COLORADO SCHOOL OF MINES
EARTH • ENERGY • ENVIRONMENT

Division for Planetary Sciences of the AMERICAN ASTRONOMICAL SOCIETY

THE UNIVERSITY OF ARIZONA

3D high-resolution radar imaging of small body interiors

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1: Science objectives

The Rosetta mission to comet 67P/C-G revolutionized comet science, but left major questions on the table. Are cometary nuclei primordial (Masseroni et al. 2015) or are they collisionally-evolved as predicted by modern theories of planet formation? What is the source of their activity? The CONSERT radar experiment sounded the interior of a nucleus for the first time (Kofman et al. 2015), showing it to be transparent to a depth of kilometers at 90 MHz. Although Rosetta did not resolve internal structure, it demonstrated the feasibility of a 3D global reflection imaging investigation at 5-15 MHz.

The **Comet Radar Explorer** mission (Asphaug et al. 2014) will acquire a dense network of in-phase radar echoes from orbit to obtain a high resolution 3D image. It will directly reveal the internal record of comet nucleus accretion and collisional evolution, and allow us to understand what drives cometary activity and geology. Full wavefield methods facilitate high quality imaging of the comet interiors, particularly if the comet nucleus is characterized by complex structure and large contrasts of physical properties (Sava et al. 2015). Knowledge of the comet shape and all-around orbital radar acquisition enable accurate and computationally-efficient 3D tomography.

Using realistic numerical experiments, we argue that

1. interior comet imaging is intrinsically 3D; conventional SAR processing does not satisfy imaging/resolution requirements;
2. interior tomography can be performed progressively with data acquired through successive orbits around the comet;
3. exploiting the known exterior shape of the comet facilitates cost-effective wavefield tomography;
4. multiscale wavefield tomography provides detailed information about the 3D internal physical properties;
5. imaging resolution exceeds the wavelength limits corresponding to low-frequency radar waves;
6. monostatic tomography yields 3D models comparable with what could be accomplished by more complex systems.

- Asphaug, E., et al., 2014, The Comet Radar Explorer Mission, AAS/Division for Planetary Sciences Meeting Abstracts. Vol. 46.
- Kofman, W., et al., 2015, Properties of the 67P/Churyumov-Gerasimenko interior revealed by CONSERT radar, Science, 349.6247, aab0639.
- Massironi, Matteo, et al., 2015, Two independent and primitive envelopes of the bilobate nucleus of comet 67P, Nature, 526.7573, 402
- Sava, P.C. et al., 2015, Radio Reflection Imaging of Asteroid and Comet Interiors I: Acquisition and Imaging Theory, Advances in Space Research, v. 55, 2149-2165.

2: Orbital acquisition

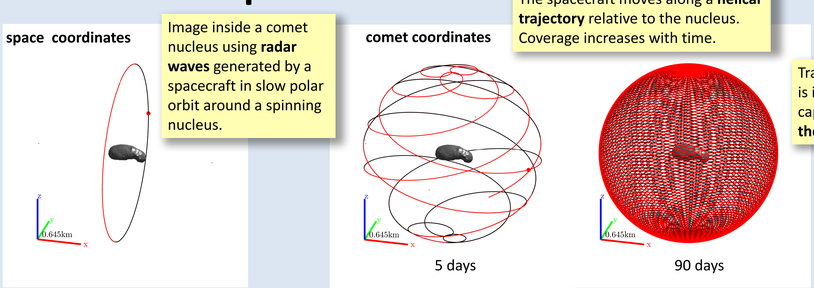
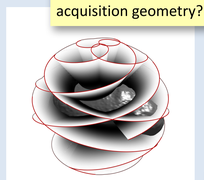


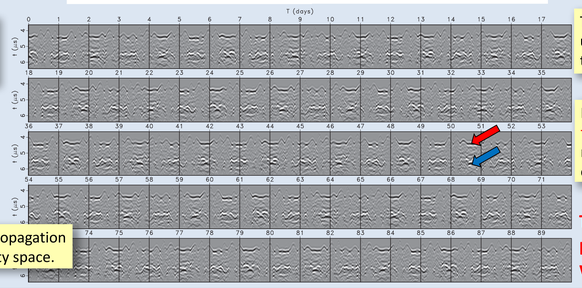
Image inside a comet nucleus using **radar waves** generated by a spacecraft in slow polar orbit around a spinning nucleus.

The spacecraft moves along a **helical trajectory** relative to the nucleus. Coverage increases with time.

Track-based processing (e.g. SAR) is inadequate – reflections are captured, from all directions, i.e. **there is no clutter**.

Fold sequential data in order to account for the 3D helical acquisition geometry?





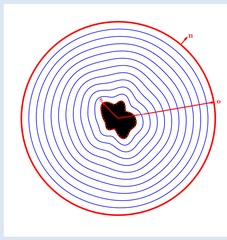
The data are sampled uniformly in acquisition time (not in position).

Reflections from the front and back of the nucleus surface dominate the data.

Long propagation in empty space.

The comet nucleus imaging problem is intrinsically 3D, with all-around illumination.

3: Wavefield datuming



$$h(\mathbf{o},t) = \int_{\Omega} d\mathbf{s}\; d(\mathbf{s},t) * g(\mathbf{o},\mathbf{s},t)$$

Remap data to/from the comet surface.

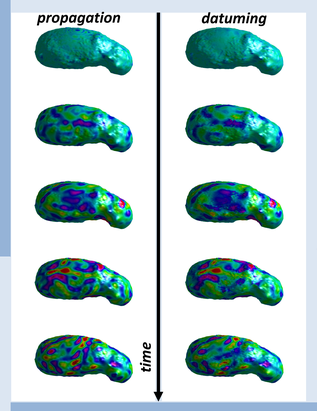
Propagate using Green's functions.

$$g(\mathbf{o},\mathbf{s},t) = A(\mathbf{o},\mathbf{s},\mathbf{n})\,\delta\left(t - \frac{\|\mathbf{o}-\mathbf{s}\|}{c}\right)$$

Account for spreading and antenna radiation.

$$A(\mathbf{o},\mathbf{s},\mathbf{n}) = \frac{W(\mathbf{o},\mathbf{s},\mathbf{n})}{4\pi\|\mathbf{o}-\mathbf{s}\|c^2}$$

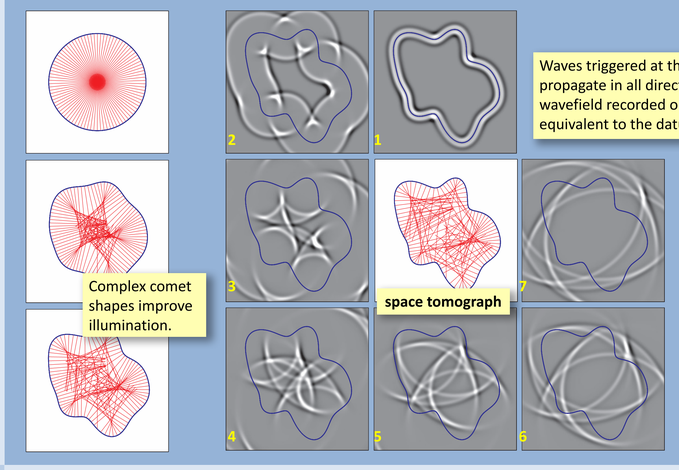
$$W(\mathbf{o},\mathbf{s},\mathbf{n}) = \mathbf{n}\cdot(\mathbf{o}-\mathbf{s})$$



datuming operator	$\mathbf{h} = \mathbf{G}\,\mathbf{d}$
inverse problem	$J(\mathbf{d}) = \frac{1}{2}\|\mathbf{h} - \mathbf{G}\,\mathbf{d}\|^2 + \frac{1}{2}\|\epsilon\mathbf{R}\mathbf{d}\|^2$
	datuming *regularization*
least-squares solution	$\mathbf{d} = (\mathbf{G}^t\mathbf{G} + \epsilon^2\mathbf{R}^t\mathbf{R})^{-1}\mathbf{G}^t\mathbf{d}$

Relocate the data to the comet surface + fill acquisition gaps.

Wavefield datuming removes propagation in empty space, and achieves computational speed-up.



Waves triggered at the surface propagate in all directions. The wavefield recorded on the surface is equivalent to the datumed wavefield.

Complex comet shapes improve illumination.

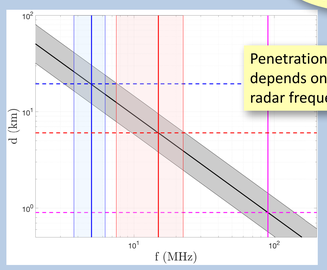
space tomograph

4: Depth of investigation

Scalar EM wave equation
μ – permeability
ϵ – permittivity
σ – conductivity

$$\mathbf{L}u \equiv \mu\epsilon\frac{\partial^2 u}{\partial t^2} + \mu\sigma\frac{\partial u}{\partial t} - \boldsymbol{\nabla}^2 u = f(\mathbf{x},t)$$

low conductivity = **weak attenuation**



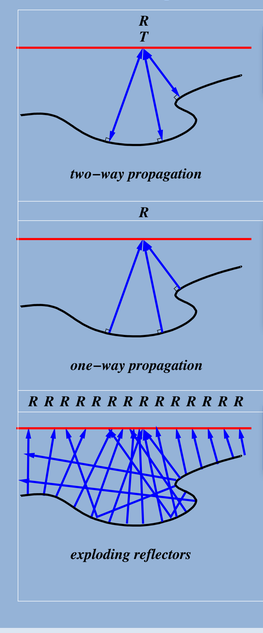
Penetration depth depends on the radar frequency.

Scalar EM wave equation
v – velocity

$$\mathbf{L}u \equiv \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \boldsymbol{\nabla}^2 u = f(\mathbf{x},t)$$

Low frequency radar waves return reflections from the comet back-side.

5: Exploding reflector model



A wave propagating from transmitter ***T*** returns to receiver ***R=T*** if it reflects at 90° on an interface (Snell's law).

The propagation paths before/after reflection are identical. Two-way propagation can be simulated with sources on the reflector and one-way propagation.

An entire monostatic dataset can be simulated with all reflectors "exploding" at the same initial time.

The exploding reflector model transforms a monostatic reflection experiment into a transmission tomographic experiment.

6: Wavefield tomography

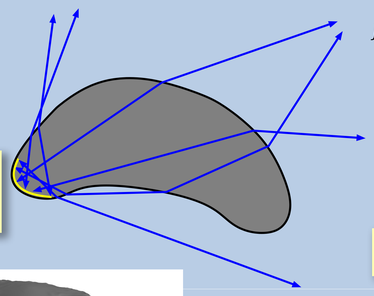


Image the back-side reflectors through the comet interior.

$$J(\mathbf{m}) = \frac{1}{2}\left\|\mathbf{d}^{dtm} - \mathbf{d}^{pre}(\mathbf{m})\right\|^2$$

Match wavefield simulated from the comet surface with the wavefields datumed from orbit.

$$\mathbf{d}^{pre} = \mathbf{u}(\mathbf{x}=\mathbf{s})$$

Restrict the simulated wavefield to the surface.

$$m\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} - \boldsymbol{\nabla}^2 u(\mathbf{x},t) = f(\mathbf{s},t)$$

Simulate wavefields using the current model.

Build sources from the surface reflectivity and the (known) radar wavelet.

$$f(\mathbf{s},t) = r(\mathbf{s})\,w(t)$$

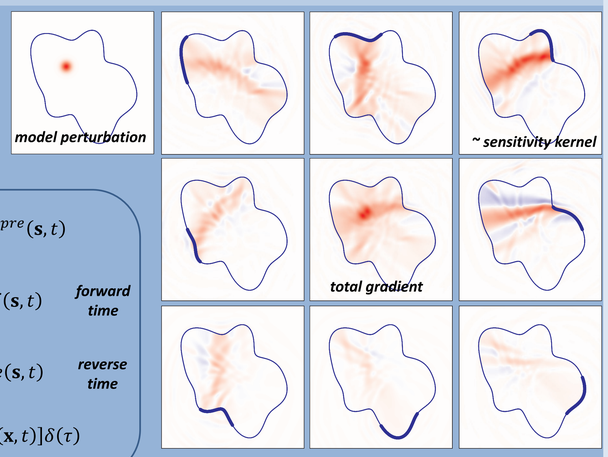
$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k\,\boldsymbol{\nabla}J(\mathbf{m}_k)$$

Iteratively update the model using non-linear optimization techniques.

Formulate wavefield tomography by imposing the condition that the back-side surface reflector is imaged at its known position.

7: Adjoint state method

Compute efficiently the gradient of the objective function.



1: compute the adjoint source	$e(\mathbf{s},t) = d^{dtm}(\mathbf{s},t) - d^{pre}(\mathbf{s},t)$	
2: simulate the state wavefield	$\mathbf{L}u \equiv m\frac{\partial^2 u}{\partial t^2} - \boldsymbol{\nabla}^2 u = f(\mathbf{s},t)$	*forward time*
3: simulate the adjoint wavefield	$\mathbf{L}^t a \equiv m\frac{\partial^2 a}{\partial t^2} - \boldsymbol{\nabla}^2 a = e(\mathbf{s},t)$	*reverse time*
4: compute the gradient	$\boldsymbol{\nabla}J(\mathbf{x}) = \sum_{\tau}[\ddot{u}(\mathbf{x},t) \star a(\mathbf{x},t)]\delta(\tau)$	

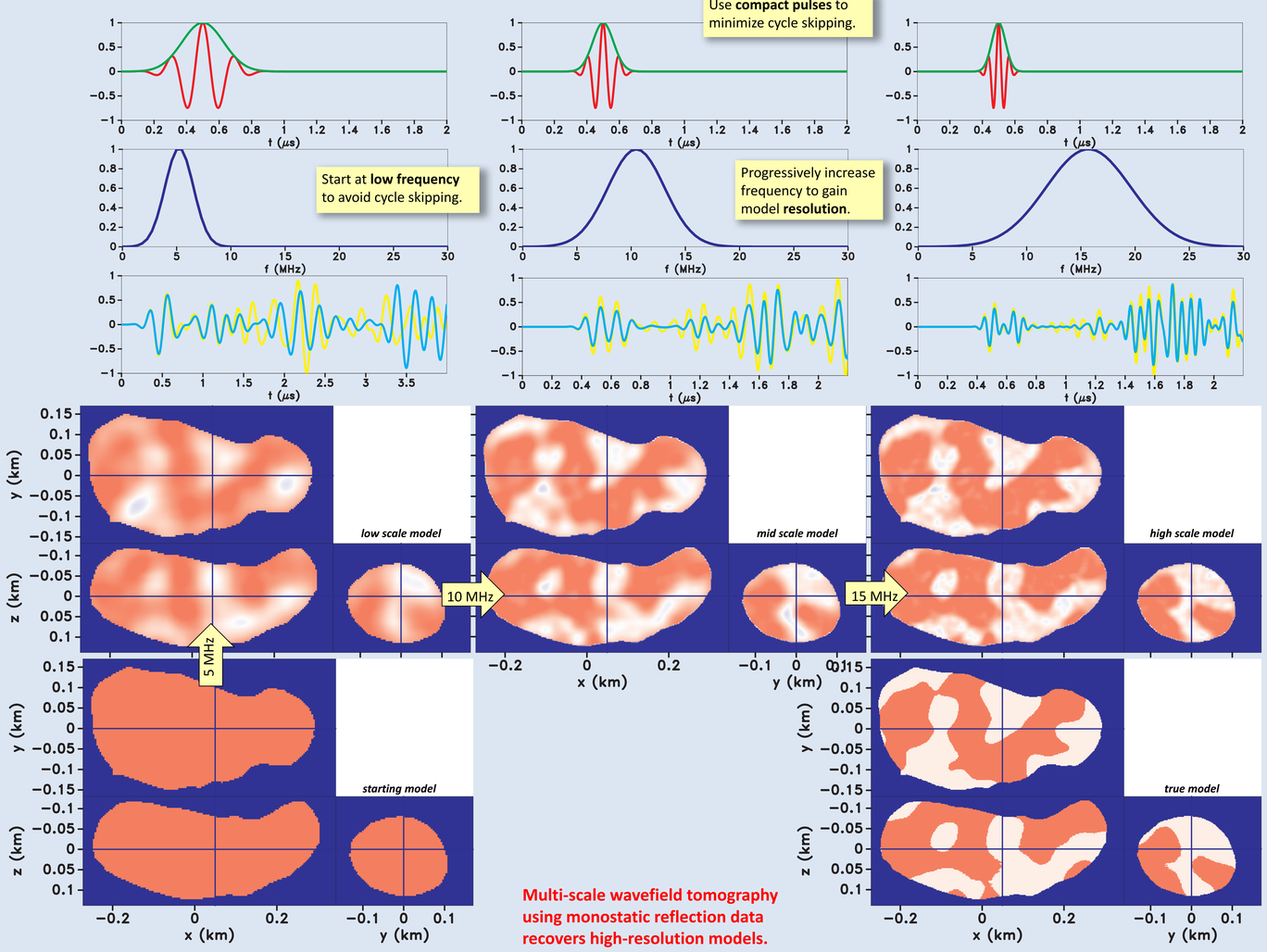
The adjoint state method effectively evaluates the gradient of the tomographic objective function.

8: 3D example

Use **compact pulses** to minimize cycle skipping.

Start at **low frequency** to avoid cycle skipping.

Progressively increase frequency to gain model **resolution**.



Multi-scale wavefield tomography using monostatic reflection data recovers high-resolution models.