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Anisotropic elastic wavefield imaging using the energy norm

Daniel Rocha[1], Nicolay Tanushev[2], and Paul Sava[1]

ABSTRACT

Based on the energy conservation principle, we derive a scalar imaging condition for anisotropic elastic wavefield migration. Compared with conventional imaging conditions that correlate displacement components or potentials from source and receiver wavefields, the proposed imaging condition does not suffer from polarity reversal, which degrades the image quality after stacking over shots. Our imaging condition also accounts for the directionality of the wavefields in space and time, leading to the attenuation of backscattering artifacts, which commonly appear in elastic reverse time migration images in the presence of strong model contrasts. In addition, our new imaging condition does not require wave-mode decomposition, which demands significant additional cost for elastic wavefields in anisotropic media. To properly image structures, we rely on the anisotropy parameters used in migration, as one would do for any other imaging condition. Our imaging condition is suitable for arbitrary anisotropy. We show the successful application of the anisotropic energy imaging condition by performing numerical experiments on simple and complex geologic models. We compare its quality with conventional counterparts by simulating complex geologic settings with vertical or tilted transverse isotropy.

INTRODUCTION

We frame our paper in the context of (1) an elastic earth, supporting the propagation of P- and S-waves, and (2) an anisotropic earth, causing wave phenomena such as phase-velocity variation with the propagation direction. Although the seismic exploration industry has used acoustic and isotropic assumptions for a long time, the search for more authentic images and subsurface information, such as fracture distribution, encourages multicomponent elastic-wavefield imaging (Chang and McMechan, 1987; Balch and Erdemir, 1994; Denli and Huang, 2008; Yan and Sava, 2011b; Yan and Xie, 2012; Du et al., 2014; Gong et al., 2016). At the same time, data acquired with larger offsets and the increasing number of acquired azimuths enable one to properly account for the influence of anisotropy to image complex geologic structures (Mikhailov et al., 2001; Jing et al., 2006; Lu et al., 2010; Zhang and McMechan, 2011; Li et al., 2012; McGarry and Qin, 2013; Hu et al., 2014; Wang et al., 2014).

Wavefield imaging, using either the acoustic or the elastic wave equation, is implemented in two steps: (1) wavefield extrapolation in the subsurface, using recorded data and an earth model, and (2) the application of an imaging condition to extract the earth's reflectivity from the extrapolated wavefields (Claerbout, 1971; Etgen, 1988; Yan and Sava, 2009). If a two-way elastic wave equation is used in the wavefield-extrapolation step, followed by the zero-lag crosscorrelation of the wavefields as the imaging condition, the procedure is called elastic reverse time migration (Chang and McMechan, 1987; Hokstad et al., 1998). In recent years, many elastic-imaging conditions have been proposed by exploiting the multicomponent aspect of the elastic wavefield and, for the isotropic case, by possibly decomposing the displacement fields into P- and S-wave potentials (Dellinger and Etgen, 1990; Zhe and Greenhalg, 1997; Yan and Sava, 2007; Yan and Xie, 2010; Duan and Sava, 2014a). For anisotropic media, wave-mode decomposition during wavefield extrapolation is feasible but requires additional computational cost and robust techniques (Yan and Sava, 2009; Zhang and McMechan, 2010; Yan and Sava, 2011a; Cheng and Fomel, 2014). Considering the high cost of wave-mode decomposition, the straight correlation of the source and receiver displacement fields for each component is a convenient imaging condition for elastic wavefields in anisotropic media (Jiang, 2016; Kim et al., 2016). However, for 3D imaging, this imaging condition generates nine images (considering all possible correlations among source and receive wavefield components) with a mixture of wave modes, and this large set of images poses serious difficulties for interpretation.

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Another problem exhibited by displacement imaging conditions is polarity reversal, which occurs if one correlates displacement components from opposite directions. The main disadvantage of polarity reversal is the degradation of image quality after stacking over experiments. Polarity reversal can be corrected by different techniques (Balch and Erdemir, 1994; Yan and Sava, 2008; Du et al., 2014; Duan and Sava, 2014a), but all such approaches are either approximate (e.g., reversal at zero offset) or costly.

Compared with the acoustic case, elastic imaging involves another issue in addition to polarity reversal. Injection of elastic data into a model (implementing wavefield backpropagation) creates nonphysical (or "fake") wave modes during the wavefield extrapolation step, leading to artifacts after the application of an imaging condition (Yan and Sava, 2007; Ravasi and Curtis, 2013; Duan and Sava, 2014a). These artifacts degrade image quality, and might be present even after stacking, masking weak reflections in the final image (Duan and Sava, 2014b). This type of artifact is caused by displacement injection into the model and is beyond the scope of this paper.

Analogous to the acoustic case, a second type of artifact appears if the elastic model contains sharp interfaces. These interfaces create backscattered reflections at the wavefield extrapolation step, and conventional correlation of the multicomponent wavefields creates low-wavenumber artifacts in the image at the imaging condition step (Youn and Zhou, 2001; Yoon and Marfurt, 2006; Guitton et al., 2007; Denli and Huang, 2008; Chen and Huang, 2014; Diaz and Sava, 2015; Rocha et al., 2016b). Ideally, the backscattering artifacts should be attenuated during imaging, without the use of post-imaging artificial filters, such as the Laplacian operator (Zhang and Sun, 2009).

Considering these issues, we seek to facilitate interpretation and provide a concise description of the subsurface structures by defining an imaging condition that yields an attribute of the earth's reflectivity into a single image without wave-mode decomposition, without polarity reversal, and without backscattering artifacts. Our proposed imaging condition is formulated using the energy conservation principle (Yu, 1964; Ben-Menahem and Singh, 1981; Webster, 2000; Rocha et al., 2016a), and it extends the previous work on isotropic elastic wavefield imaging (Rocha et al., 2016b).

THEORY

We use energy conservation laws analogous to the acoustic and isotropic elastic cases (Rocha et al., 2016a, 2016b) to derive a function that measures the energy of a wavefield in anisotropic media. This energy function also allows us to form an imaging condition for extrapolated elastic wavefields from sources and receivers.

Elastic wave equation

For an anisotropic medium enclosed by a physical domain $\Omega \subset \mathbb{R}^3$, we can write the equation of motion with no external sources (Aki and Richards, 2002):

$$\rho\ddot{\mathbf{U}} = \nabla \cdot \underline{\mathbf{t}}. \tag{1}$$

In equation 1, the displacement field is a function of space ($\mathbf{x}$), time (t), and the experiment index (e): $\mathbf{U}(e, \mathbf{x}, t)$ for $\mathbf{x} \in \Omega$ and $t \in [0, T]$, where T is the maximum acquisition time. The experiment index (e) usually represents a shot record, but it could also represent a plane wave or other type of shot encoding (Romero et al., 2000; Liu et al., 2006; Godwin and Sava, 2013). The superscript dot indicates time differentiation. The density is a function of space $\rho(\mathbf{x})$. The underlying bar indicates that the stress is a second-order tensor as a function of space, time, and experiment index $\underline{\mathbf{t}}(e, \mathbf{x}, t)$. One needs to use certain relations involving material properties to turn equation 1 into a wave equation. Using a general form of linear Hooke's law

$$\underline{\mathbf{t}} = \underline{\underline{\mathbf{c}}}\,\underline{\mathbf{e}}, \tag{2}$$

where $\underline{\mathbf{e}}(e, \mathbf{x}, t)$ is the strain tensor, $\underline{\underline{\mathbf{c}}}(\mathbf{x})$ is the stiffness tensor, and the underlying double bar indicates that the stiffness is a fourth-order tensor. One can rewrite the equation of motion in equation 1 as a function of the strain tensor

$$\rho\ddot{\mathbf{U}} = \nabla \cdot (\underline{\underline{\mathbf{c}}}\,\underline{\mathbf{e}}). \tag{3}$$

The strain tensor is a function of the displacement:

$$\underline{\mathbf{e}} = \frac{1}{2}(\nabla\mathbf{U} + \nabla\mathbf{U}^{\mathrm{T}}). \tag{4}$$

Therefore, using the symmetry of the stiffness tensor, we can rewrite equation 3 as a wave equation:

$$\rho\ddot{\mathbf{U}} = \nabla \cdot [\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}]. \tag{5}$$

Equation 5 can be used to extrapolate an elastic wavefield for arbitrary sources, assuming that we know the spatial distribution of the stiffness tensor $\underline{\underline{\mathbf{c}}}(\mathbf{x})$.

Elastic wavefield energy

Analogous to the acoustic and isotropic elastic cases disscussed in Rocha et al. (2016a, 2016b), we develop the energy conservation expression for elastic wavefields in anisotropic media by taking a dot product of the time derivative of the displacement wavefield (i.e., particle velocity) with the corresponding wave equation, followed by integration over the entire physical domain Ω:

$$\int_{\Omega} \rho\dot{\mathbf{U}} \cdot \ddot{\mathbf{U}} d\mathbf{x} = \int_{\Omega} \dot{\mathbf{U}} \cdot [\nabla \cdot (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})] d\mathbf{x}. \tag{6}$$

Rearranging the term on the left side by the chain rule leads to

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega} \rho\|\dot{\mathbf{U}}\|^2 d\mathbf{x} = \int_{\Omega} \dot{\mathbf{U}} \cdot [\nabla \cdot (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})] d\mathbf{x}. \tag{7}$$

Using the chain rule again on the right side leads to

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega} \rho\|\dot{\mathbf{U}}\|^2 d\mathbf{x} = \int_{\Omega} \nabla \cdot [(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})\dot{\mathbf{U}}] d\mathbf{x} - \int_{\Omega} \nabla\dot{\mathbf{U}} : (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}) d\mathbf{x}. \tag{8}$$

The first term on the right side turns into a surface integral over the boundary using the divergence theorem (Boas, 2006). Because we assume homogeneous boundary conditions for the wavefield and its derivatives, this surface integral goes to zero:

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega}\rho\|\dot{\mathbf{U}}\|^2 d\mathbf{x} = -\int_{\Omega}\nabla\dot{\mathbf{U}}:(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})d\mathbf{x}. \tag{9}$$

Developing the remaining term on the right side by using the properties of the Frobenius product (Appendix A), we obtain

$$\int_{\Omega}\nabla\dot{\mathbf{U}}:(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})d\mathbf{x} = \int_{\Omega}\mathrm{Tr}[\nabla\dot{\mathbf{U}}^{\mathrm{T}}(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})]d\mathbf{x}, \tag{10}$$

$$= \frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega}\mathrm{Tr}[\nabla\mathbf{U}^{\mathrm{T}}(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U})]d\mathbf{x}, \tag{11}$$

$$= \frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega}(\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{U}d\mathbf{x}, \tag{12}$$

where $\mathrm{Tr}(\cdot)$ evaluates the trace of a matrix. Substituting equation 12 in equation 9, we have

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega}\rho\|\dot{\mathbf{U}}\|^2 + (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{U}d\mathbf{x} = 0. \tag{13}$$

Therefore, the function

$$E(t) = \frac{1}{2}\int_{\Omega}\rho\|\dot{\mathbf{U}}\|^2 + (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{U}d\mathbf{x} \tag{14}$$

is conserved in time.

Physical interpretation

The first term in the integrand of equation 14 represents the kinetic energy of the wavefield, and the second term is the strain energy (Slawinski, 2003), which represents the potential energy of the wavefield. Therefore, equation 13 states the familiar fact that the total energy $E(t)$ is conserved in time. Substituting the stiffness coefficients for the isotropic case in equation 14, we obtain the energy function for the isotropic case shown in Rocha et al. (2016b) (Appendix B):

$$E(t) = \frac{1}{2}\int_{\Omega}\rho\|\dot{\mathbf{U}}\|^2 + \lambda\|\nabla\cdot\mathbf{U}\|^2 + \mu[\nabla\mathbf{U}:\nabla\mathbf{U} + \nabla\mathbf{U}:\nabla\mathbf{U}^{\mathrm{T}}]d\mathbf{x}. \tag{15}$$

The individual terms in equations 14 and 15 are also related to nonlinear inversion theory as shown by Hokstad et al. (1998) and Zhu et al. (2009).

Imaging condition

Considering two wavefields $\mathbf{U}$ and $\mathbf{V}$, and the elastic energy norm in equation 14, we can define the following inner product:

$$\langle\mathbf{U},\mathbf{V}\rangle_E = \int_0^T\int_{\Omega}\rho\dot{\mathbf{U}}\cdot\dot{\mathbf{V}} + (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{V}d\mathbf{x}. \tag{16}$$

We propose a new elastic imaging condition based on the inner product in equation 16 between source and receiver wavefields, followed by summation over time and experiments:

$$I_E = \sum_{e,t}[\rho\dot{\mathbf{U}}\cdot\dot{\mathbf{V}} + (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{V}]. \tag{17}$$

Here, $\mathbf{U}(e,\mathbf{x},t)$ and $\mathbf{V}(e,\mathbf{x},t)$ are the source and receiver vector wavefields, respectively, and $I_E(\mathbf{x})$ is a scalar image obtained from the elastic wavefields. The imaging condition in equation 17 can conveniently be formulated for orthorhombic and vertical transverse isotropic media (Appendix C). We can describe this imaging condition as a dot product between the following vectors:

$$\square\mathbf{U} = \{\rho^{1/2}\dot{\mathbf{U}}, \underline{\underline{\mathbf{c}}}^{1/2}(\nabla\mathbf{U})\}, \tag{18}$$

$$\square\mathbf{V} = \{\rho^{1/2}\dot{\mathbf{V}}, \underline{\underline{\mathbf{c}}}^{1/2}(\nabla\mathbf{V})\}. \tag{19}$$

The tensor $\underline{\underline{\mathbf{c}}}^{1/2}$ is the square root of $\underline{\underline{\mathbf{c}}}$, which exists and is unique because $\underline{\underline{\mathbf{c}}}$ is positive definite (Shao and Lu, 2009). Equations 18 and 19 define multidimensional vectors with 12 components (in contrast with the 3C vectors in 3D space). The first three components of the vectors in equations 18 and 19 are proportional to the wavefield time derivatives, $\dot{\mathbf{U}}$ and $\dot{\mathbf{V}}$, and the other nine components are derived from the displacement gradients $\nabla\mathbf{U}$ and $\nabla\mathbf{V}$. We can call $\square\mathbf{U}$ and $\square\mathbf{V}$ "energy" vectors because their magnitudes are equal to the energy norm in equation 14 for a fixed point in space.

Therefore, we can rewrite equation 17 as

$$I_E = \sum_{e,t}\square\mathbf{U}\cdot\square\mathbf{V}. \tag{20}$$

This expression is analogous to the similar imaging condition developed for acoustic and isotropic elastic wavefields (Rocha et al., 2016a, 2016b), and it has similar physical interpretation and application, as discussed in the next section.

Backscattering attenuation

Vectors $\square\mathbf{U}$ and $\square\mathbf{V}$ are related to the polarization and propagation direction of the elastic wavefields $\mathbf{U}$ and $\mathbf{V}$. Consider a plane wavefield $\mathbf{U}$ given by

$$\mathbf{U} = \mathbf{u}_0\cos[\omega(\mathbf{p}\cdot\mathbf{x} - t)], \tag{21}$$

where $\mathbf{u}_0$ is the polarization vector, $\mathbf{p}$ is the slowness vector, and ω is the frequency. We assume that ω is large compared with the size of the medium, and that the vectors $\mathbf{u}_0$ and $\mathbf{p}$ are slowly varying in space and time, which makes the spatial and temporal derivatives of $\mathbf{u}_0$ and $\mathbf{p}$ small compared with ω. We can rewrite equation 21 in index form as

$$U_i = u_i\cos[\omega(p_m x_m - t)], \tag{22}$$

where indices $i,m = \{1,2,3\}$ represent the Cartesian components of wavefield $\mathbf{U}$. We follow the Einstein convention and all repeated

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indices are summed over. Substituting the plane-wave definition in equation 22 as a trial solution of the elastic equation in 5, and collecting the highest order term in ω, we have

$$\omega^2 \sin[\omega(p_m x_m - t)][\rho u_i - u_k c_{ijkl} p_j p_l] = 0, \tag{23}$$

where $j, k, l = \{1, 2, 3\}$. Because equation 23 must hold for any ω, we obtain the Christoffel equation

$$\rho u_i = u_k c_{ijkl} p_j p_l. \tag{24}$$

The dot product between equation 24 and the polarization vector $\mathbf{u}_0$ yields

$$\rho u_i u_i - u_i u_k c_{ijkl} p_j p_l = 0. \tag{25}$$

Using equation 22, the term $\dot{\mathbf{U}}$ in index form is

$$\dot{U}_i = \omega u_i \sin[\omega(p_m x_m - t)], \tag{26}$$

and $\nabla\mathbf{U}$ in index form is

$$U_{i,j} = -\omega u_i p_j \sin[\omega(p_m x_m - t)], \tag{27}$$

where the subscript indices after the comma symbol indicate the spatial derivation with respect to one Cartesian component. Therefore, we can rewrite the norm of $\square\mathbf{U}$, defined in equation 18, in index form as

$$\|\square\mathbf{U}\|^2 = \rho \dot{U}_i \dot{U}_i + c_{ijkl} U_{i,j} U_{k,l}. \tag{28}$$

Using equations 26 and 27, the individual terms in equation 28 are

$$\rho \dot{U}_i \dot{U}_i = \rho \omega^2 u_i u_i \sin^2[\omega(p_m x_m - t)], \tag{29}$$

$$c_{ijkl} U_{i,j} U_{k,l} = \omega^2 u_i u_k c_{ijkl} p_j p_l \sin^2[\omega(p_m x_m - t)]. \tag{30}$$

Hence, the norm of $\square\mathbf{U}$ is

$$\|\square\mathbf{U}\|^2 = \omega^2[\rho u_i u_i + u_i u_k c_{ijkl} p_j p_l] \sin^2[\omega(p_m x_m - t)]. \tag{31}$$

We seek to define an imaging condition that attenuates the correlation between waves from the source and receiver wavefields propagating along the same path and with the same polarization, i.e., elastic backscattering. Therefore, elastic backscattering events are characterized by the same plane waves: $V_i = U_i = u_i \cos[\omega(p_m x_m - t)]$. Defining $(\square\mathbf{V})^\dagger$ as

$$(\square\mathbf{V})^\dagger = \{-\rho^{1/2}\dot{\mathbf{V}}, \underline{\mathbf{c}}^{1/2}(\nabla\mathbf{V})\}, \tag{32}$$

we compute the dot product between $\square\mathbf{U}$ and $(\square\mathbf{V})^\dagger$ for backscattering events as

$$\square\mathbf{U} \cdot (\square\mathbf{V})^\dagger = \omega^2[\rho u_i u_i - u_i u_k c_{ijkl} p_j p_l] \sin^2[\omega(p_m x_m - t)]. \tag{33}$$

Using the relation in equation 25, we obtain

$$\square\mathbf{U} \cdot (\square\mathbf{V})^\dagger = 0, \tag{34}$$

i.e., the dot product is equal to zero everywhere except at the locations where reflectors exist or different wave modes interact because the vectors $\mathbf{p}$ and $\mathbf{u}_0$ are different for $\mathbf{U}$ and $\mathbf{V}$ at these locations. Therefore, the dot product in equation 34 nullifies the events that propagate along the same path and have the same polarization. Such events include reflection backscattering, diving, direct, and head waves from the same wave modes. Therefore, the imaging condition

$$I_E^\dagger = \sum_{e,t} \square\mathbf{U} \cdot (\square\mathbf{V})^\dagger \tag{35}$$

attenuates backscattering artifacts in elastic RTM images.

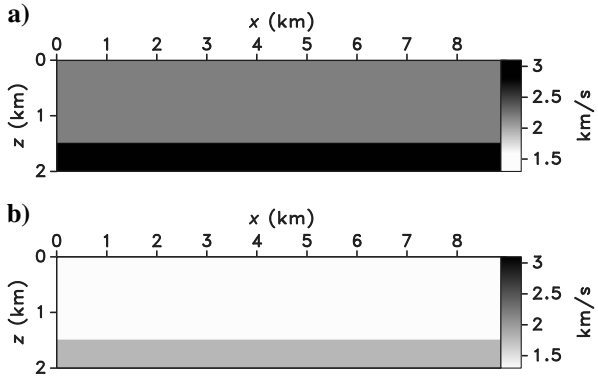


Figure 1. Model parameters useds for the experiments in Figure 2: (a) P and (b) S velocity along the symmetry axis. The anisotropy parameters $\epsilon = 0.4$, $\delta = 0.3$, and $\nu = 26^\circ$ are constant for the entire model.

EXAMPLES

Using a simple model with a horizontal reflector at $z = 1.5$ km and a vertical displacement source function (Figure 1), we test the proposed imaging condition and compare it with conventional imaging conditions (Figure 2). We refer to the zero-lag correlation between displacement field components as conventional imaging conditions (Yan and Sava, 2007). Figure 1 also shows the model parameters that we use in this experiment following the convention from Thomsen (1986). The presence of the reflector in the migration velocity leads to a velocity contrast, causing backscattering artifacts in the conventional images (Figure 2a–2c). In addition, the polarity reversal at normal incidence occurs when different components of the displacement field are correlated (Figure 2b). In contrast, the energy imaging condition from equation 35 effectively attenuates the backscattering artifacts and shows no polarity reversal at the imaged reflector (Figure 2d). Figure 2d only contains artifacts that are caused by the nonphysical (fake) modes generated during the wavefield extrapolation. These artifacts are characterized by the correlation of events with different polarization and/or propagation direction. Based on equation 33, the energy image outputs a nonzero dot product between the energy vectors $\square\mathbf{U}$ and $(\square\mathbf{V})^\dagger$ for these artifacts. For a multiple-shot experiment, the fake mode

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artifacts are attenuated after stacking over shots, whereas backscattering artifacts and polarity reversal effects are not.

Figures 3, 4, 5, and 6 show the Marmousi II (Martin et al., 2002) acquisition geometry, model parameters used to simulate synthetic data, the migration model parameters, and the resulted images. As seen in Figure 3, the synthetic data are obtained from a horizontal line of receivers that record the displacement field at every grid point of the water bottom ($z = 0.5$ km), and from 76 pressure sources located near the surface of the water layer ($z = 0.05$ km) with a horizontal spacing of 150 m. This acquisition geometry resembles a multicomponent ocean bottom seismic survey. The migration model parameters are smoothed versions of the true model parameters (Figures 4 and 5) and contain sharp interfaces to sufficiently create backscattering reflections during the wavefield extrapolation. Such reflections generate artifacts when using conventional imaging conditions. As expected, all conventional images in Figure 6a–6c contain backscattering artifacts, and Figure 6b also shows the effects of a nonconstructive stack due to polarity reversal artifacts. In contrast, we attenuate all of these artifacts in Figure 6d using the imaging condition from equation 35. Similar to the simple experiment in Figure 2d, using our new imaging condition, the individual elastic images for shots have only fake mode artifacts, which are attenuated after stacking over shots, thus leading to a good-quality image, as seen in Figure 6d.

Figures 7 and 8 show a 3D elastic experiment with constant model parameters above a horizontal reflector at $z = 0.21$ km. The model consists of a 3D tilted transverse isotropy (TTI) anisotropy symmetry system, and the tilt is determined by a 30° angle with the vertical axis, and an azimuth of 45° with respect to the x-axis. We use a vertical dislocation source at the surface in the middle of the model ($x = y = 0.45$ km and $z = 0$ km) to generate the source wavefield. The receiver wavefield is extrapolated from receivers at every grid point from the surface ($z = 0$ km). The correlation between displacement components from the source and receiver wavefields generate images with strong backscattering artifacts

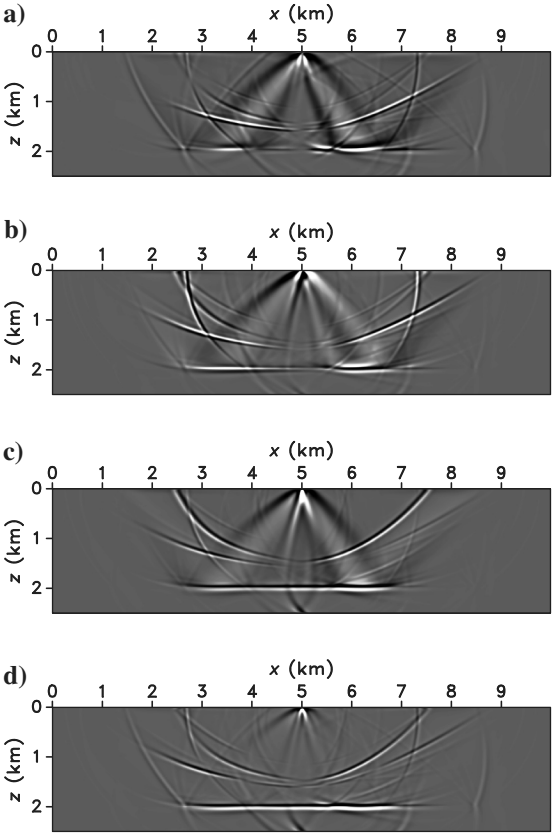


Figure 2. Imaging conditions using displacement components for a horizontal reflector with a vertical displacement source at $x = 5$ km: (a) U_xV_x, (b) U_zV_x, and (c) U_zV_z image, where $\{U_x, U_z\}$ and $\{V_x, V_z\}$ are the horizontal and vertical components of the source and receiver wavefields, respectively. Artifacts present in all images occur due to the fake modes (caused by injection of vector fields). Polarity reversal at normal incidence occurs for the image in panel (b), and backscattering artifacts occur for images in (a-c). Compared with the other images, the energy image in panel (d) is the only one with attenuated backscattering artifacts and without polarity reversal at normal incidence.

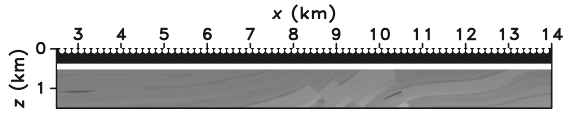


Figure 3. Acquisition geometry used in the Marmousi II experiment (Figures 4–6). The white dots indicate the positions of the 76 pressure sources ($z = 0.05$ km), and the white line at the water bottom indicates multicomponent receivers that record the displacement field ($z = 0.5$ km).

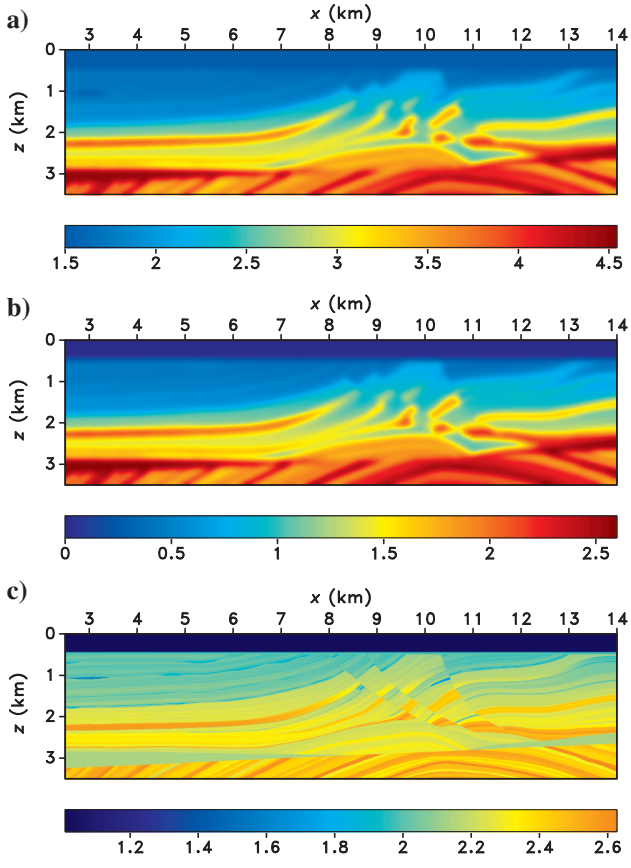


Figure 4. Migration model parameters used for the Marmousi II experiment. (a) P- and (b) S-wave velocities along the local symmetry axis (V_{P0} and V_{S0}, in km/s). (c) Stratigraphic density model used to generate synthetic data (in g/cm^3).

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and polarity-reversal effects (Figure 8a–8c). Using the energy imaging in equation 35, we form an image with attenuated artifacts and no polarity reversal (Figure 8d).

DISCUSSION

The main advantage of our method is that we are able to image the subsurface using anisotropic and elastic wavefields without wave-mode decomposition. Substantial improvements have been achieved in recent years through methods based on wave-mode decomposition (Zhang and McMechan, 2010; Yan and Sava, 2011a; Cheng and Fomel, 2014; Sripanich et al., 2015; Wang et al., 2016). Such methods are still computationally intense, as demonstrated by the fact that the vast majority of publications on this subject are limited to decomposing a few wavefield snapshots instead of running complete shot-record migrations. For instance, Cheng and Fomel (2014) use low-rank approximation to implement wave-mode decomposition that has a computational cost of order $O(MN_{\mathbf{x}} \log N_{\mathbf{x}})$, where $N_{\mathbf{x}}$ is the spatial dimension of the model and M is the rank, which can vary from 7 to 17 for complex models (depending on the desired accuracy). Alternatively, our energy imaging condition requires the same spatial derivatives as those in Helmholtz decomposition, which has a computational cost of order $O(N_{\mathbf{x}})$.

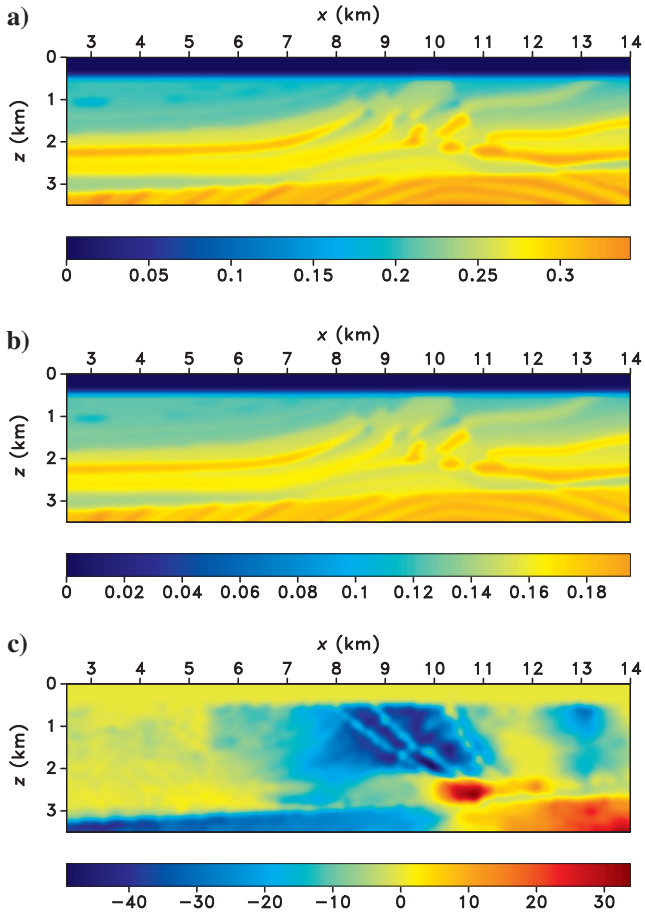


Figure 5. Migration anisotropy parameters used for the Marmousi II experiment. (a) ϵ, (b) δ, and (c) local tilt angle ν (in degrees).

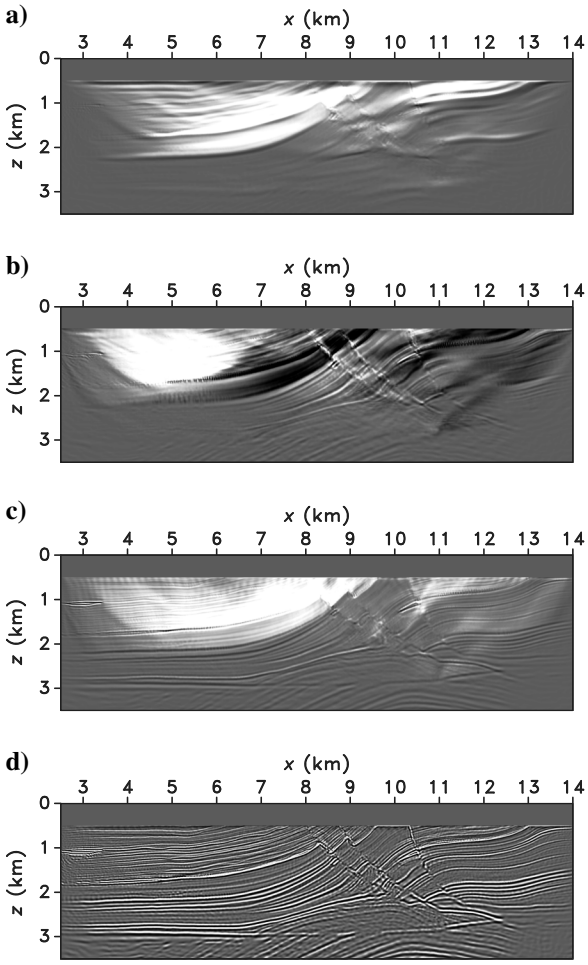


Figure 6. Images obtained from the Marmousi II experiment: conventional images using the correlation between displacement components (a) U_xV_x, (b) U_xV_z, and (c) U_zV_z; (d) energy image using the imaging condition in equation 35. Polarity reversal effects are visible in panel (b) by the change in polarity from the low-wavenumber artifacts. The energy image in panel (d) is the only one without low-wavenumber backscattering artifacts.

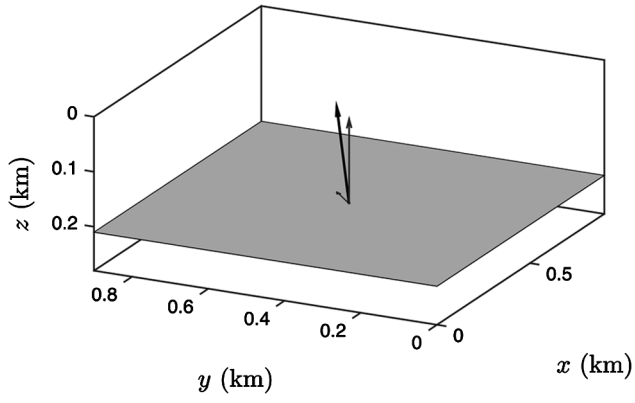


Figure 7. Schematic representation of the 3D TTI model used in the experiment of Figure 8. The black arrow indicates the tilted symmetry axis. The gray arrows indicate the projections of this vector onto the vertical axis and onto the reflection plane. Model parameters in the original system are $V_{P0} = 2.3$ km/s, $V_{S0} = 1.3$ km/s, $\rho = 2.0$ g/cm^3, $\epsilon = 0.4$, $\delta = 0.1$, and $\gamma = 0.0$.

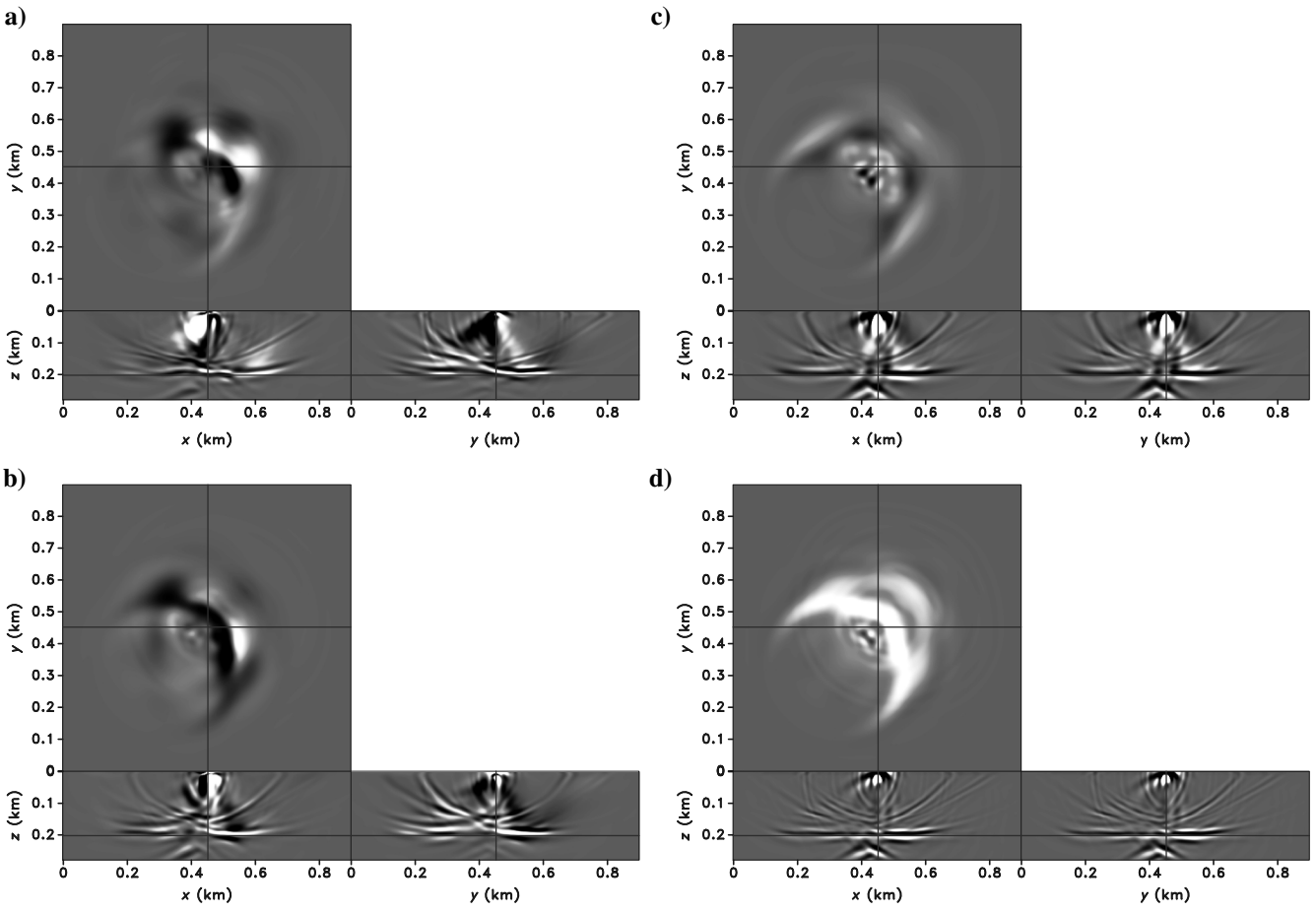


Figure 8. Images using the correlation between displacement components: (a) U_xV_y, (b) U_xV_z, and (c) U_zV_z; and an image using the energy imaging condition in panel (d). Artifacts due to the fake modes are present in all images. Backscattering artifacts and polarity reversal occur in the conventional images in panels (a-c) but not in the energy image in panel (d).

However, the absence of wave-mode decomposition in our method limits interpretation by not providing a suitable relationship between P- and S-wave impedance contrasts, which is systematically studied in reservoir characterization. Our energy imaging condition reflects the balance of energy between the incoming source wavefield and the reflected receiver wavefield. The amplitude information from the energy image and its possible contribution to reservoir characterization are not clear and are subjects for future research.

CONCLUSION

The anisotropic energy imaging condition is applicable for arbitrary anisotropy, produces a scalar image without the need for wave-mode decomposition, and does not suffer from polarity reversal, as is the case for displacement images using different components of the source and receiver wavefields. We describe this imaging condition as the scalar product between the energy vectors $\square\mathbf{U}$ and $\square\mathbf{V}$, which are built using conventionally extrapolated wavefields $\mathbf{U}$ and $\mathbf{V}$. As inferred from the definition of these vectors, the proposed imaging condition accounts for wavefield directionality, incorporating the wavefield propagation and polarization directions. By exploiting this directionality, we are able to attenuate the backscattering artifacts in the imaging process, and obtain robust, inexpensive, and high-quality elastic images.

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APPENDIX A

FROBENIUS PRODUCT AND NORM

The Frobenius norm in linear algebra generalizes the more conventional vector norm concept. Consider an arbitrary matrix $A_{M\times N}$, this norm is defined as (Golub and Loan, 1996)

$$\|A\|_F = \sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}|a_{ij}|^2}. \tag{A-1}$$

One can also define a norm of a tensor field using the inner product of associated matrices. Given tensor fields A and B, the Frobenius product is defined as

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$$\langle A, B\rangle_F = A:B = \sum_{i=1}^{M}\sum_{j=1}^{N} a_{ij}b_{ij}, \tag{A-2}$$

where M and N are the dimensions of the tensors A and B. This product can also be written in matrix form as

$$\langle A, B\rangle_F = A:B = \mathrm{Tr}(A^{\mathrm{T}}B) = \mathrm{Tr}(AB^{\mathrm{T}}). \tag{A-3}$$

Then, the Frobenius norm of a tensor A is written as

$$\|A\|_F = \sqrt{\mathrm{Tr}(AA^{\mathrm{T}})}. \tag{A-4}$$

APPENDIX B

STRAIN-ENERGY FUNCTION IN ISOTROPIC MEDIA

In a general elastic medium subject to infinitesimal strains, the strain-energy function is (Slawinski, 2003)

$$E(\epsilon) = \frac{1}{2}c_{ijlk}\epsilon_{kl}\epsilon_{ij}, \tag{B-1}$$

where c_{ijlk} is an element of the stiffness tensor $\underline{\underline{\mathbf{c}}}$, and ϵ_{ij} is an element of the strain tensor $\underline{\mathbf{e}}$. The repeated indices i, j, k, and l imply summation, following the Einstein convention (Robert, 2005). For an isotropic medium, the stiffness coefficients are (Aki and Richards, 2002)

$$c_{ijlk} = \lambda\delta_{ij}\delta_{kl} + \mu\delta_{ik}\delta_{jl} + \mu\delta_{il}\delta_{jk}, \tag{B-2}$$

where δ_{ij} is the Kronecker delta function, and λ and μ are the Lamé parameters. Substituting equation B-2 in equation B-1, we obtain

$$E(\epsilon) = \frac{1}{2}[\lambda\delta_{ij}\delta_{kl} + \mu\delta_{ik}\delta_{jl} + \mu\delta_{il}\delta_{jk}]\epsilon_{kl}\epsilon_{ij}. \tag{B-3}$$

Considering the Kronecker delta function is equal to unity for equal indices and zero otherwise, we develop each term and obtain

$$E(\epsilon) = \frac{1}{2}[\lambda\epsilon_{ii}\epsilon_{kk} + \mu\epsilon_{ij}\epsilon_{ij} + \mu\epsilon_{ji}\epsilon_{ij}]. \tag{B-4}$$

These implicit summations can be written in matrix form as

$$E(\mathbf{e}) = \frac{1}{2}[\lambda\mathrm{Tr}[\mathbf{e}]\mathrm{Tr}[\mathbf{e}] + \mu(\mathbf{e}:\mathbf{e}) + \mu(\mathbf{e}:\mathbf{e}^{\mathrm{T}})]. \tag{B-5}$$

As the strain tensor is symmetric, we obtain

$$E(\mathbf{e}) = \frac{1}{2}[\lambda(\mathrm{Tr}[\mathbf{e}])^2 + 2\mu(\mathbf{e}:\mathbf{e})]. \tag{B-6}$$

The strain tensor $\mathbf{e}$ is a function of the wavefield derivatives:

$$\mathbf{e} = \frac{1}{2}[\nabla\mathbf{U} + \nabla\mathbf{U}^{\mathrm{T}}]. \tag{B-7}$$

Substituting equation B-7 in equation B-6, we obtain

$$\begin{aligned} E(\mathbf{e}) &= \frac{1}{2}\left[\lambda\|\nabla\cdot\mathbf{U}\|^2 + 2\mu\left(\frac{1}{2}(\nabla\mathbf{U}+\nabla\mathbf{U}^{\mathrm{T}}):\frac{1}{2}(\nabla\mathbf{U}+\nabla\mathbf{U}^{\mathrm{T}})\right)\right], \\ &= \frac{1}{2}\left[\lambda\|\nabla\cdot\mathbf{U}\|^2 + \frac{1}{2}\mu(\nabla\mathbf{U}:\nabla\mathbf{U} + 2\nabla\mathbf{U}:\nabla\mathbf{U}^{\mathrm{T}} + \nabla\mathbf{U}^{\mathrm{T}}:\nabla\mathbf{U}^{\mathrm{T}})\right], \\ &= \frac{1}{2}[\lambda\|\nabla\cdot\mathbf{U}\|^2 + \mu(\nabla\mathbf{U}:\nabla\mathbf{U} + \nabla\mathbf{U}:\nabla\mathbf{U}^{\mathrm{T}})], \end{aligned} \tag{B-8}$$

which corresponds to the potential energy terms of equation 15.

APPENDIX C

IMAGING CONDITIONS FOR ORTHORHOMBIC AND VERTICAL TRANSVERSE ISOTROPIC MEDIA

The most general imaging condition for two anisotropic elastic wavefields $\mathbf{U}(\mathbf{x}, t)$ and $\mathbf{V}(\mathbf{x}, t)$ is

$$I_E = \sum_{e,t}[\rho\dot{\mathbf{U}}\cdot\dot{\mathbf{V}} + (\underline{\underline{\mathbf{c}}}\nabla\mathbf{U}):\nabla\mathbf{V}]. \tag{C-1}$$

In orthorhombic media, equation C-1 becomes

$$\begin{aligned} I_E = {} & \rho[\dot{U}_1\dot{V}_1 + \dot{U}_2\dot{V}_2 + \dot{U}_3\dot{V}_3] \\ & + C_{11}U_{1,1}V_{1,1} + C_{22}U_{2,2}V_{2,2} + C_{33}U_{3,3}V_{3,3} \\ & + C_{12}[U_{1,1}V_{2,2} + U_{2,2}V_{1,1}] \\ & + C_{13}[U_{1,1}V_{3,3} + U_{3,3}V_{1,1}] \\ & + C_{23}[U_{2,2}V_{3,3} + U_{3,3}V_{2,2}] \\ & + C_{44}[U_{2,3}V_{2,3} + U_{2,3}V_{3,2} + U_{3,2}V_{2,3} + U_{3,2}V_{3,2}] \\ & + C_{55}[U_{1,3}V_{1,3} + U_{1,3}V_{3,1} + U_{3,1}V_{1,3} + U_{3,1}V_{3,1}] \\ & + C_{66}[U_{1,2}V_{1,2} + U_{1,2}V_{2,1} + U_{2,1}V_{1,2} + U_{2,1}V_{2,1}], \end{aligned} \tag{C-2}$$

where C_{ij} for $i, j = \{1, 2, 3\}$ are the stiffness coefficients in Voigt notation, $U_{i,j}$ or $V_{i,j}$ represents the jth derivative of the ith component from the wavefield $\mathbf{U}$ or $\mathbf{V}$. In wavefield imaging, the density of the medium $\rho(\mathbf{x})$ is often unknown; therefore, we can rewrite equation C-2 without the density term as

$$\begin{aligned} I_E = {} & \dot{U}_1\dot{V}_1 + \dot{U}_2\dot{V}_2 + \dot{U}_3\dot{V}_3 \\ & + A_{11}U_{1,1}V_{1,1} + A_{22}U_{2,2}V_{2,2} + A_{33}U_{3,3}V_{3,3} \\ & + A_{12}[U_{1,1}V_{2,2} + U_{2,2}V_{1,1}] \\ & + A_{13}[U_{1,1}V_{3,3} + U_{3,3}V_{1,1}] \\ & + A_{23}[U_{2,2}V_{3,3} + U_{3,3}V_{2,2}] \\ & + A_{44}[U_{2,3}V_{2,3} + U_{2,3}V_{3,2} + U_{3,2}V_{2,3} + U_{3,2}V_{3,2}] \\ & + A_{55}[U_{1,3}V_{1,3} + U_{1,3}V_{3,1} + U_{3,1}V_{1,3} + U_{3,1}V_{3,1}] \\ & + A_{66}[U_{1,2}V_{1,2} + U_{1,2}V_{2,1} + U_{2,1}V_{1,2} + U_{2,1}V_{2,1}], \end{aligned} \tag{C-3}$$

where A_{ij} represents stiffness coefficients normalized by the density (in velocity squared units).

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For vertical traverse isotropic media, one can reduce the number of stiffness coefficients by using $C_{22}=C_{11}, C_{12}=C_{11}-2C_{66}, C_{23}=C_{13}$, and $C_{44}=C_{55}$. Therefore,

$$\begin{aligned} I_E &= \rho[\dot{U}_1\dot{V}_1+\dot{U}_2\dot{V}_2+\dot{U}_3\dot{V}_3] \\ &+ C_{11}[U_{1,1}V_{1,1}+U_{2,2}V_{2,2}+U_{1,1}V_{2,2}+U_{2,2}V_{1,1}] \\ &+ C_{33}U_{3,3}V_{3,3} \\ &+ C_{13}[U_{1,1}V_{3,3}+U_{3,3}V_{1,1}+U_{2,2}V_{3,3}+U_{3,3}V_{2,2}] \\ &+ C_{55}[U_{2,3}V_{2,3}+U_{2,3}V_{3,2}+U_{3,2}V_{2,3}+U_{3,2}V_{3,2} \\ &+ U_{1,3}V_{1,3}+U_{1,3}V_{3,1}+U_{3,1}V_{1,3}+U_{3,1}V_{3,1}] \\ &+ C_{66}[U_{1,2}V_{1,2}+U_{1,2}V_{2,1}+U_{2,1}V_{1,2} \\ &+ U_{2,1}V_{2,1}-2U_{1,1}V_{2,2}-2U_{2,2}V_{1,1}]. \end{aligned} \tag{C-4}$$

Using Thomsen's (1986) parameters and the P- and S-wave velocities at the symmetry axis, we can rewrite equation C-4 as

$$\begin{aligned} I_E &= \dot{U}_1\dot{V}_1+\dot{U}_2\dot{V}_2+\dot{U}_3\dot{V}_3 \\ &+ (1+2\epsilon)v_{P0}^2[U_{1,1}V_{1,1}+U_{2,2}V_{2,2}+U_{1,1}V_{2,2}+U_{2,2}V_{1,1}] \\ &+ v_{P0}^2U_{3,3}V_{3,3}+\left[\sqrt{(v_{P0}^2-v_{S0}^2)(v_{PN}^2-v_{S0}^2)}-v_{S0}^2\right] \\ &\times[U_{1,1}V_{3,3}+U_{3,3}V_{1,1}+U_{2,2}V_{3,3}+U_{3,3}V_{2,2}] \\ &+ v_{S0}^2[U_{2,3}V_{2,3}+U_{2,3}V_{3,2}+U_{3,2}V_{2,3}+U_{3,2}V_{3,2} \\ &+ U_{1,3}V_{1,3}+U_{1,3}V_{3,1}+U_{3,1}V_{1,3}+U_{3,1}V_{3,1}] \\ &+ (1+2\epsilon)v_{S0}^2[U_{1,2}V_{1,2}+U_{1,2}V_{2,1}+U_{2,1}V_{1,2} \\ &+ U_{2,1}V_{2,1}-2U_{1,1}V_{2,2}-2U_{2,2}V_{1,1}], \end{aligned} \tag{C-5}$$

where v_{P0} and v_{S0} are the P- and S-wave velocities along the symmetry axis, v_{PN} is the normal moveout P-wave velocity, and ϵ and γ are the Thomsen's parameters.

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