3D passive wavefield imaging using the energy norm

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ABSTRACT

In passive monitoring of microseismicity, full-wavefield imaging offers a robust approach for estimating source locations and subsurface properties. With multicomponent data backpropagated with the 3D elastic wave equation, the coexistence of P- and S-modes at the source location allows for application of migration-based imaging conditions to identify the source position. Correlation between decomposed P- and S-wavefields is the most common imaging condition used in passive elastic wavefield imaging. However, this operation produces a nodal plane at the source location that can lead to inaccurate source localization, and requires Helmholtz decomposition, which is costly for anisotropic media. We propose an imaging condition for passive wavefield imaging based on energy conservation and is related to the Lagrangian operator of the elastic wave equation. Our imaging condition not only compares the different modes present in the displacement field, but also produces a strong and focused correlation at the source location without requiring wave-mode decomposition at each time step. Numerical experiments on 3D synthetic and field data demonstrate the advantages of the proposed imaging condition (compared to PS correlation with decomposed wave modes), its sensitivity with respect to velocity inaccuracy, and its quality and efficacy in estimating the source location.

Key words: passive seismic, microseismic, imaging condition, conservation of energy, multicomponent, elastic imaging, 3D

1 INTRODUCTION

Passive seismic monitoring uses signals caused by either natural or induced seismicity to infer earthquake locations, source mechanisms and subsurface properties. Its differences with respect to conventional exploration seismology mainly consist of the absence of a controlled source, but also of distinct array geometries, continuous recording times, and low signal-to-noise ratios, among others. Although the source locations for passive seismic scenario are unknown a priori, one can apply imaging and inversion methods adapted from active source seismic scenarios to generate an image, showing the estimated source location, and/or to reconstruct a 3D earth model, with information about subsurface physical properties (Duncan and Eisner, 2010; Maxwell et al., 2010; Xuan and Sava, 2010; Behura et al., 2013; Blias and Grechka, 2013; Douma and Snieder, 2015; Witten and Shragge, 2015; Bazargani and Snieder, 2016; Witten and Shragge, 2017a,b).

In oil and gas exploration, passive seismic monitoring is called microseismic monitoring (Warpinski et al., 2012; Maxwell, 2014; Grechka and Heigl, 2017), since this type of induced seismicity typically has low orders of moment magnitudes (i.e., $M_w \leq 0$). Microseismic monitoring is a powerful technique for obtaining production attributes from unconventional reservoirs, especially when investigating hydraulic fracturing. Where fluid injection into a reservoir induces microseismicity observable from surface or borehole monitoring arrays, one can use recorded data to estimate microseismic source locations and thereby gain insight as to where hydraulic fracturing is occurring in the subsurface (Maxwell, 2010; Michel and Tsvankin, 2013). Beyond source location estimation, microseismic monitoring (using a catalog of detected events) facilitates other useful applications such as estimating source mechanisms, potentially providing geomechanical information such as fault and fracture orientation (Jeremic et al., 2014; Zhebel and Eisner, 2015).

One common approach for analyzing microseismic data involves applying a Kirchhoff-based migration algorithm to stack windowed events along traveltime trajectories and over all possible source initiation times, assuming a sufficiently accurate velocity model (Kao and Shan, 2004; Baker et al., 2005). However, such approach does not stack over the full

waveform and imply strong assumptions about velocity model accuracy and associated wave propagation effects (e.g., neglecting multipathing) that are not appropriate for realistic scenarios. Alternatively, approaches based on wave-equation migration have gained interest for passive seismic imaging (Artman et al., 2010; Xuan and Sava, 2010; Witten and Shragge, 2015; Nakata and Beroza, 2016) since they use the full waveform of these signals (beyond stacking along traveltime trajectories), accurately handle wave propagation in complex earth models, and offer a range of imaging conditions that effectively stack over the full microseismic waveform.

Full-wavefield imaging adapted to the problem of estimating microseismic source locations is usually implemented in two steps: (1) time-reversed wavefield extrapolation (or more simply, backpropagation) of the recorded wavefield into an earth model; and (2) application of an imaging condition to extract the source location and/or origin time from the extrapolated wavefield (McMechan, 1982; Gajewski and Tessmer, 2005; Xuan and Sava, 2010; Nakata and Beroza, 2016). Generally, the imaging condition involves computing the zero-lag autocorrelation of the 4D wavefield, collapsing it into a 3D image volume. Assuming a correct velocity model and proper treatment of source radiation patterns, the peak amplitude of the image corresponds to the true source location; however, not satisfying these assumptions can often lead to significant mislocation of imaged events. To address velocity model inaccuracy, it is insufficient to rely solely on zero-lag wavefield autocorrelation to provide information for velocity improvement. As explored in active seismic scenarios (Rickett and Sava, 2002; Sava and Fomel, 2006; Sava and Vasconcelos, 2011; Yang and Sava, 2015) and more recently in passive seismic ones (Witten and Shragge, 2015), wavefield correlation beyond zero lag generates so-called extended images (since they represent an extension of zero-lag images), which provide the sufficient and necessary information to improve the migration velocities within the adjoint-state tomography framework (Witten and Shragge, 2017a).

For multicomponent microseismic data, one typically employs wave extrapolation and imaging procedures that exploit distinct elastic wave modes. The colocation of P- and S-wavefields at the source excitation point allows for a PS imaging condition to be implemented in three steps: (1) elastic (or pseudo-elastic) backpropagation of the recorded multicomponent wavefield; (2) wave-mode decomposition of the backpropagated displacement field; and (3) crosscorrelation of the two P- and S-wavefields (Artman et al., 2010; Witten and Artman, 2010). However, especially in 3D applications incorporating anisotropy of the medium, step (2) is computationally expensive and thus impedes a quick and robust imaging implementation as it must be repeated at each extrapolation time step. Another shortcoming of the elastic PS correlation is that it does not properly handle the different polarization directions at the locus of maximum P- and S-phase correlation, producing a nodal plane (i.e., a zero) at the actual source location. Ideally (and especially in the presence of low signal-to-noise ratio), one would expect to encounter the peak amplitude at the source position, which otherwise poses both interpretation as well as inversion challenges.

We aim to address these shortcomings of the multicomponent elastic PS imaging condition that are detrimental to both source location identification and migration velocity analysis of microseismic data. By substituting PS correlation with energy correlation (Rocha et al., 2016b, 2017), we propose an imaging condition for passive seismic applications that has five key advantages over conventional PS imaging condition: (1) naturally accounts for source radiation patterns; (2) produces a peak image amplitude at the estimated source location; (3) precludes the need for wave-mode decomposition at each time step; (4) downweights the correlation between identical wave modes away from the source while enhancing the correlation of different wave modes at the source location; and (5) offers a straightforward complement to extended image attributes that are important for microseismic migration velocity analysis based on the temporal separation of P- and S-wave arrivals. For the purpose of improving image quality, we implement wave-mode separation - advantage (3) - only once in the data domain.

In this paper, we review the theory of elastic modeling and associated conventional imaging conditions; introduce the energy imaging condition, originally developed for active-source wavefield imaging - elastic reverse time migration (ERTM), and detail how we adapt it to passive seismic scenarios. Then, we present the extended-domain versions of PS and energy imaging conditions, and show an optional novel methodology applied in our numerical experiments to mitigate imaging artifacts, albeit with an increase in cost. We demonstrate our methodology with synthetic examples of increasing complexity, as well as with field data. The paper concludes with a discussion of the merits and caveats of our method, and with suggestions for further refinement of the method.

2 THEORY

In this section, we review the elastic wave equations used in full-wavefield imaging, as well as the most commonly employed imaging conditions for multicomponent passive data. Then, we propose an imaging condition based on the energy norm for elastic wavefields. Finally, we introduce the extended version of our proposed imaging condition that is potentially important for velocity estimation applications.

2.1 Elastic modeling

The elastic wave equation in the absence of external sources may be written as (Aki and Richards, 2002)

$$\rho\ddot{\mathbf{U}} = \nabla \cdot \left[\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}\right] \ , \tag{1}$$

where $\mathbf{U}\left(\mathbf{x},t\right)$ is the displacement field as a function of space $(\mathbf{x})$ and time (t), $\rho(\mathbf{x})$ is the medium density, and $\underline{\underline{\mathbf{c}}}(\mathbf{x})$ is the 4th-rank stiffness tensor. A superscript dot on the displacement field indicates first-order time differentiation. If we assume an

isotropic and slowly varying medium, equation 1 reduces to

$$\rho\ddot{\mathbf{U}} = (\lambda + 2\mu)\,\nabla\,(\nabla \cdot \mathbf{U}) - \mu\nabla \times (\nabla \times \mathbf{U}) \ , \quad (2)$$

where $\lambda\,(\mathbf{x})$ and $\mu\,(\mathbf{x})$ are Lamé parameters. Equation 2 can be written as a function of velocities by substituting $V_P^2 = \frac{\lambda+2\mu}{\rho}$ and $V_S^2 = \frac{\mu}{\rho}$:

$$\ddot{\mathbf{U}} = V_P^2\nabla\,(\nabla \cdot \mathbf{U}) - V_S^2\nabla \times (\nabla \times \mathbf{U}) \ , \quad (3)$$

where $V_P\,(\mathbf{x})$ and $V_S\,(\mathbf{x})$ are the P- and S-wave velocities, respectively. Using multicomponent data recorded at the receivers, a displacement field $\mathbf{U}$ can be extrapolated forward or backward in time throughout a subsurface model using either equations 1 or 3, depending on the assumptions about anisotropy of the medium. Although more realistic in nature, incorporating anisotropy involves complex phenomena such as shear-wave splitting (Crampin, 1985; Alford, 1986), which introduces two shear wave modes with different polarizations and phase velocities in wavefield extrapolation. Without loss of generality, we assume isotropy in the following numerical experiments so as to analyze only a single shear-wave mode (and for clarity purposes). We discuss the extension of our imaging approach to anisotropic media in the Discussion section.

2.2 Passive imaging conditions: autocorrelation

Different imaging conditions acting directly on the wavefield displacement have been proposed for estimating the source locations in passive wavefield imaging (Steiner et al., 2008; Jeremic et al., 2015). For instance, Steiner et al. (2008) propose to use the absolute value of the particle velocity, i.e. the time derivative of the displacement vector field, while Saenger (2011) presents an imaging condition based on the product between stress and strain fields. More commonly, one implements wave-mode decomposition of the extrapolated displacement field into its P- and S-wavefield constituents, typically implemented using Helmholtz decomposition operators (Dellinger and Etgen, 1990; Yan and Sava, 2009):

$$P = \nabla \cdot \mathbf{U} \ , \quad (4)$$

$$\mathbf{S} = \nabla \times \mathbf{U} \ , \quad (5)$$

where $P\,(\mathbf{x}, t)$ is a scalar wavefield containing the compressional wave mode, and $\mathbf{S}\,(\mathbf{x}, t)$ is a vector wavefield containing the transverse wave mode.

Because a scalar wavefield image is generally preferable to a vector or tensor image, but the S-wavefield from this decomposition remains a 3D vector field, passive imaging conditions can use the energy densities E_P and E_S of the decomposed P- and S-wavefields (Morse and Feshbach, 1953):

$$E_P\,(\mathbf{x}, t) = (\lambda + 2\mu)\,\|P\| \ , \quad (6)$$

$$E_S\,(\mathbf{x}, t) = \mu\|\mathbf{S}\| \ . \quad (7)$$

Separated wave modes and their energy densities allow one to implement autocorrelation passive imaging conditions:

$$I_{PP}\,(\mathbf{x}) = \sum_t P\,(\mathbf{x}, t)\,P\,(\mathbf{x}, t) \equiv \sum_t E_P^2\,(\mathbf{x}, t) \ , \quad (8)$$

$$I_{SS}\,(\mathbf{x}) = \sum_t \mathbf{S}\,(\mathbf{x}, t) \cdot \mathbf{S}\,(\mathbf{x}, t) \equiv \sum_t E_S^2\,(\mathbf{x}, t) \ . \quad (9)$$

2.3 Passive imaging conditions: cross-correlation

A recurrent problem with the imaging conditions in equations 8 and 9 is that the correlation of the wavefield with itself produces low-wavenumber content along the propagation path, e.g. between the source in the subsurface and the receivers on the surface or in a borehole. The same problem occurs with imaging conditions that directly use the displacement field in the absence of wave-mode decomposition. This poses significant challenges for microseismic migration and inversion-based analyses that require tightly focused event images.

Alternatively, one can use the different modes of the displacement field to define an imaging condition free of low-wavenumber autocorrelation artifacts. For a non-scattering medium and assuming finite source bandwidth, extrapolated P- and S-waves from a single microseismic event propagate at different speeds and only coexist in space and time at the vicinity of the source location and at initiation time. Exploiting this fact, one can define a PS imaging condition (Artman et al., 2010):

$$\mathbf{I}_{PS}\,(\mathbf{x}) = \sum_t P\,(\mathbf{x}, t)\,\mathbf{S}\,(\mathbf{x}, t) \ , \quad (10)$$

where $\mathbf{I}_{PS}\,(\mathbf{x})$ is a vector image, whose individual components represent the correlation between $P\,(\mathbf{x}, t)$ and the corresponding components of $\mathbf{S}\,(\mathbf{x}, t)$. Yan and Sava (2008) also propose a similar PS imaging condition, but in the context of elastic reverse time migration (ERTM). However, one drawback is that, in a 3-D experiment, three images are available instead of a scalar one concisely showing the source location. Considering this, one could conveniently compute a scalar PS image using the S-wave energy density in equation 7:

$$I_{PS}\,(\mathbf{x}) = \sum_t P\,(\mathbf{x}, t)\,E_S\,(\mathbf{x}, t) \ . \quad (11)$$

For isotropic media, equation 11 is quite straightforward to implement; however, this is not the case in anisotropic media for which wave-mode decomposition is significantly more intensive to compute (Dellinger and Etgen, 1990; Zhang and McMechan, 2010; Yan and Sava, 2011; Cheng and Fomel, 2014). Also, the PS correlation - either in equation 10 or 11 - produces a zero amplitude at the source location and an associated nodal plane, whose positive and negative side lobes suggest the source localization at the zero crossing. Although pre-processing techniques exist to address this problem (Eisner et al., 2008; Ay et al., 2012; Witten and Shragge, 2015), they lead to lower-frequency wavefields and loss of spatial resolution.

2.4 Energy norm imaging conditions

In contrast, the energy norm (Rocha et al., 2016a,b) allows one to define a pair of imaging conditions for many applications, with no difficulty in its implementation regardless of anisotropy of the medium. In that context, this pair of imaging conditions is defined as (Rocha et al., 2017)

$$I_E^H(\mathbf{x}) = \sum_t \left[\rho\dot{\mathbf{U}}\cdot\dot{\mathbf{V}} + \left(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}\right) : \nabla\mathbf{V}\right], \quad (12)$$

$$I_E^L(\mathbf{x}) = \sum_t \left[\rho\dot{\mathbf{U}}\cdot\dot{\mathbf{V}} - \left(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}\right) : \nabla\mathbf{V}\right], \quad (13)$$

where $\mathbf{U}(\mathbf{x},t)$ and $\mathbf{V}(\mathbf{x},t)$ are two state wavefields, and the symbol : indicates the Frobenius product (Golub and Loan, 1996). The imaging condition in equation 12 represents the temporal integration of the total energy product between the two wavefields, while that in equation 13 is a differential energy measure that attenuates events within the two wavefields sharing the same polarization and propagation direction. The correlation between such events generates the aforementioned detrimental low-wavenumber artifacts. Rocha et al. (2017) successfully use the imaging condition in equation 13 for ERTM (where $\mathbf{U}(\mathbf{x},t)$ and $\mathbf{V}(\mathbf{x},t)$ are the source and receiver wavefields), suppressing these artifacts and concisely imaging subsurface reflectors.

For passive imaging, where we extrapolate a single wavefield containing all wave modes, we can also define a pair of autocorrelation imaging conditions by establishing $\mathbf{U} = \mathbf{V}$ in equations 12 and 13. With such substitution, the individual terms have a special meaning in reference to the energy of the elastic wavefield: the first and second terms in equations 12 and 13 represent the kinetic and potential energies of the single wavefield, respectively. Therefore, equation 12 is related to the Hamiltonian operator (superscript H stands for Hamiltonian), which measures total wavefield energy, while equation 13 is related to the Lagrangian operator (superscript L stands for Lagrangian), defined as the difference between kinetic and potential energy terms (Ben-Menahem and Singh, 1981). We note that Saenger (2011) uses the potential term alone as an imaging condition, but here we go beyond by combining kinetic and potentials terms to effectively mitigate low-wavenumber content away from the source and to generate the temporal integral of the wavefield Lagrangian at the source location.

Although the passive imaging condition in equation 13 involves autocorrelation of the displacement field $\mathbf{U}$, the correlation between the same wave modes is attenuated as shown by Rocha et al. (2016b, 2017). In contrast, at the source, the correlation between P- and S-wave modes is preserved, producing a peak amplitude directly related to the source mechanism (see Appendix A) instead of a nodal plane as the PS correlation. Therefore, the passive energy imaging condition correlates different modes using the displacement field directly, without wave-mode decomposition at each extrapolation time step.

2.5 Passive imaging conditions: extended domain

The passive imaging conditions presented above are based on zero-lag wavefield correlation; thus, they can be considered as special cases of a more general correlation-based comparison between wavefields. As discussed previously, extending wavefield correlation beyond zero lag is useful since it provides additional information such as velocity accuracy (Sava and Vasconcelos, 2011). The temporal and spatial dependences of the wavefields allow one to compute space-lag image gathers (Rickett and Sava, 2002) and/or time-lag image gathers (Sava and Fomel, 2006). Extended imaging conditions (EICs) are also applicable to passive imaging as shown by Witten and Shragge (2015) for extrapolated scalar wavefields.

For wave-mode decomposed elastic wavefields, we define

$$I_{\alpha_1\alpha_2}(\mathbf{x},\boldsymbol{\lambda},\tau) = \sum_t \alpha_1(\mathbf{x}+\boldsymbol{\lambda},t+\tau)\,\alpha_2(\mathbf{x}-\boldsymbol{\lambda},t-\tau)\ , \quad (14)$$

where $\boldsymbol{\lambda}$ and τ are the spatial and temporal lags, respectively. Symbols α_1 and α_2 stand for either $P(\mathbf{x},t)$ or components of $\mathbf{S}(\mathbf{x},t)$ from equation 10; however, this can generate many possible extended images for elastic potentials when using the S-vector potential components. Exploiting the scalar S-wave energy density leads to a single extended image similar to equation 11. For the energy imaging condition in equation 13, we adapt the approach used in equation 14 to write

$$I_E^L(\mathbf{x},\boldsymbol{\lambda},\tau) = \sum_t \Big[\rho(\mathbf{x}+\boldsymbol{\lambda})\dot{\mathbf{U}}(\mathbf{x}+\boldsymbol{\lambda},t+\tau)\cdot\dot{\mathbf{U}}(\mathbf{x}-\boldsymbol{\lambda},t-\tau) - \underline{\underline{\mathbf{c}}}(\mathbf{x}+\boldsymbol{\lambda})\nabla\mathbf{U}(\mathbf{x}+\boldsymbol{\lambda},t+\tau) : \nabla\mathbf{U}(\mathbf{x}-\boldsymbol{\lambda},t-\tau)\Big], \quad (15)$$

where we note that ρ and $\underline{\underline{\mathbf{c}}}$ are similarly shifted leaving the resulting image volumes asymmetric. We can also obtain the extended imaging condition for the Hamiltonian form by changing the sign between the two term inside brackets in equation 15.

In general, one can estimate the velocity inaccuracy on extended image gathers by evaluating focusing quality around zero lag. The more focused the energy is around zero lag, the closer the migration velocity is to the true velocity, as shown in many cases for active source seismic wavefield tomography (Albertin et al., 2005; Yang and Sava, 2011; Diaz et al., 2013; Yang et al., 2013; Yang and Sava, 2015). More recently, Witten and Shragge (2017a,b) use extended images in passive source wavefield tomography and show how calculating a single EIC at each estimated event location can assist with the elastic velocity inversion procedure. In this paper, besides evaluating the quality of the energy image with the correct velocity, we investigate the extended images for both PS and energy imaging conditions to investigate sensitivity to velocity inaccuracy and thereby infer potential application to the microseismic velocity estimation problem.

3 METHODOLOGY

Although the energy imaging conditions in equations 12 and 13 exploit the interaction of backpropagated P- and S- direct arrivals, we need to address some caveats that arise during practical implementation of elastic time-reversal wavefield extrapolation: limited aperture and the presence of fake modes artifacts. In general, acquired seismic data are incomplete and possibly spatially aliased, causing truncation artifacts (especially at the edges of the receiver acquisition patch) during backpropagation. Truncations are characterized with the same wave mode as the original data events, but have different propagation directions that can cause spurious artifacts when applying the energy imaging condition. With a straightforward numerical experiment in the Examples section, we illustrate how the truncations create such artifacts - and demonstrate how they can be mitigated.

A second class of artifact appearing in both PS and energy images arises from the injection of single-mode data events as wavefield displacement into the extrapolation model. This generates both P- and S-wave modes regardless of their single-mode character, which are commonly called "fake modes" since they do not represent true data events (Yan and Sava, 2008; Ravasi and Curtis, 2013; Duan and Sava, 2015).

In order to mitigate both types of artifacts in the PS and energy images and obtain a better source location estimation result, we choose to first separate P- and S-wave arrivals in the data domain and generate two elastic wavefields, one with each individual wave mode arrival. Following this approach leads to PS and energy images with wave-mode separated wavefields. Then, such images may be subtracted from the corresponding PS and energy images that use both P- and S-arrivals as input to generate a final image I free of artifacts

$$I = I^{(P,S)} - I^{(P)} - I^{(S)} \,, \tag{16}$$

where $I^{(P)}$ and $I^{(S)}$ are images using P- and S-wave arrivals only as input, respectively, while $I^{(P,S)}$ uses both arrivals. We compute all image terms in equation 16 with the same imaging condition, either PS (I_{PS}) or energy (I_E): the superscripts with parentheses only indicate images formed with P- and/or S-data events. By following this novel approach of mitigating extrapolation-related artifacts, we obtain an energy image free of correlated truncation events and with attenuated fake-mode artifacts.

Separating P- and S-wave arrivals in the multicomponent data domain is computationally inexpensive, but does introduce a preprocessing step that requires user input in determining the windowing parameters. Only one additional wavefield extrapolation is necessary, considering that we can generate wavefield $\mathbf{U}$ - from the P-arrival only - and wavefield $\mathbf{V}$ - from the S-arrival only. A third composite wavefield, $\mathbf{W} = \mathbf{U} + \mathbf{V}$, is effectively equal to extrapolation using both P- and S-arrivals by considering linearity of the extrapolators. Witten and Shragge (2017a) also require wave-mode separation in the data domain and two wavefield extrapolations for their acoustic passive imaging and inversion implementations. It is important to note that this data-domain separation approach would be more difficult for a swarm of microseismic events containing overlapping P- and S-wave arrivals.

4 3-D NUMERICAL EXAMPLES

In this section, we present energy and PS images showing the estimated source position for correct constant velocity models. Then, we analyze the imprint of velocity inaccuracy on both conventional and extended images. We examine the benefits of applying the energy imaging condition in a more complex 3D synthetic velocity model with realistic acquisition configuration. We conclude by applying the method to a microseismic event from a 3-D field dataset.

4.1 Experiment 1: focusing with correct velocity

Our first numerical experiment investigates the imaging conditions developed above in an idealized microseismic monitoring experiment where the true velocity model is constant and known ($V_P = 3.0$km/s and $V_S = 1.8$km/s). In this scenario, we expect that a well-devised imaging condition should deliver a strong and focused correlation at the correct event location. Figure 1(a) shows the acquisition and model geometry of such experiment where multicomponent receivers are distributed at the surface with a realistic fixed spacing ($\Delta x = \Delta y = 0.25$km, covering a 16km^2 area). We locate the microseismic event at the center of the 3-D model ($x = y = z = 2$km) and assume a stress source mechanism oriented at 45° with respect to the x-axis. For this scenario, the source moment tensor has nonzero components of $\tau_{zz} = 1$ and $\tau_{xx} = -1$. Using a Ricker wavelet with peak frequency of 16Hz and model grid spacing of $\Delta x = \Delta y = \Delta z = 0.01$km, we generate 3D synthetic elastic data, followed by backpropagation and the application of the various imaging conditions.

Figures 1(b) and 1(c) show the images obtained using the autocorrelation of the P- and S-wavefields (equations 8 and 9), respectively. In either image, the low-wavenumber content arising due to wavefield autocorrelation contaminates the whole image space (especially its shallow part), and effectively prevents an identifiable focus at the source location. Alternatively, the cross-correlation of P- and S-wave modes (equation 11) generates the image in Figure 1(d) with minimal low-wavenumber content and a strong amplitude event at the correct source location. The energy imaging condition based on the Hamiltonian operator I_E^H (Figure 1(e), using equation 12) represents the total energy stacked over time, which results in a strong low-wavenumber pattern similar to those in the autocorrelation images (Figures 1(b) and 1(c)). In contrast, the energy imaging condition based on the Lagrangian operator I_E^L (Figure 1(f), using equation 13) shows a result similar to the PS image, with a strong and focused correlation at the source location. In all images, one should consider smearing and truncation artifacts when estimating a source location using wavefield imaging methods for passive seismic, since sparse and band-limited data acquisition prevents extrapolated waves from collapsing into an idealized focus.

To obtain all images in Figure 1, we use the procedure explained in the methodology section, i.e. computing three separated energy images such as $I = I^{(P,S)} - I^{(P)} - I^{(S)}$. This is necessary because truncation events correlate with the true events and create a strong artifact at the center of the energy image. Figures 2(a) and 2(b) show schematic representations of the truncation wavefronts away and close to the source location, respectively. In Figure 2(c), we show the vertical component of the back-propagated elastic wavefield at a snapshot prior to events focusing at the source location. Besides the true wave events (P- and S-backpropagated microseismic arrivals), truncations exist and interact with the true events close to the focusing location and time. Note how the S-wave truncations interfere both with themselves and with the incoming P-wave event. In addition, the fake P-wave mode also appears strong in Figure 2(c), but generates artifacts mostly in the shallow area (down to 1km in depth). Figure 2(d) shows the same energy image as in Figure 1(f), but without the subtraction of such imaging artifacts. We can note residual low-wavenumber content and the different correlation at the source in Figure 2(d) as compared to Figure 1(f). This experiment in a constant-velocity medium demonstrates how (1) low-wavenumber artifacts from wavefield autocorrelation impede a visual interpretation of the source location; (2) truncation events substantially alter the imaged events; and (3) imaging conditions that exploit different wave modes in the displacement field localize the source by producing a focused correlated event at the true source position, especially the energy imaging condition which produces a focused peak at the source location.

4.2 Experiment 2: sensitivity to changes in velocity and extended images

Our second experiment investigates the sensitivity of the different imaging conditions when using the incorrect velocity. We expect that the correlation between P- and S-waves occurs at a different location and time than for the correct model scenarios; however, we would still expect a coherent pattern useful for interpretation and migration velocity analysis. In Figures 3 and 4, we compute PS and energy images for different velocities using the same experimental setup as in Figure 1. We create incorrect constant velocities by globally increasing or decreasing the original velocities by 8%. For improved visualization, we show zoomed slices from the original image volumes. The images exhibit misfocusing due to the incorrect velocities, and show a correlation point either too shallow or too deep with respect to the correct source location. We note that all PS images have a zero crossing at the point of maximum correlation (e.g., at $x = y = z = 2$km in Figure 3(e)). Though present in Experiment 1, but more noticeable here, the usage of the absolute value of the S-vector potential creates zero-crossing patterns throughout the images due to the existence of nodal planes. However, for energy images computed with incorrect velocities (Figure 4), we observe a more straightforward and interpretable moveout pattern from the unfocused events relative to the PS images because of their fewer side lobes, as well as an absence of visible nodal planes. This misfocusing of event images for incorrect velocities is important in the context of assessing velocity and source location errors.

We also compute extended images with correct and incorrect velocities for both PS (Figure 5) and energy imaging conditions (Figure 6). These extended image examples are generated with the same incorrect velocities and displayed similarly to Figures 3 and 4. For simplicity, we evaluate these extended images at a single spatial point located at the correct source position and with spatial lags only (with axes $\lambda_x, \lambda_y, \lambda_z$), thus generating the 3D volumes shown in Figures 5 and 6. As discussed above, we expect to observe unfocused events in the extended domain when velocities are incorrect, and a focused event at zero lag when both P- and S-wave velocities are correct. Indeed, we note that these misfocusing attributes exist in the extended domain for both imaging conditions, but are expressed differently. Compared to the extended PS image, the energy one shows significantly fewer lobes and no nodal planes, and therefore, exhibits imaged waveforms more straightforward to interpret. However, whereas the extended energy image exhibits symmetric unfocused events with respect to the λ_z axis, the extended PS image shows a single unfocused event either at positive or negative λ_z values. This indicates that, although the events in the extended energy image exhibit more straightforward waveforms, interpreting the direction of the required velocity update (either positive or negative) is more challenging. For instance, by looking at the extended energy images when we perturb only the S-wave velocity by $-\%8$ or $+\%8$ (Figures 6(d) and 6(f)), we note a similar dislocation from zero lag but different focusing attributes. Therefore, the extend energy image correctly provides the magnitude of the velocity perturbation, but apparently has lower sensitivity with respect to the perturbation sign. Based on such differing sensitivity, we infer that the two extended imaging conditions would provide complementary information for microseismic migration velocity analysis.

4.3 Experiment 3: SEG/EAGE 3-D overthrust model

Our third numerical test investigates how the imaging conditions compare using a more complex velocity model - SEG/EAGE 3-D overthrust (Aminzadeh et al., 1994). We place the source at the center of the model ($x = y = 3.048$km, $z = 1.073$km in Figure 7(a)), and use a non-uniform distribution of 192 multicomponent receivers at elevations ranging between $z = -213$m and $z = -423$m (Figure 7(c)). These receiver coordinates mimic the acquisition geometry of the following field data example. Because the original 3D overthrust model contains only the P-wave velocity, we arbitrarily create a S-wave velocity model using an oscillatory PS velocity ratio as a function of depth (Figure 7(b)). We model and extract synthetic data at the 192 receiver locations, and compare PS and energy images using a source mechanism different from the previous two numerical experiments. The corresponding source moment tensor has nonzero components of $\tau_{zz} = 1$, $\tau_{yy} = -2$, and $\tau_{xx} = 1$.

Figure 8 shows the PS and energy images for this numer-

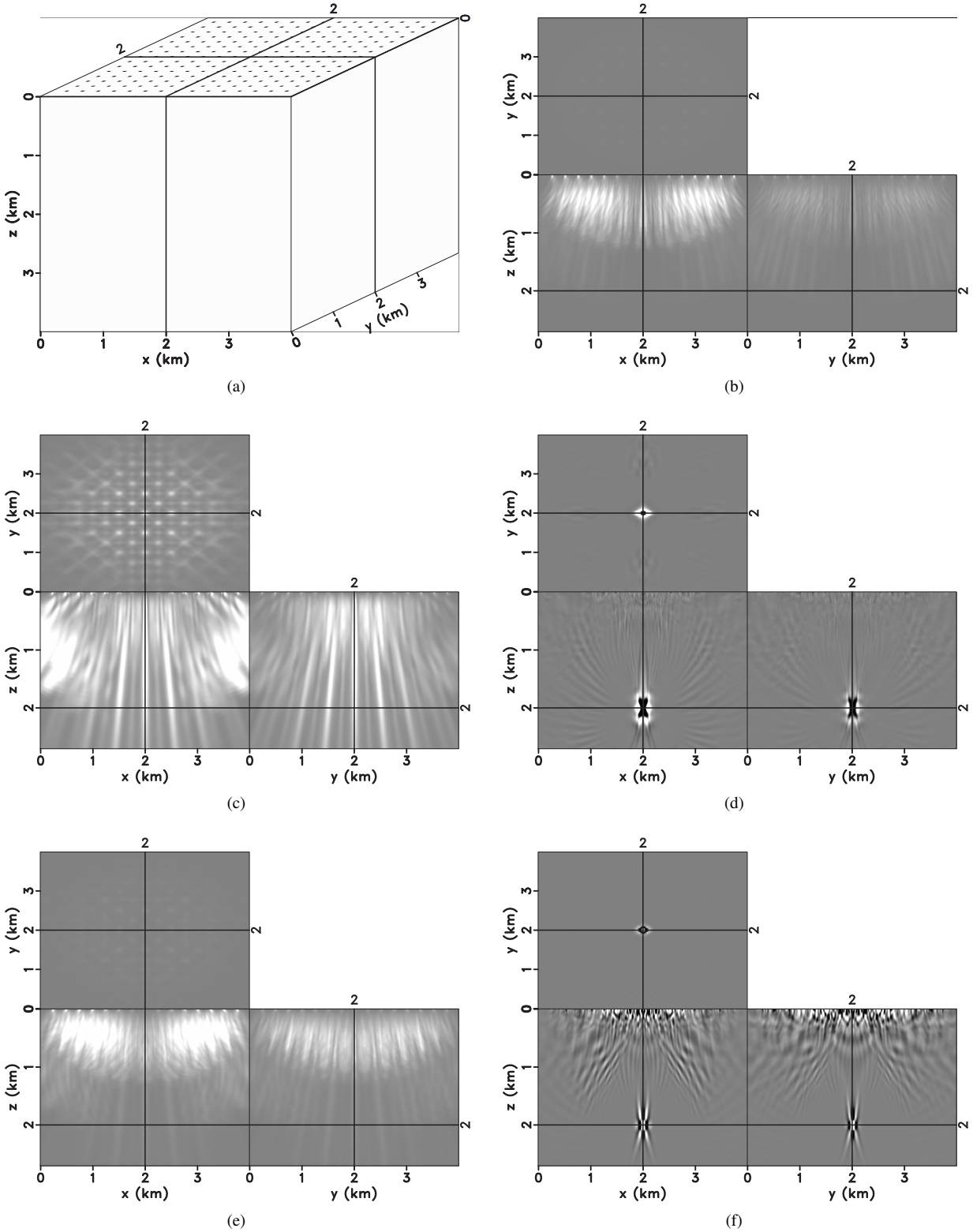


Figure 1. Constant-velocity experiment that consists of (a) an earth model with $4\times4\times4$ km^3 and multicomponent receivers (black dots) spaced by 0.25km at the surface. (b) PP, (c) SS, (d) PS and energy images using equation 12 (e) and equation 13 (f). The PS and energy images have strong and focused peaks at the source location compared to all other images that exhibit strong low-wavenumber artifacts.

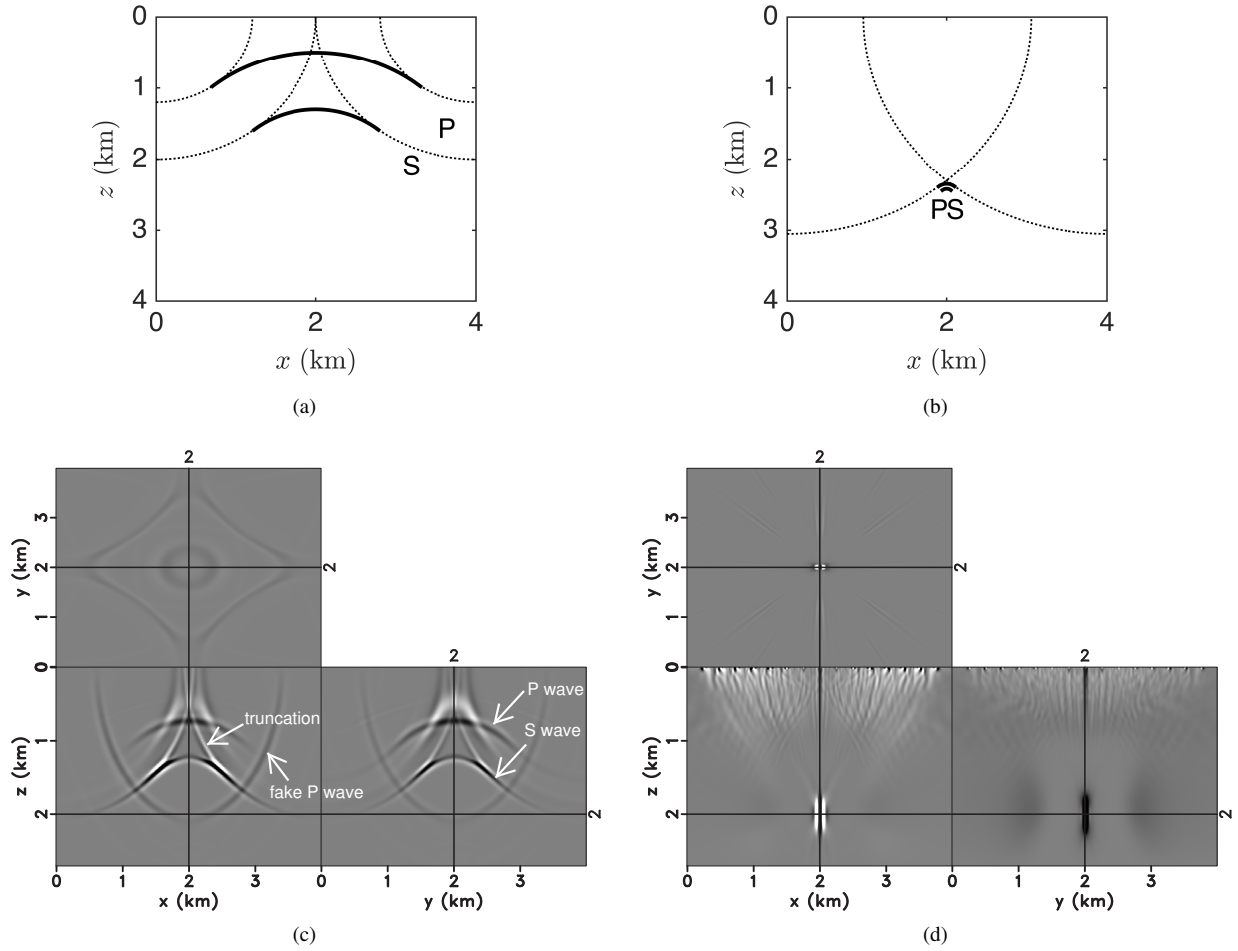


Figure 2. Schematic representation of (a) P- and S- waves backpropagated with truncation events (dashed lines). Close to their focus points, (b) P- and S- waves correlate with their respective truncation events, which have the polarity of their original wave but different propagation directions. (c) Snapshot of the vertical component from the back-propagated elastic wavefield. As indicated by the arrows, note the true backpropagated events and the associated artificial extrapolated events. (d) Energy image with the strong artifact caused by the truncation, as opposed to the energy image without such artifact (Figure 1(f)).

ical experiment. Figures 8(a) and 8(b) show the entire image volumes, while Figures 8(c) and 8(d) zoom around the source location. Originally, both PS and energy images have strong artifacts in the shallow subsurface due to the aforementioned caveats (i.e., truncated acquisition, non-existent modes created by displacement injection) from elastic wavefield extrapolation; however, by applying a straightforward exponential gain with depth to both images to mitigate these effects, we obtain the images shown in Figures 8(a) and 8(b). The image locations colored in red represent clipped colorbar amplitudes, and are generally consistent with the estimated source locations for both images (Figure 8(c) and 8(d)). The PS image (Figure 8(c)) exhibits the clipped amplitudes about 200m away from the source location, mainly due to the zero crossing that occurs at the known source location. Meanwhile, the energy image (Figure 8(d)) has the clipped amplitudes within one grid cell of the true source location. Therefore, by comparing the zoomed images in Figures 8(c) and 8(d), we observe more consistent focusing of the energy image relative to the PS image, and the misplacement of the clipped peak amplitude for the PS image. As discussed in the theory section, data preprocessing could handle polarization issues with the PS correlation, but at the cost of a lower resolution image.

To observe focusing in the extended domain, we generate extended images for both imaging conditions using the correct velocity only (Figures 8(e) and 8(f)). As previously, the extended PS image has a nodal plane at zero lag and exhibits substantial defocusing away from zero lag even though the velocities are exact. The extended energy image shows a more focused event at zero lag without nodal planes. The $\lambda_x - \lambda_y$ plane of the energy image in Figure 8(f) might suggest that the source has its primary stress component in the $y-$axis due to the elongation direction of the imaged event. However, val-

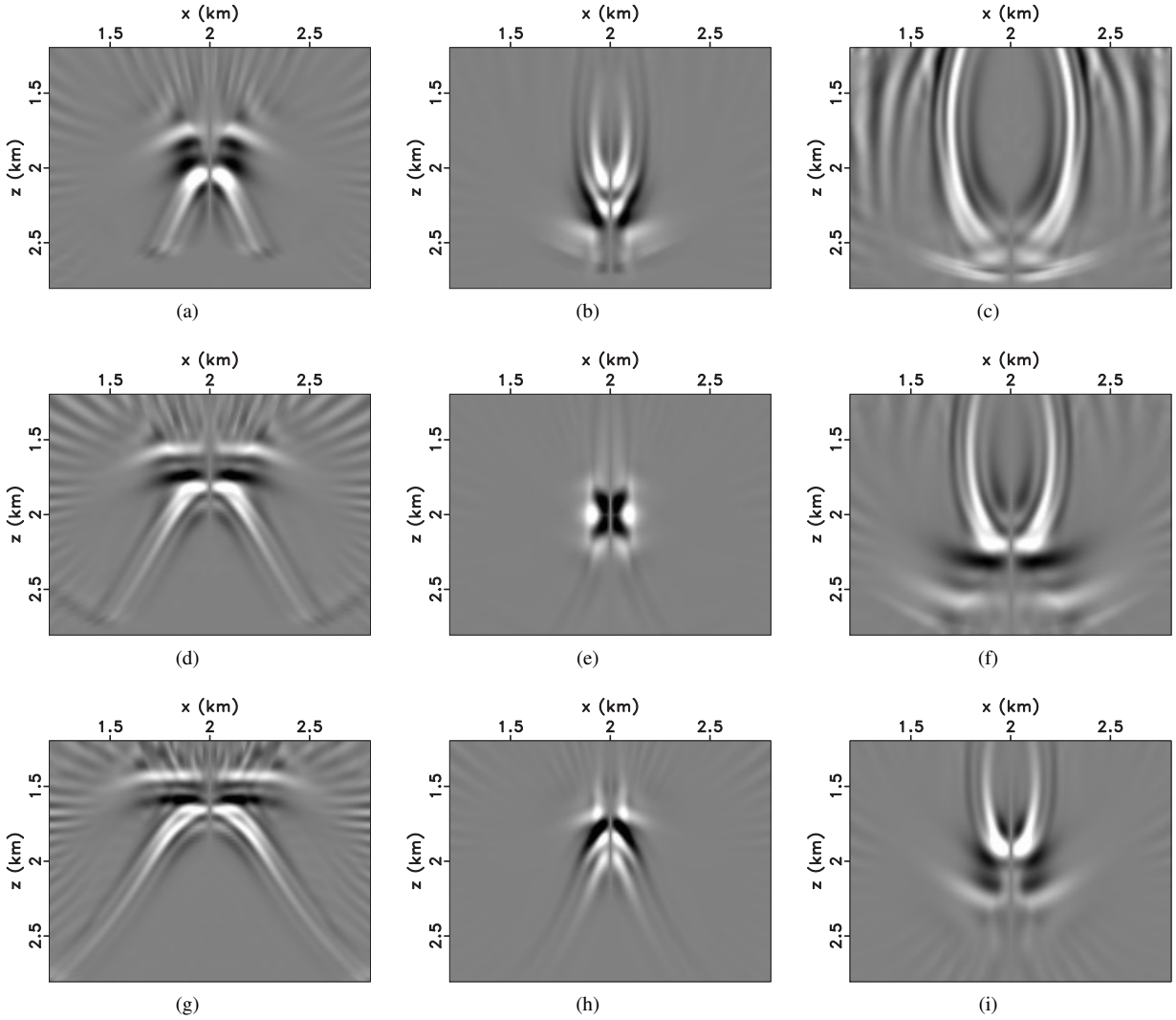


Figure 3. PS images with incorrect velocities with respect to a $V_P = 3.0$km/s and $V_S = 1.8$km/s. Top, middle and bottom rows correspond to $-8\%, 0\%, +8\%$ changes in the P-wave velocity; and left, middle and right columns correspond to $-8\%, 0\%, +8\%$ changes in the S-wave velocity. Note the unfocused events with respect to the focused correlation in the central image (which uses the correct velocity).

idating a direct extraction of the source mechanism warrants further research beyond the scope of this paper.

The SEG/EAGE 3D overthrust experiment demonstrates that the direct correlation between Helmholtz decomposed P- and S- wavefields produces strong imaging artifacts with amplitudes surpassing the correlation at the true source position. The energy imaging condition involves an indirect correlation of P- and S- waves that produces a stronger correlation at the source location compared to the imaging artifacts, both in the conventional and extended domains. The difference between these two imaging conditions becomes more evident with increasing model complexity.

4.4 Experiment 4: field data example

The final experiment uses microseismic field data recorded by a passive seismic acquisition campaign during a hydraulic stimulation program in eastern Ohio, USA. Witten and Shragge (2017b) present a thorough description of the experiment, dataset and subsequent pre-processing. The receiver array consists of 192 multicomponent (3C) geophones whose geometry is shown in Figure 7(c). We use P- and S-wave velocity models (Figure 9) obtained by the image-domain velocity inversion described in Witten and Shragge (2017b). From a data catalog of 28 high-S/N microseismic events, we select one microseismic event (single P- and S-arrivals) of magnitude $M_w = 0.24$. The processing applied prior to backpropagation and imaging involves different steps compared to the acoustic

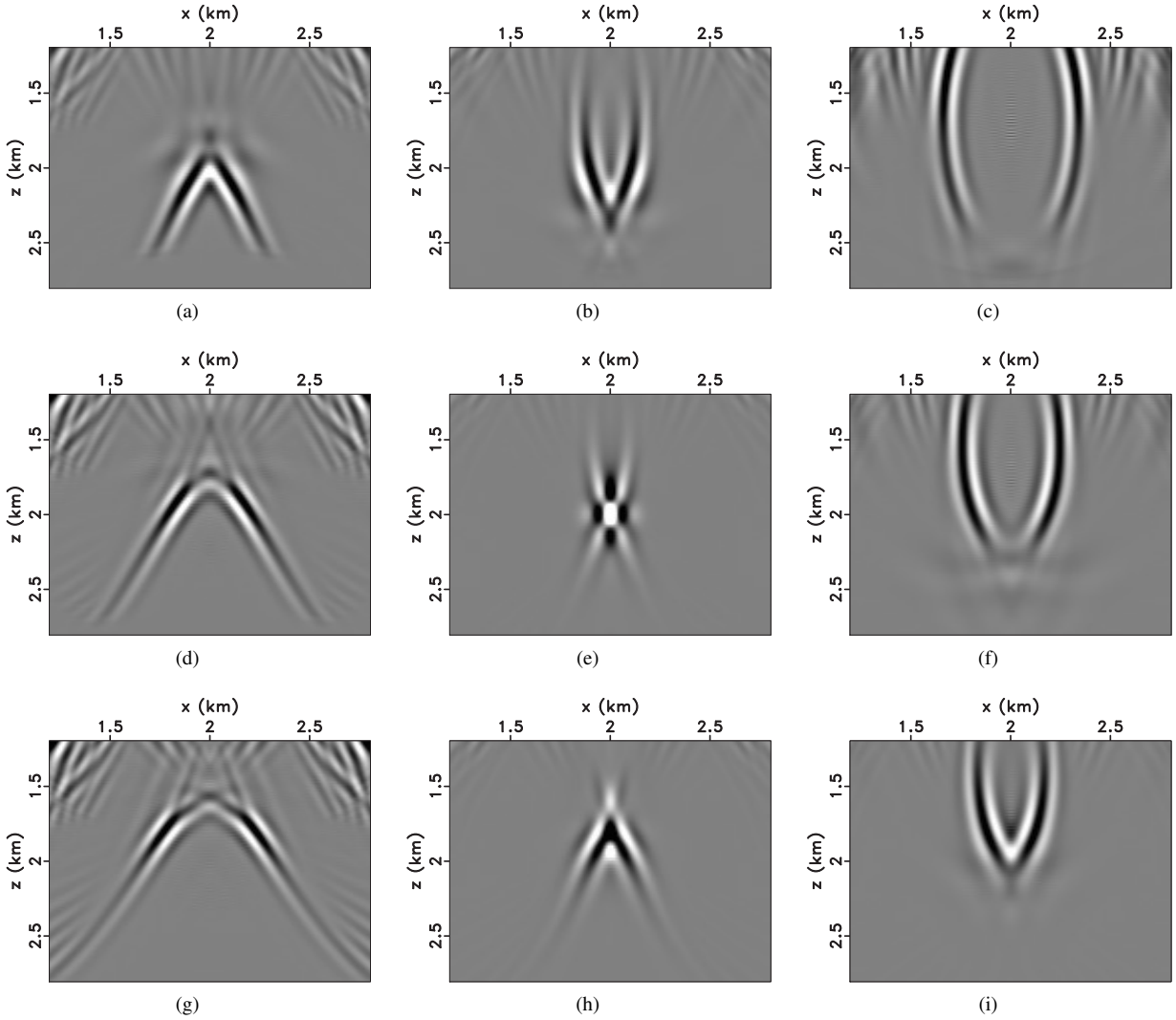


Figure 4. Energy images with incorrect velocities, computed and displayed as in Figure 3. Compared to the PS images in Figure 3, the energy correlation produces unfocused imaged events that are more straightforward to interpret and without zero crossings.

implementation from Witten and Shragge (2017b): (1) rotating the northing and easting components to x, y components consistent with the velocity model geometry (by an angle of 41°); (2) applying a bandpass filter from 4 to 30Hz (with no trace normalization, nor application of envelope filter); and (3) implementing stronger weighting of early P- and S-wave arrivals. Figure 10 shows the multicomponent dataset processed using such approach.

We inject the microseismic event shown in Figure 10 into the velocity model (Figure 9), and apply the PS and energy imaging conditions. We use the same image subtraction procedure ($I = I^{(P,S)} - I^{(P)} - I^{(S)}$) and exponential gain with depth as in Experiment 3. Figure 11 presents the final results of the imaging procedure. Because in this field dataset example we do not know the true source location, we elect to present image slices corresponding to the point of maximum amplitude for each imaging condition clipped using the same percentile (Figure 11(a) and 11(b)). As observed in the image slices, the discrepancy between the estimated PS and energy source locations is $[\Delta x, \Delta y, \Delta z] = [366, 146, 91]$m. The PS image in Figure 11(a) exhibits more artifacts away from its presumed source location (especially on the $x - y$ plane) when compared to the energy image in Figure 11(b). Looking at the zoomed images (Figures 11(c) and 11(d)), we note a further advantage of the energy image relative to the PS image: most of the correlated energy is positive. Finally, the extended energy image (Figure 11(f)) exhibits a more centralized focusing at zero-lag when compared to the extended PS image (Figure 11(e)). Although there is residual energy beyond zero lag, both extended images indicate that the provided velocities obtained by image-domain tomographic inversion are sufficiently consistent and, therefore, accurate by the self-

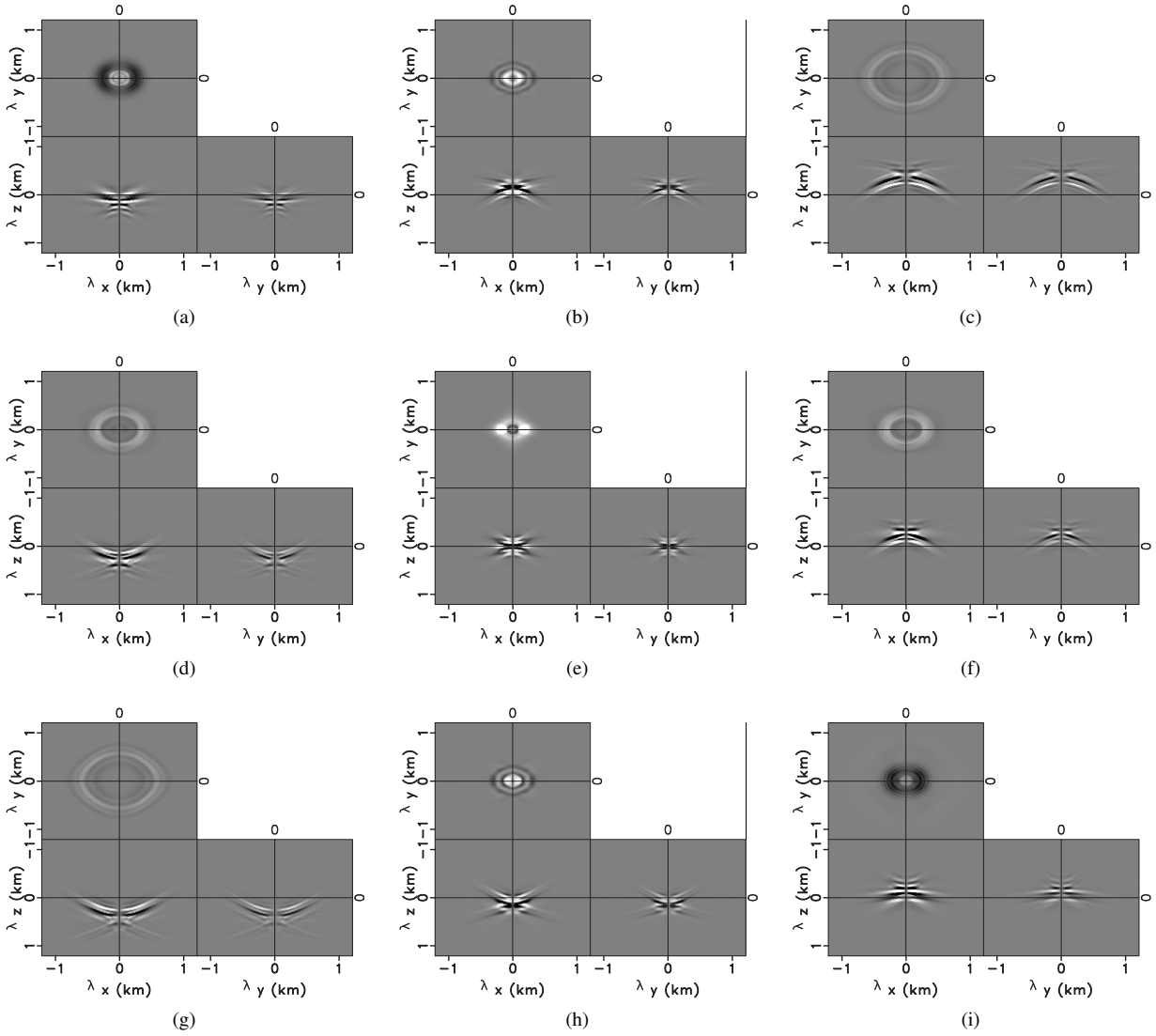


Figure 5. Extended PS images with incorrect velocities. Top, middle and bottom rows correspond to -8%, 0%, $+8\%$ changes in the P-wave velocity; and left, middle and right columns correspond to -8%, 0%, $+8\%$ changes in the S-wave velocity. Note the unfocused events with respect to the focused correlation in the central image (which uses the correct velocity).

consistency principle (Witten and Shragge, 2017a). Overall, the field data example demonstrates that the energy imaging condition behaves well in the presence of noise, realistic acquisition geometry, as well as possible residual errors in the velocity models. This experiment also illustrates that elastic wavefield imaging methods are more effective for real surface passive seismic with relatively dense receiver geometry, after the complementary image provided by the energy correlation.

5 DISCUSSION

Based on the numerical examples presented in this paper, the energy imaging condition not only serves as an alternative procedure for obtaining source location estimates and examining velocity accuracy, but offers some improvements relative to the conventional PS imaging condition. The more directly interpretable images obtained by the proposed imaging condition not only reduces uncertainty of the event location, but also helps velocity estimation based on migration velocity analysis methodology (i.e., semblance principle). This interpretational simplicity is due to the absence of nodal planes and associated polarity changes, which are ubiquitous in PS correlation. The penalty function commonly used in image-domain wavefield tomography (Symes and Carazzone, 1991; Mulder and ten Kroode, 2002; Yang and Sava, 2015; Witten and Shragge, 2017a) requires extended-domain images with fewer artifacts and better delineated events. In the context of passive imag-

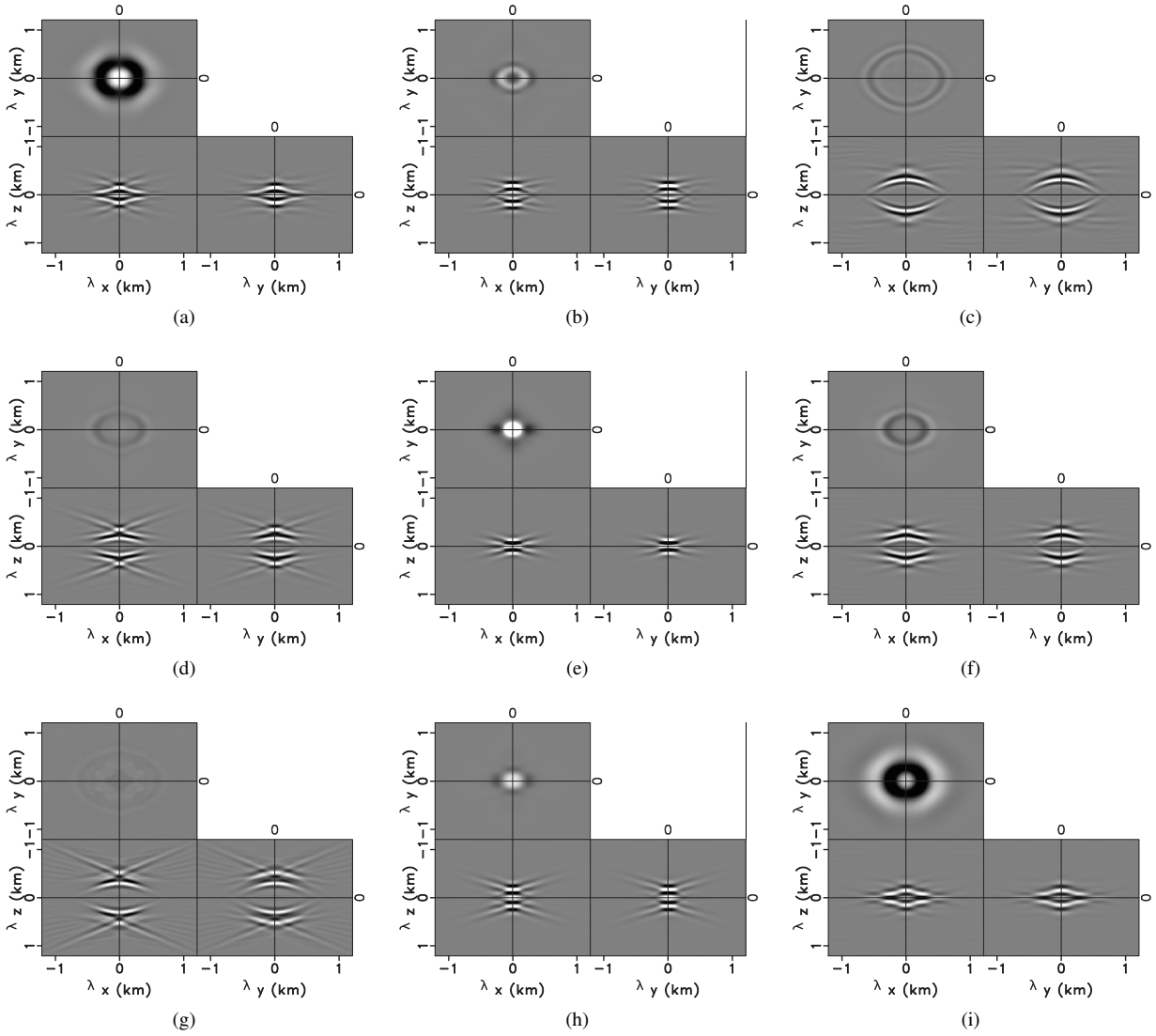


Figure 6. Extended energy images with incorrect velocities, organized similarly to Figure 5. Compared to the PS images in Figure 5, the energy correlation produces unfocused images that are more straightforward to interpret, are symmetric with respect to the λ_z axis, and contain no zero crossings.

ing and inversion, our examples show that such characteristics are enhanced by the energy correlation. However, computing tomographic velocity updates requires reliable magnitude and direction information, and the latter appears to not be sufficiently distinguishable by the energy correlation alone. In addition, for both imaging conditions, differentiating between P- and S-wave velocity inaccuracies is only possible with complementary information from autocorrelation P- and S-wave imaging conditions, as shown previously in Witten and Shragge (2017a). Thus, we view these various extended imaging conditions could play a complementary - but important - role in applications of microseismic migration velocity analysis.

The fact that the energy correlation formalism introduced above is able to handle anisotropy is not demonstrated here; however, the pathway for incorporating anisotropy should be clear based on the theory presented in this paper. Current wave-mode decomposition methods during wavefield extrapolation for anisotropic media remain computationally intensive (Zhang and McMechan, 2010; Yan and Sava, 2011; Cheng and Fomel, 2014; Sripanich et al., 2015; Wang et al., 2016; Rocha et al., 2017). Until they become more efficient, the energy correlation serves as a formal and cost-effective method for correlating different wave modes for anisotropic elastic passive imaging. Our imaging condition does not add a significant computational burden since it requires the same spatial derivatives as those in Helmholtz decomposition and straightforward temporal derivatives (identical to those computed at

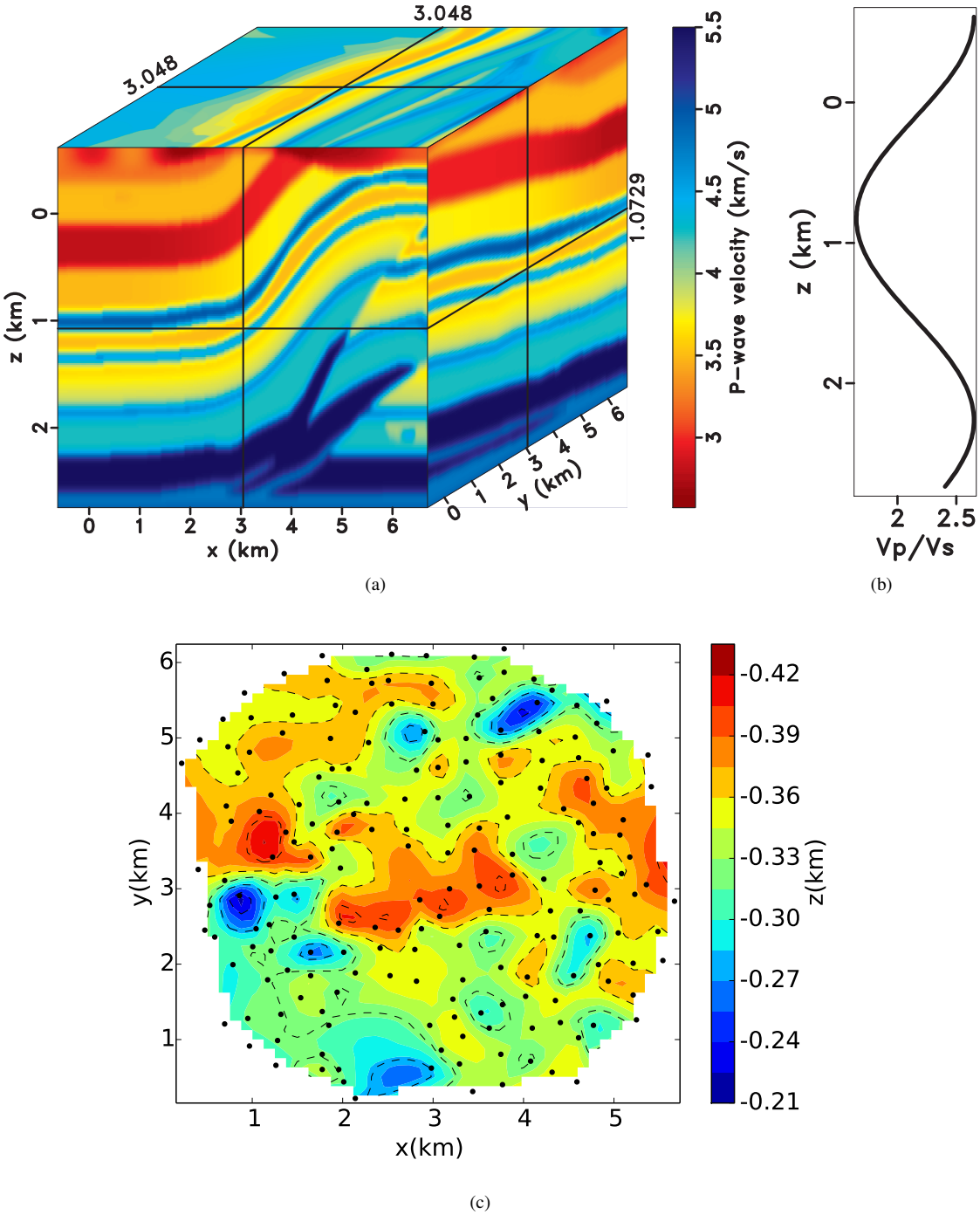


Figure 7. SEG/EAGE overthrust experiment: (a) P-wave velocity from the 3D overthrust model and (b) PS velocity ratio. (c) Receiver coordinates used for the overthrust and field acquisition experiments shown in Figures 9 and 11.

each elastic extrapolation time step). Moreover, the extended energy image is sufficiently computed at a single common image point (CIP) for migration velocity analysis (Witten and Shragge, 2017a).

We also advocate the usage of elastic wavefield extrapolation as opposed to acoustic extrapolation, which is applied to the same field dataset in a related paper (Witten and Shragge, 2017b). Acoustic time-reversal involves normalization of each individual trace and computing the complex envelope of the seismic signals, thereby altering the amplitude and phase of

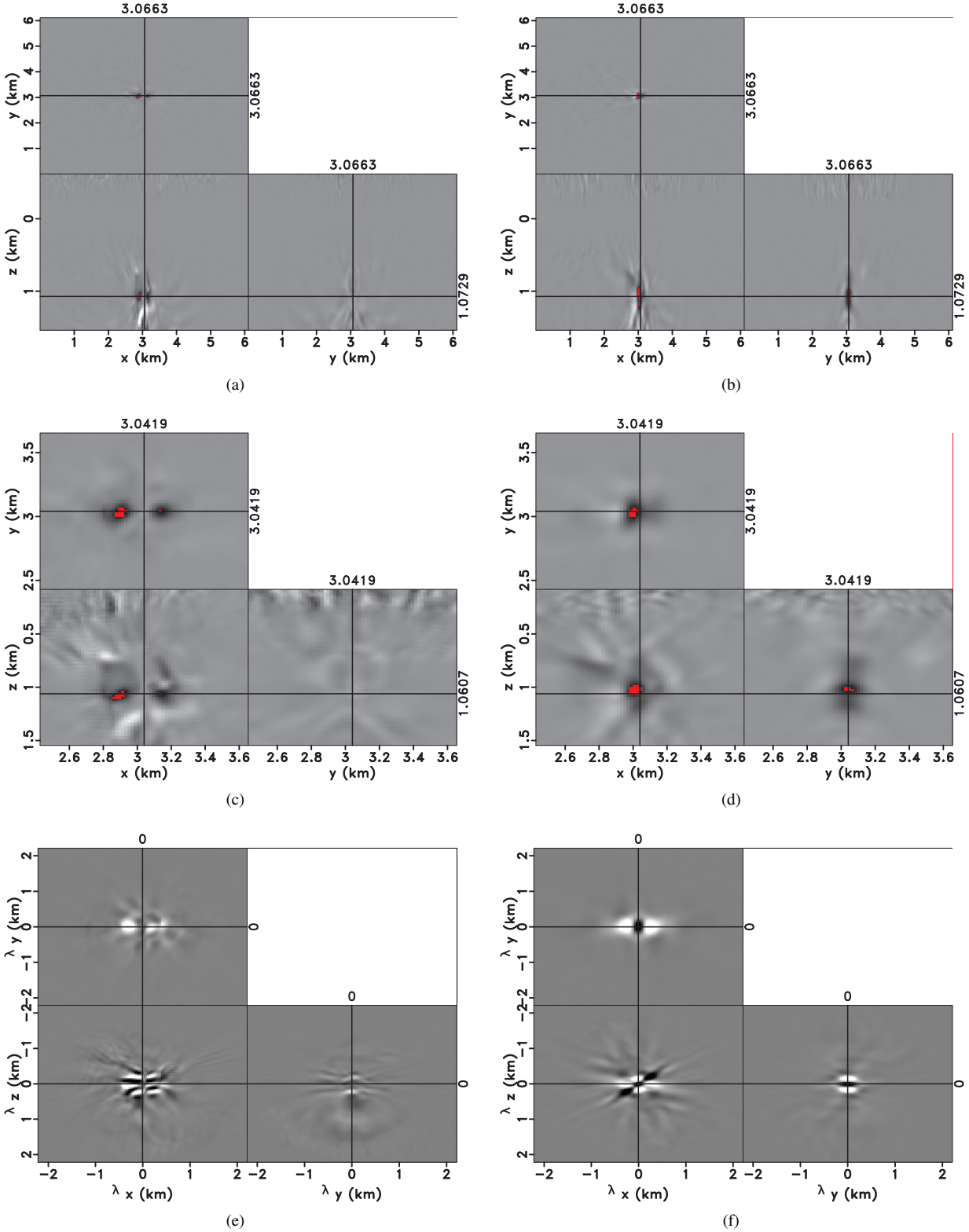
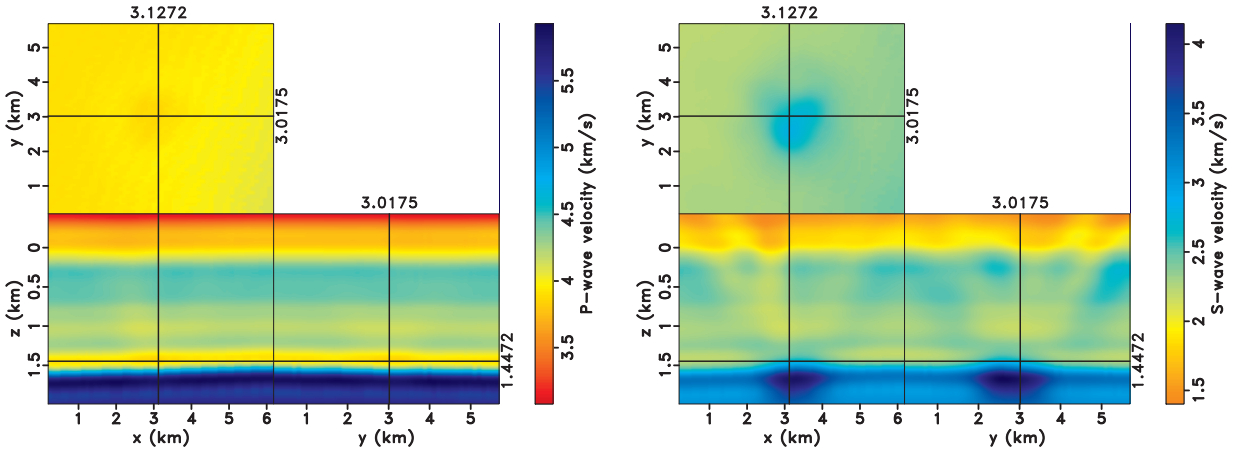


Figure 8. SEG/EAGE overthrust experiment: (a) PS and (b) energy images for a stress source with $\tau_{xx} = -2, \tau_{yy} = 1, \tau_{zz} = 1$. Zoomed (c) PS and (d) energy images, as well as extended (e) PS and (f) energy images. The energy correlation exhibits peak amplitudes (in red) that are closer to the source location in (d) when compared to (c), and are more focused in the extended domain in (f) when compared to (e).



(a) (b)

Figure 9. Field data experiment: (a) P- and (b) S-wave velocity models.

the acquired signals. While essential for acoustic velocity analysis, such procedure undermines any possible cotemporal radiation pattern analysis (i.e., through joint microseismic velocity analysis and moment-tensor inversion). Acoustic extrapolation assumes single-component data and, therefore, does not exploit the polarization of seismic waves. Elastic extrapolation does not involve these issues and offers finer imaging sampling considering the same frequency band from data. However, it remains substantially more expensive than its acoustic counterpart especially for 3D models, though it is achievable at the scale of a small cluster or even a high-end workstation when using GPU-based 3-D elastic propagators (Weiss and Shragge, 2013).

6 CONCLUSIONS

For passive wavefield imaging with surface-recorded multicomponent data, the energy imaging condition offers an elegant solution for locating seismic sources within an arbitrary earth model. Based on energy conservation for extrapolated wavefields, our imaging condition represents the temporal integral of the Lagrangian operator (which is the difference between kinetic and potential wavefield energy terms) and produces an image that is theoretically related to the source mechanism. For simple models, we demonstrate that the energy image exhibits better attributes than PS images in terms of image quality and velocity sensitivity. For more realistic synthetic and field data examples, the improved results from the energy imaging condition are similarly illustrated despite the introduction of field data noise, sparser and non-uniform receiver geometry, and more complex earth models. The preclusion of wave-mode decomposition during extrapolation in our method permits its implementation to anisotropic earth models without substantial additional cost or complication, as opposed to the conventional PS imaging condition that generally relies on Helmholtz decomposition.

In summary, we have validated the five advantages stated in the Introduction: (1) handling radiation patterns without the generation of nodal planes; (2) producing a peak amplitude at the source location; (3) precluding wave-mode decomposition at each time step; (4) downweighting identical wave modes and enhancing distinct ones during correlation; and (5) offering complementary extended image attributes for migration velocity analysis. Future work involves exploring the further benefits of the energy imaging condition for anisotropic media, potential inferences on focal mechanism and its inversion, and the development of a microseismic migration velocity inversion framework using the unfocused energy on extended image gathers.

7 ACKNOWLEDGMENTS

We thank the sponsors of the Center for Wave Phenomena, whose support made this research possible. We thank Statoil USA for allowing publication and ESG/Spectraseis for delivering the field dataset. We acknowledge Ivan Lim Chen Ning, Paul Fowler and Roel Snieder for helpful discussions, and Yuting Duan for sharing her wavefield modeling codes. The reproducible numeric examples in this paper use the Madagascar open-source software package (Fomel et al., 2013) freely available from http://www.ahay.org.

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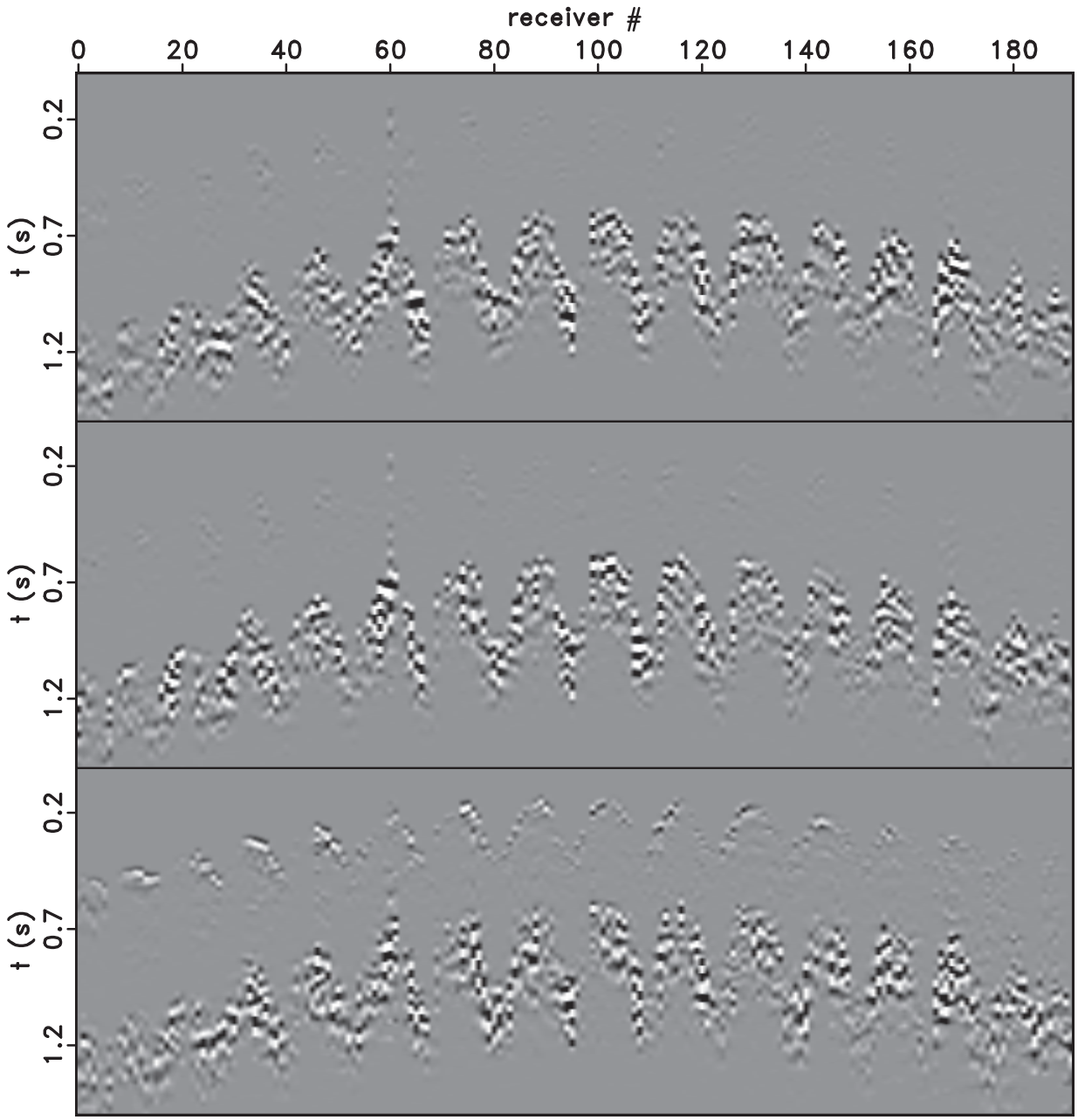


Figure 10. Field data experiment: x (top), y (middle) and z (bottom) components from the processed dataset containing a single microseismic event. Note the P- (from $t = 0$s to $t = 0.6$s) and S- (from $t = 0.6$ to $t = 1.5$s) arrivals, and how the P-wave arrival is stronger in the vertical (z) component.

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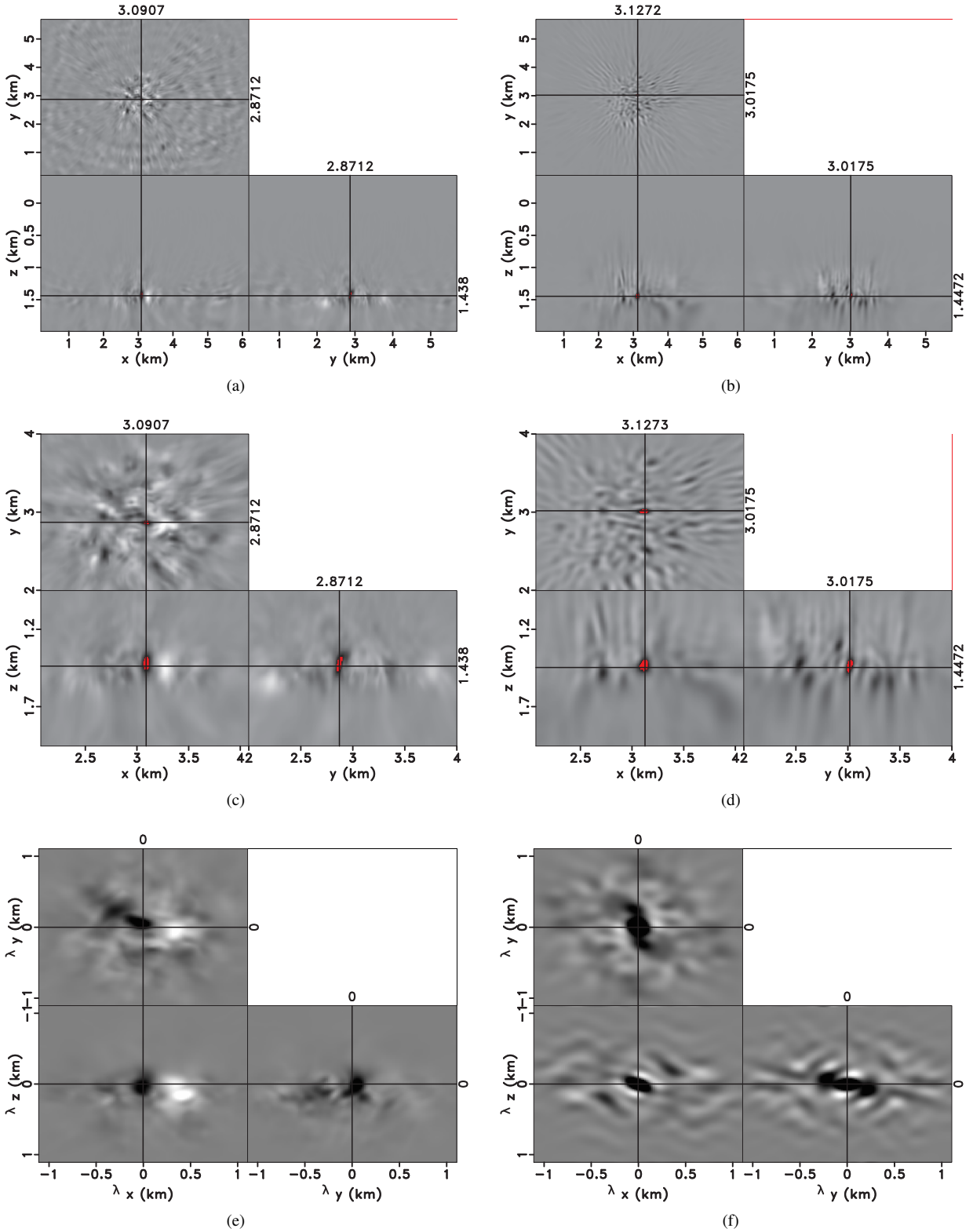


Figure 11. Field data experiment: (a) PS and (b) energy images for the field dataset. Zoomed (c) PS and (d) energy images, as well as extended (e) PS and (f) energy images. The energy correlation has peak amplitudes (in red) that are closer to the source location in (d) when compared to (c), and are more centralized in the extended domain in (f) when compared to (e).

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Appendix A - Hamilton's Principle

Considering a displacement field $\mathbf{U}(\mathbf{x},t)$, the Lagrangian density function is (Ben-Menahem and Singh, 1981)

$$\mathcal{L}(\mathbf{U},\mathbf{x},t) = \frac{1}{2}\rho\|\dot{\mathbf{U}}\|^2 - \frac{1}{2}\left(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}\right) : \nabla\mathbf{U} \ . \tag{A.1}$$

The first and second terms in the Lagrangian function represent the kinetic and potential energies of the wavefield, respectively. In a medium with no external forces, the action is defined as the Lagrangian density function integrated over time

$$A(\mathbf{U}) = \int_0^T \mathcal{L}(\mathbf{U},t)\,\mathrm{d}t \ . \tag{A.2}$$

Hamilton's variational principle states that the action is stationary under small displacements, and can be expressed as (Slawinski, 2003):

$$\delta A(\mathbf{U},\mathbf{x}) = \delta\int_0^T \mathcal{L}(\mathbf{U},t)\,\mathrm{d}t = 0 \ , \tag{A.3}$$

where δ indicates the variation of a function. The variation is permutable with a definite integral and can be defined with respect to the displacement variable (Lanczos, 1970). Hence,

$$\begin{aligned}\delta A(\mathbf{U},\mathbf{x}) &= \int_0^T \delta\mathcal{L}(\mathbf{U},t)\,\mathrm{d}t \\ &= \int_0^T [\mathcal{L}(\mathbf{U}+\delta\mathbf{U},t) - \mathcal{L}(\mathbf{U},t)]\,\mathrm{d}t = 0 \ ,\end{aligned} \tag{A.4}$$

where $\delta\mathbf{U}(\mathbf{x},t)$ is a small displacement satisfying $\delta\mathbf{U}(\mathbf{x},t=0) = \delta\mathbf{U}(\mathbf{x},t=T) = 0$.

We can characterize the Lagrangian, $\mathcal{L}(\mathbf{U},t)$, for a particle at rest from $t=0$ to $t=T$, i.e., $\mathbf{U}=0$, for $0\le t\le T$. We can consider the displacement field acting on this particular point as a small perturbance to the particle at the rest. Therefore,

$$\delta A(\mathbf{U},\mathbf{x}) = \int_0^T \mathcal{L}(\delta\mathbf{U},t)\,\mathrm{d}t = 0 \ . \tag{A.5}$$

This implies that, in the absence of external work from sources and in the presence of small displacements, the energy imaging condition defined as the integral of the Lagrangian density function is zero.

In a volume V that contains sources, the action is defined as (Yu, 1964):

$$A(\mathbf{U},\mathbf{x}) = \int_0^T \int_V [\mathcal{L}(\mathbf{U},t)\,\mathrm{d}V + W]\,\mathrm{d}t \ , \tag{A.6}$$

where the external work W is

$$W = \int_V \rho \mathbf{F} \cdot \mathbf{U} \mathrm{d}V + \int_S (\underline{\mathbf{t}} \cdot \mathbf{U}) \cdot \mathbf{n} \mathrm{d}S \,, \tag{A.7}$$

where $\mathbf{F}(\mathbf{x}, t)$ represents a body force field and $\underline{\mathbf{t}}(\mathbf{x}, t)$ an external stress field acting on the surface of the considered volume. Applying the variation operator on equation A.7 and using the divergence theorem, we obtain

$$\delta W = \int_V \left[\rho \mathbf{F} \cdot \delta \mathbf{U} + \nabla \cdot (\underline{\mathbf{t}} \cdot \delta \mathbf{U})\right] \mathrm{d}V \,. \tag{A.8}$$

Applying the variation operator on equation A.6 and using equation A.8 leads to

$$\delta A(\mathbf{U}, \mathbf{x}) = \int_0^T \int_V \left[\mathcal{L}(\delta \mathbf{U}, t) + \rho \mathbf{F} \cdot \delta \mathbf{U} + \nabla \cdot (\underline{\mathbf{t}} \cdot \delta \mathbf{U})\right] \mathrm{d}V \mathrm{d}t = 0 \,. \tag{A.9}$$

Consider a sufficient small volume such that it contains only one discrete point. In a point where either a displacement or a stress source exists, Equation A.9 suggests that the energy imaging condition (defined as the integral of the Lagrangian density function) is equivalent to the action of existing sources

$$I_{EN}(\mathbf{x}) = \int_0^T \mathcal{L}(\delta \mathbf{U}, t) \, \mathrm{d}t = -\int_0^T \rho \mathbf{F} \cdot \delta \mathbf{U} + \nabla \cdot (\underline{\mathbf{t}} \cdot \delta \mathbf{U}) \, \mathrm{d}t \,. \tag{A.10}$$

The energy imaging, for waves generated by a displacement point force, becomes

$$I_{EN}(\mathbf{x}) = \int_0^T \mathcal{L}(\delta \mathbf{U}, t) \, \mathrm{d}t = -\int_0^T \rho \mathbf{F} \cdot \delta \mathbf{U} \mathrm{d}t \,, \tag{A.11}$$

and by a stress source, generally described as a double couple system of forces, becomes

$$I_{EN}(\mathbf{x}) = \int_0^T \mathcal{L}(\delta \mathbf{U}, t) \, \mathrm{d}t = -\int_0^T \nabla \cdot (\underline{\mathbf{t}} \cdot \delta \mathbf{U}) \, \mathrm{d}t \,. \tag{A.12}$$