta)$ and scaling factor $a = 100$ is applied on $\sin^2(\theta)$ (green line).

3.3 Examples

We investigate the efficiency of the proposed FWI gradient filtering by using a very simple model with a true velocity of 2.5 km/s. Figures 9-11 show experiments with a geometry consisting of one source at $x = 1$ km and one receiver at $x = 5$ km, both at depth $z = 2.25$ km. Each figure depicts the initial velocity with a different percentage of increase from the true velocity: 10% (Figure 9); 20% (Figure 10); 30% (Figure 11). In Figures 9(a), 10(a), and 11(a), the data residuals consist of two separated signals, one from the synthetic and the other from the observed direct wave. The wavefield constructed from the simulated data correlates with the source wavefield along the direct path between source and receiver, thus generating the well-known low-frequency waveform inversion sensitivity kernel. Similarly, the wavefield reconstructed from the observed data correlates with the source wavefield at all locations that match the total traveltime from the source to the receiver. This correlation harms the gradient by placing an apparent reflector where it does not exist; we intend to attenuate this correlation with the proposed exponential filter by selecting only the wavefield correlations at 180°. We can apply the exponential filter either in the image domain (Figures 9(c), 10(c), and 11(c)) or in the wavefield domain (Figures 9(d), 10(d), and 11(d)). The filter is effective in removing the reflections, though weak energy from wide-angle reflections remain. We obtain slightly improved results with filter applied in the wavefield domain (Figures 9(d), 10(d) and 11(d)).

For a similar model with a free surface, we obtain the gradients in Figures 12-14. Due to multiple paths, crosstalk

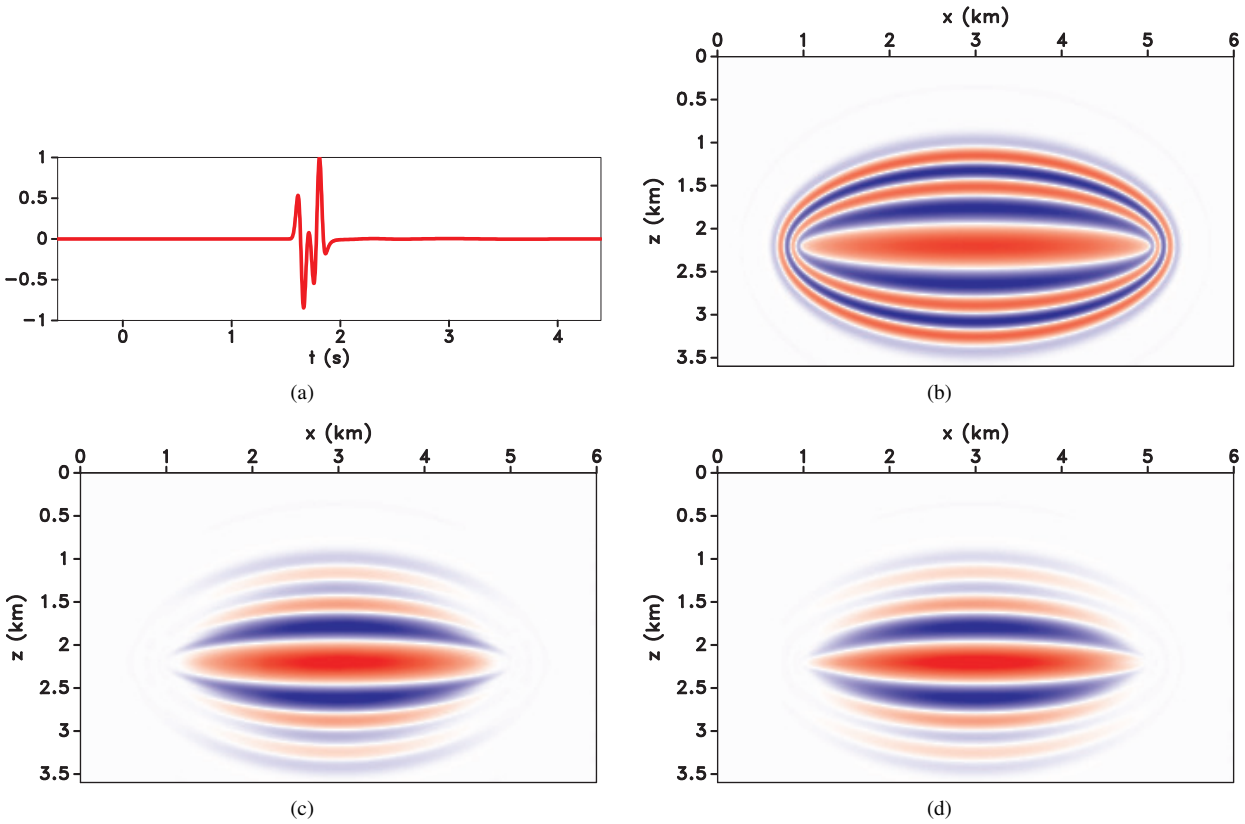


Figure 9. Experiment with an increase of 10% in velocity from the true model. (a) Data residual, (b) conventional FWI gradient, (c) energy FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. Compared to the conventional gradient, the energy imaging condition attenuates the reflection component of the gradient and emphasizes its transmission (tomographic) component.

artifacts are visible in the conventional gradient (Figures 12(b), 13(b), and 14(b)). The filtered gradients (Figures 12(c), 12(d), 13(c), 13(d), 14(c), and 14(d)) show decreased amplitude for wide-angle reflections both in the image and in the wavefield domains. The factor a in the exponent function is empirically chosen for all experiments using this filtering.

4 CONCLUSIONS

The energy imaging condition provides an elegant and effective procedure for attenuating backscattering artifacts in reverse-time migration; this method generates an image representing the projection of four-dimensional gradients ($\Box U$ and $\Box V$) onto one-another. Our new imaging condition also offers a physical explanation for the effectiveness of the Laplacian operator in attenuating events corresponding to waves propagating along the same path. The imaging condition can also be used to attenuate an arbitrary reflection angle. Future work would involve exploiting this flexibility of the imaging condition for attenuating any reflection angle to obtain an extended image in the angle domain. The new imaging condition can also be used to generate complementary images as a function of incidence and reflection angles. This complementary property between the images enables us to filter the FWI gradient in order to attenuate its reflection components and favor the transmission (tomographic) component.

5 ACKNOWLEDGMENTS

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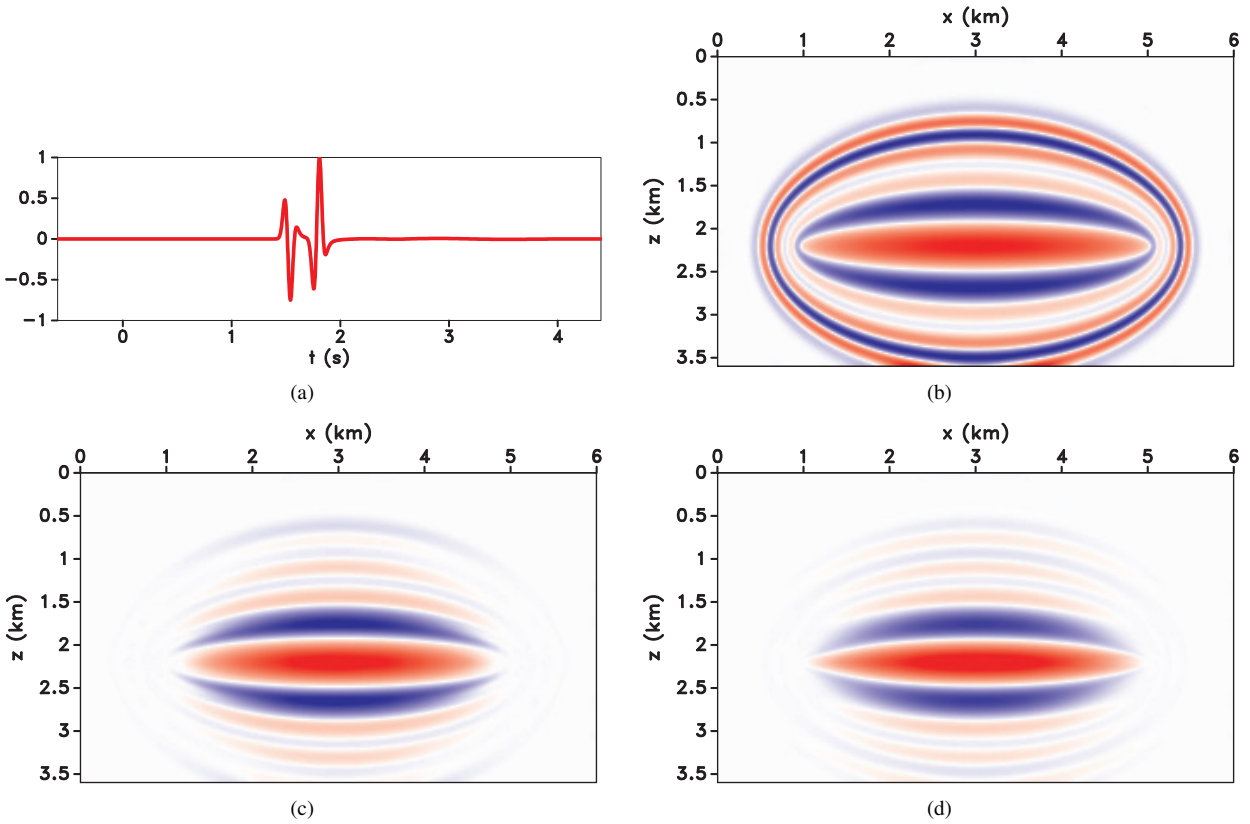


Figure 10. Experiment with an increase of 20% in velocity from the true model. (a) Data residual, (b) conventional FWI gradient, (c) energy FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. Compared to the conventional gradient, the energy imaging condition attenuates the reflection component of the gradient and emphasizes its transmission (tomographic) component.

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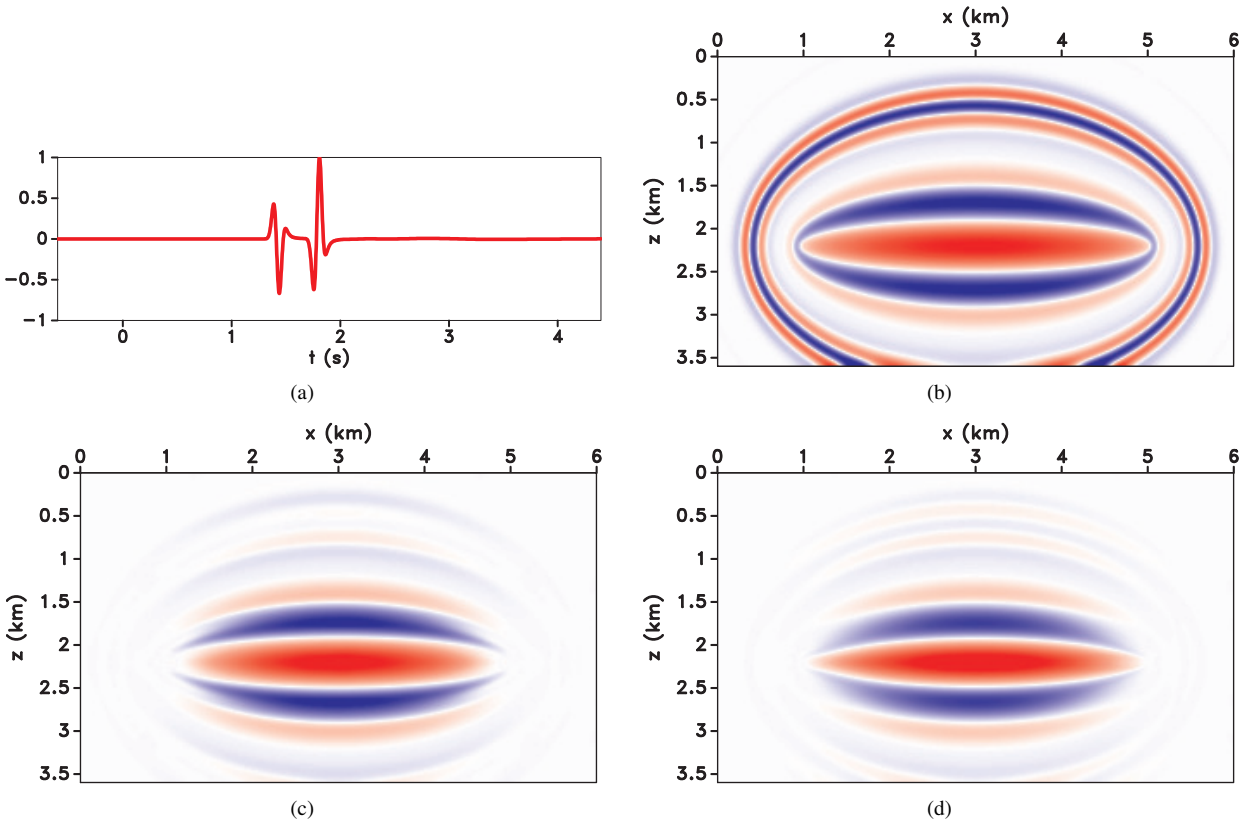


Figure 11. Experiment with an increase of 30% in velocity from the true model. (a) Data residual, (b) conventional FWI gradient, (c) energy FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. Compared to the conventional gradient, the energy imaging condition attenuates the reflection component of the gradient and emphasizes its transmission (tomographic) component.

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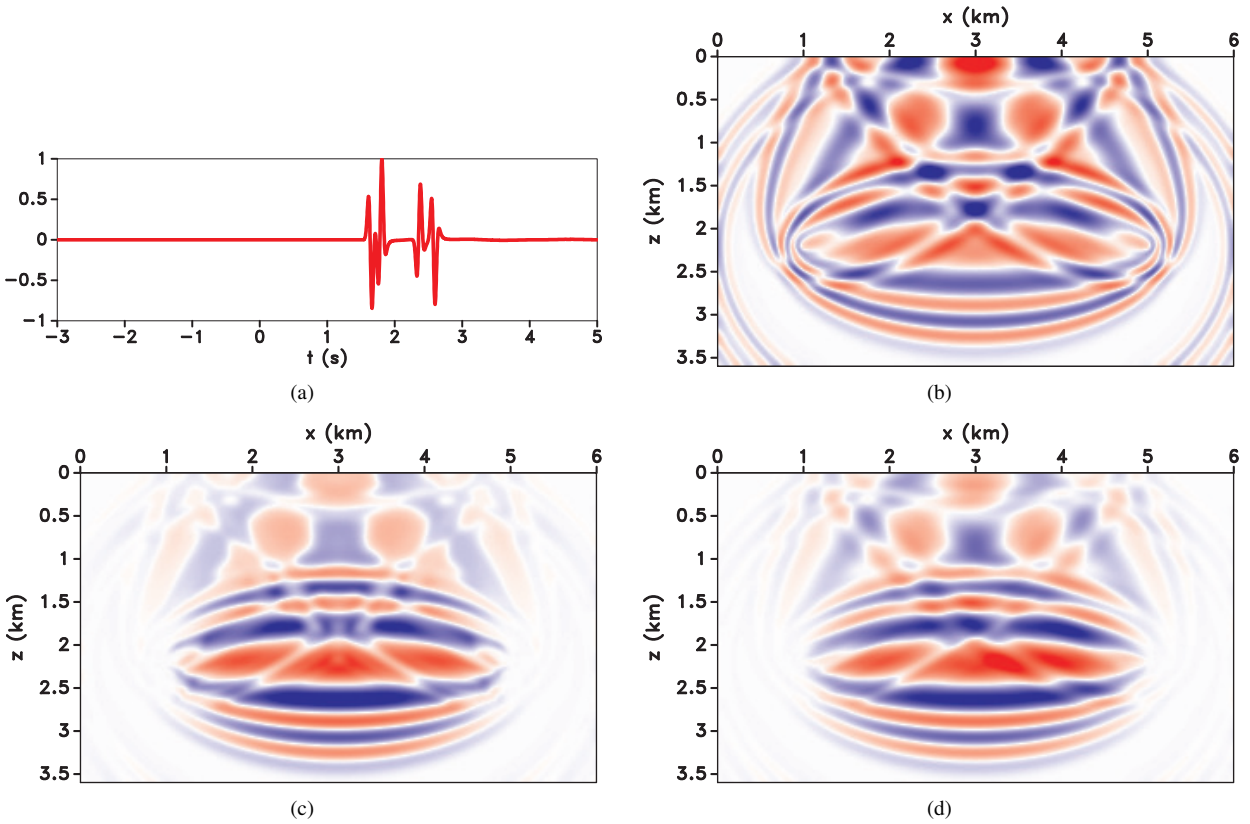


Figure 12. Experiment with an increase of 10% in velocity from the true model. A free surface is present at $x = 0$ km. (a) Data residual, (b) conventional FWI gradient, (c) energy filtered FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. The free surface generates artifacts in the gradient. Most of the reflection artifacts are attenuated.

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6 APPENDIX

6.1 Relation with the Laplacian operator

One can show that the imaging condition in equation (25) is equivalent to the Laplacian filter under certain assumptions.

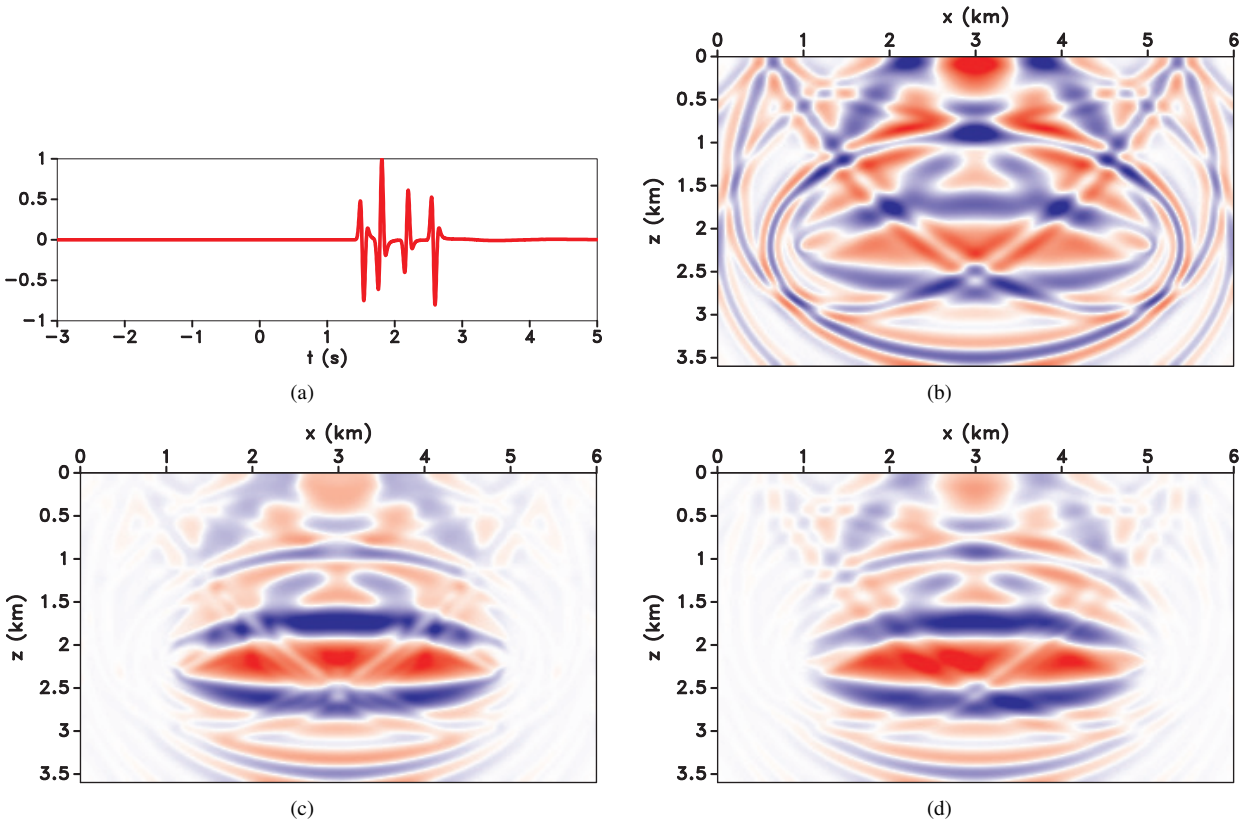


Figure 13. Experiment with an increase of 20% in velocity from the true model. A free surface is present at $x = 0$ km. (a) Data residual, (b) conventional FWI gradient, (c) energy filtered FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. The free surface generates artifacts in the gradient. Most of the reflection artifacts are attenuated.

Rewriting the imaging condition in equation (25) with continuous wavefields along time yields

$$I_E = \int_0^T \left[-\frac{1}{v^2}\frac{\partial U}{\partial t}\frac{\partial V}{\partial t} + \nabla U \cdot \nabla V \right] dt \,, \qquad \text{(A-1)}$$

where the source wavefield is oriented forward in time: $U(\mathbf{x}, t)$; and the receiver wavefield is oriented backward in time: $V(\mathbf{x}, T-t)$. Using integration by parts in the first term of the integrand, we have

$$\begin{aligned} -\int_0^T \frac{\partial U}{\partial t}\frac{\partial V}{\partial t} dt &= -\left[V\frac{\partial U}{\partial t} \right]_0^T + \int_0^T U\frac{\partial^2 V}{\partial t^2} dt \\ &= -\left[U\frac{\partial V}{\partial t} \right]_0^T + \int_0^T V\frac{\partial^2 U}{\partial t^2} dt \,. \end{aligned} \qquad \text{(A-2)}$$

Assuming homogeneous initial and final conditions, the wavefields and their derivatives evaluated at $t = 0$ and $t = T$ become zero. Substituting the remaining terms in equation (A-1) gives

$$I_E = \int_0^T \left[\frac{1}{2}U\frac{1}{v^2}\frac{\partial^2 V}{\partial t^2} + \frac{1}{2}U\frac{1}{v^2}\frac{\partial^2 V}{\partial t^2} + \nabla U \cdot \nabla V \right] dt \,. \qquad \text{(A-3)}$$

In the far field, the source and receiver wavefields satisfy the homogeneous acoustic wave-equation. Therefore,

$$I_E = \frac{1}{2}\int_0^T \left[U\nabla^2 V + U\nabla^2 V + 2\nabla U \cdot \nabla V \right] dt \,. \qquad \text{(A-4)}$$

Using calculus, we obtain

$$\nabla^2 UV = U\nabla^2 V + U\nabla^2 V + 2\nabla U \cdot \nabla V \,, \qquad \text{(A-5)}$$

and therefore

$$I_E = \frac{1}{2}\nabla^2 \int_0^T UV dt = \frac{1}{2}\nabla^2 I_{L_2} \,, \qquad \text{(A-6)}$$

which means the energy imaging condition I_E is equivalent to the Laplacian applied on the conventional image I_{L_2} by a scaling factor of $^1/_2$.

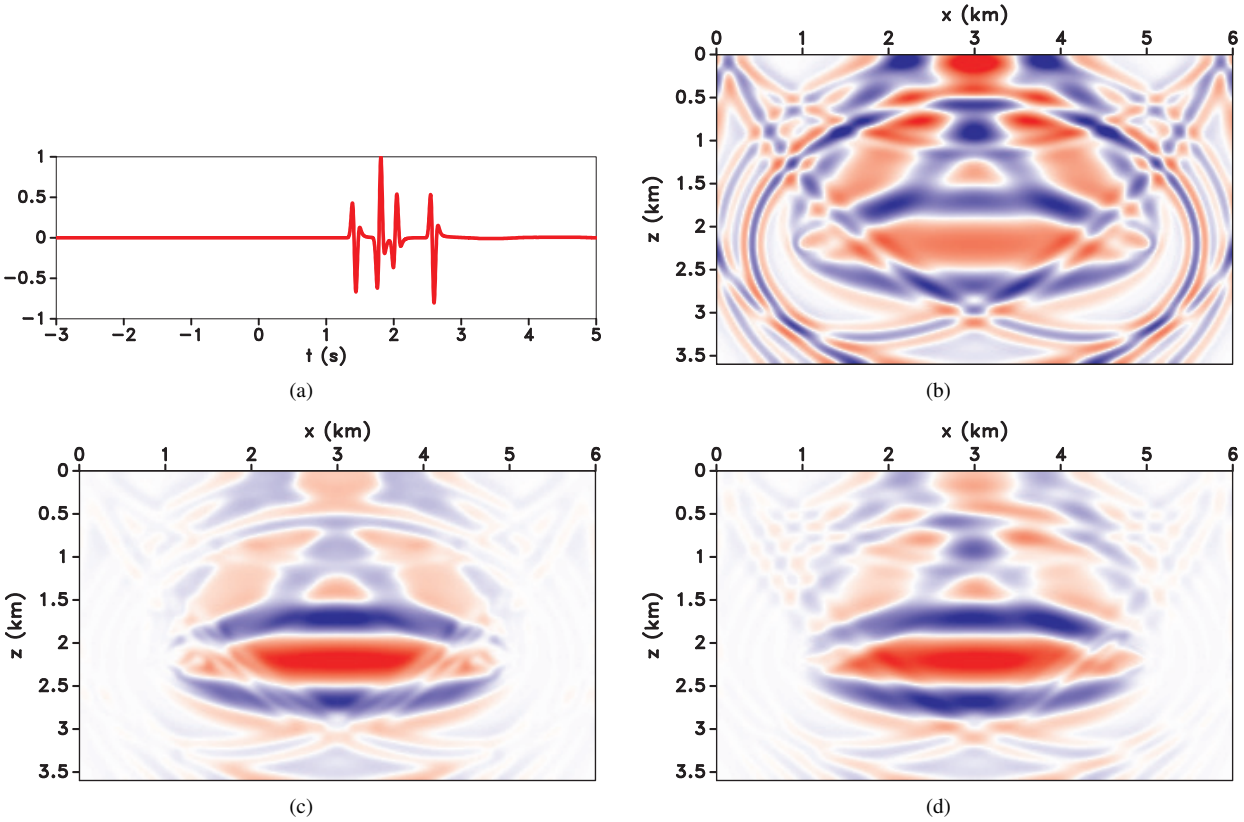


Figure 14. Experiment with an increase of 30% in velocity from the true model. A free surface is present at $x = 0$ km. (a) Data residual, (b) conventional FWI gradient, (c) energy filtered FWI gradient, with the exponential filter applied on the image, and (d) on the wavefields. The free surface generates artifacts in the gradient. Most of the reflection artifacts are attenuated.