P- and SV-mode separation in complex geology with VTI anisotropy

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ABSTRACT

The separation of wave modes to P and S from isotropic elastic wavefields is typically done using Helmholtz decomposition. However, Helmholtz decomposition using conventional divergence and curl operators in anisotropic media does not give satisfactory results and leaves the different wave modes only partially separated. The separation of anisotropic wavefields requires the use of more sophisticated operators which depend on local material parameters. Anisotropic wavefield separation operators are constructed using the polarization vectors evaluated at each point of the medium by solving the Christoffel equation for local medium parameters. These polarization vectors can be represented in the space domain as localized filtering operators, which resemble conventional derivative operators. The spatially variable "pseudo" derivative operators perform well in heterogeneous VTI media even at places of rapid velocity/density that the operators can be used to separate wavefields for VTI media with arbitrary degree of anisotropy.

Method

•Separation of scalar and vector potentials can be achieved by Helmholtz decomposition $W = \nabla \cdot \Theta + \nabla \times \Phi$

Isotropic

$$S = \nabla \times W$$

$$P = \nabla \cdot W = D_x[W_x] + D_z[W_z]$$

$$= ik_x \tilde{W}_x + ik_z \tilde{W}_z = i\,\mathbf{k} \cdot \tilde{\mathbf{W}}$$

•The operator $\nabla \cdot$ projects the wavefields onto the wave vector **k**, which represents the polarization direction for P waves.

•The operator $\nabla \times$ projects the wavefields onto the direction orthogonal to the wave vector **k**, which represents the polarization direction for S waves .

•Wave mode separation can be extended to anisotropic media by projecting the wavefields onto the directions in which the P and S modes are polarized (Dellinger and Etgen, 1990).

Anisotropic

$$S = \nabla_a \times W$$

$$P = \nabla_a \cdot W = L_x[W_x] + L_z[W_z]$$

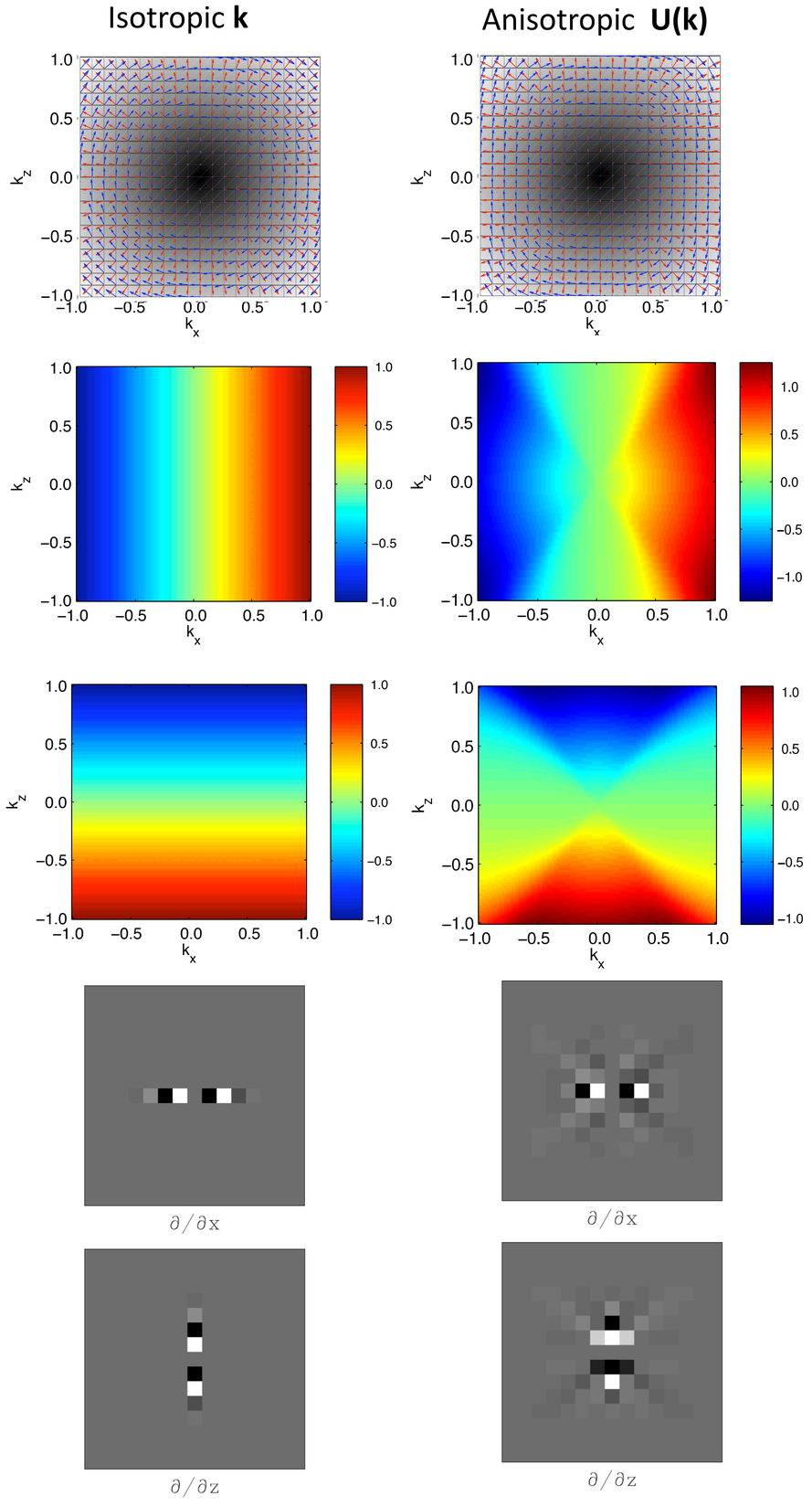
$$= iU_x \tilde{W}_x + iU_z \tilde{W}_z = i\,\mathbf{U}(\mathbf{k}) \cdot \tilde{\mathbf{W}}$$

Christoffel equation

U is the polarization vector of a certain wave type that is solved for with **Christoffel** equation

$$[\mathbf{G} - \rho V^2 \mathbf{I}]\,\mathbf{U} = 0$$

$$\begin{bmatrix} c_{11}k_x^2 + c_{55}k_z^2 - \rho V^2 & (c_{13} + c_{55})k_x k_z \\ (c_{13} + c_{55})k_x k_z & c_{55}k_x^2 + c_{33}k_z^2 - \rho V^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_z \end{bmatrix} = 0$$



polarization vectors

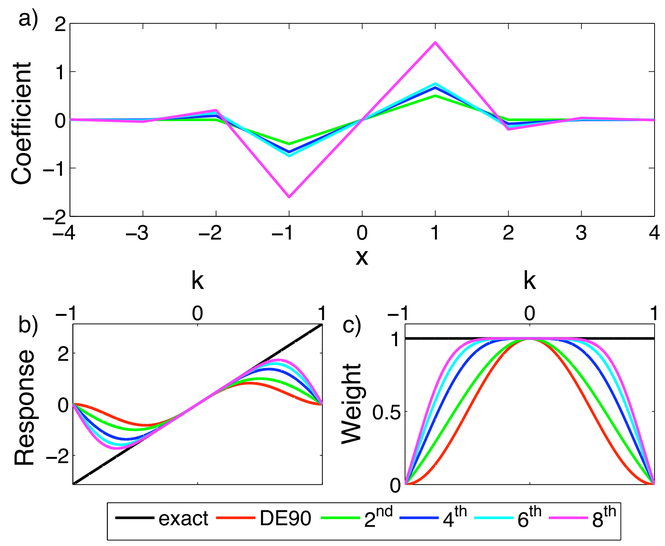
wavenumber domain separators in z direction

in z direction

space domain separators in x direction

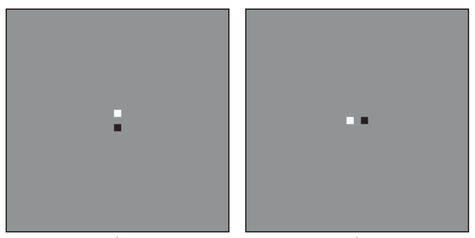
in z direction

Separators with different orders of accuracy

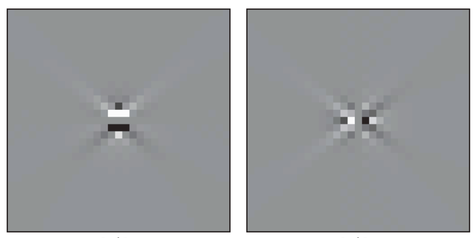


Comparison of derivative operators of different orders of accuracy (2nd, 4th, 6th, 8th order in space and cosine tape (Dellinger and Etgen, 1990)) in (a) the space domain and (b) the wavenumber domain. (c) Weights to apply to the components of the polarization vectors.

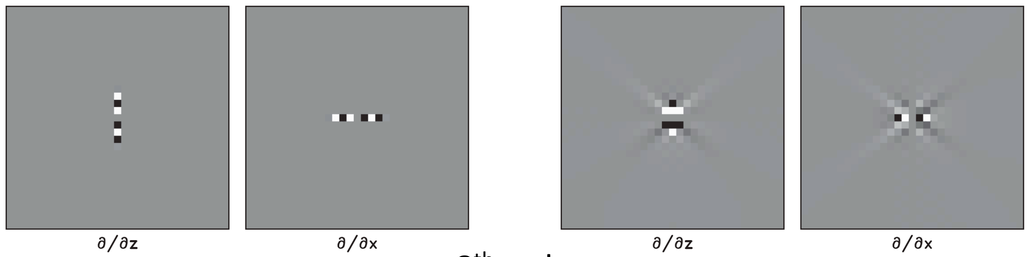
Isotropic



Anisotropic



2nd order

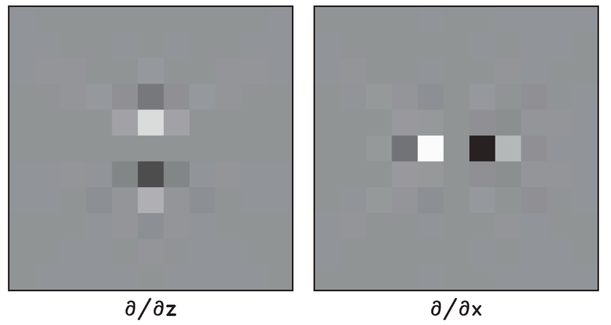


8th order

Influence of anisotropy strength on the size of separators

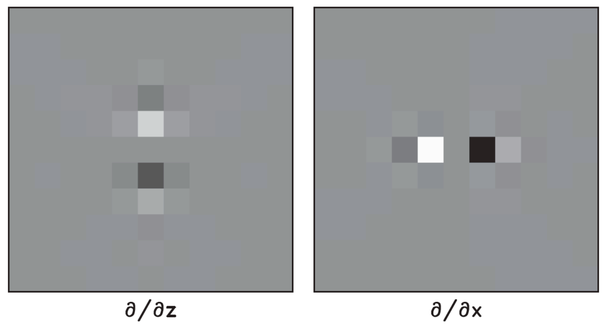
$\varepsilon = 0.25$

$\delta = -0.29$



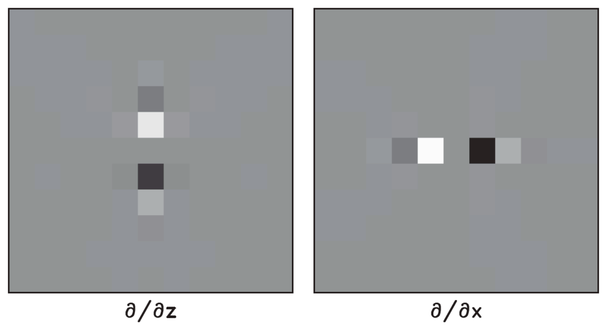
$\varepsilon = 0.54$

$\delta = 0$



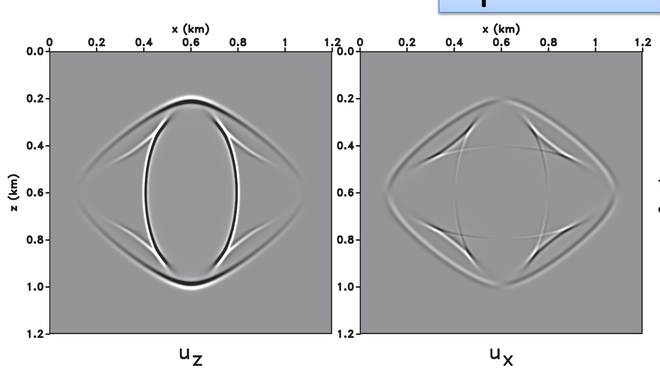
$\varepsilon = 0.2$

$\delta = 0$

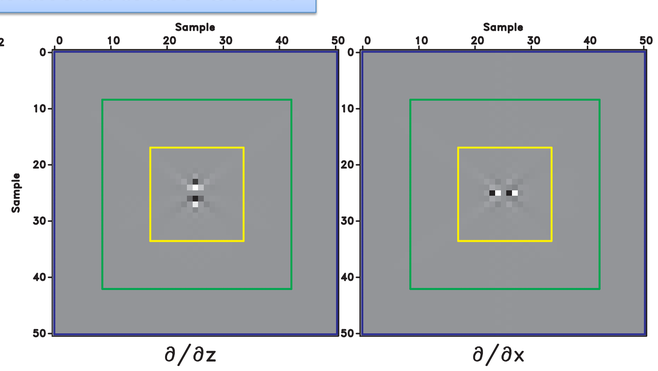


The size of the anisotropic wavefield separators is bigger when the anisotropy of the medium is strong.

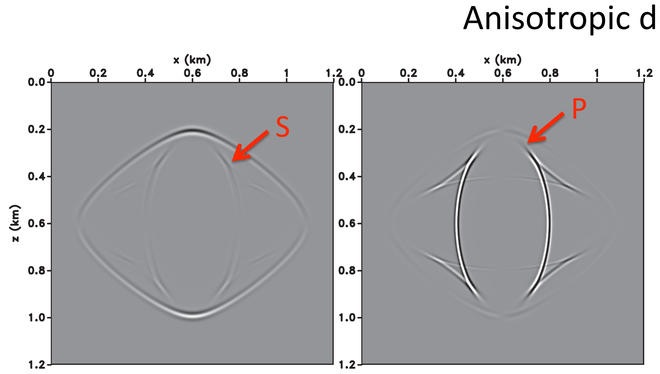
Operator truncation error



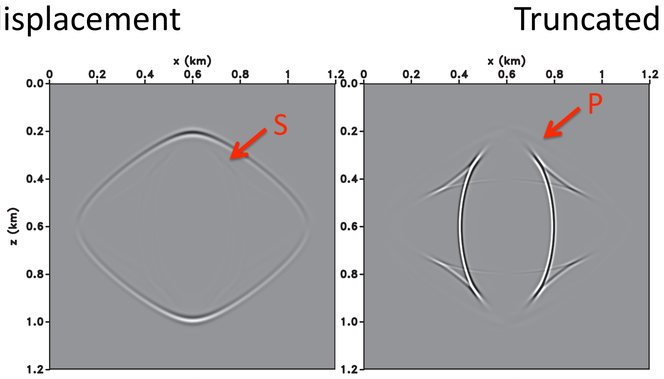
Anisotropic displacement



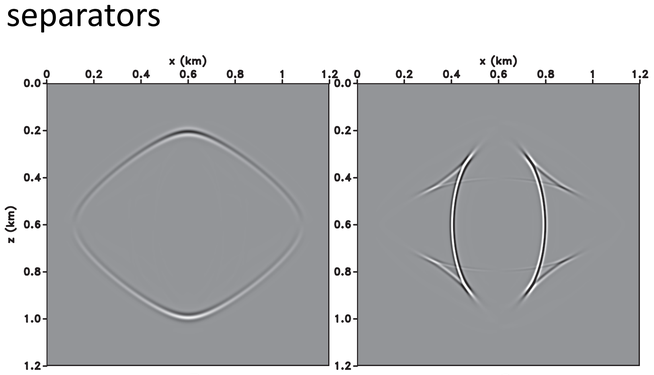
Truncated separators



Separation with operator size 11x11



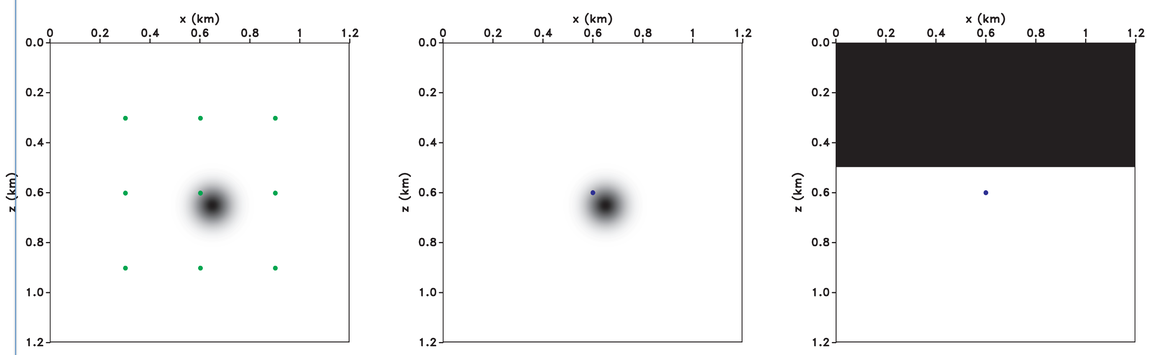
Separation with operator size 31x31



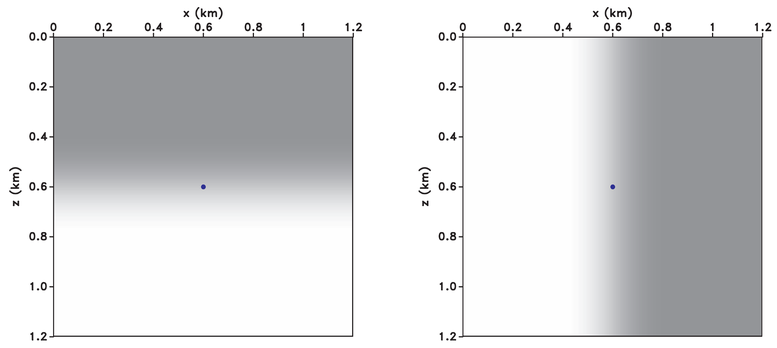
Separation with operator size 51x51

Examples

Gaussian anomaly



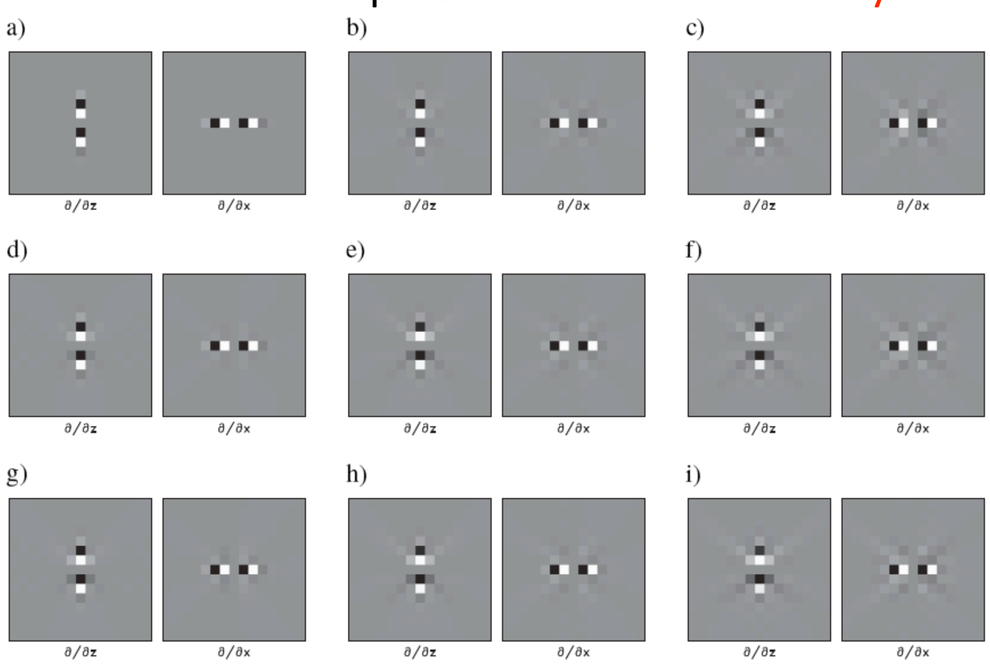
V_p V_s ρ



ε δ

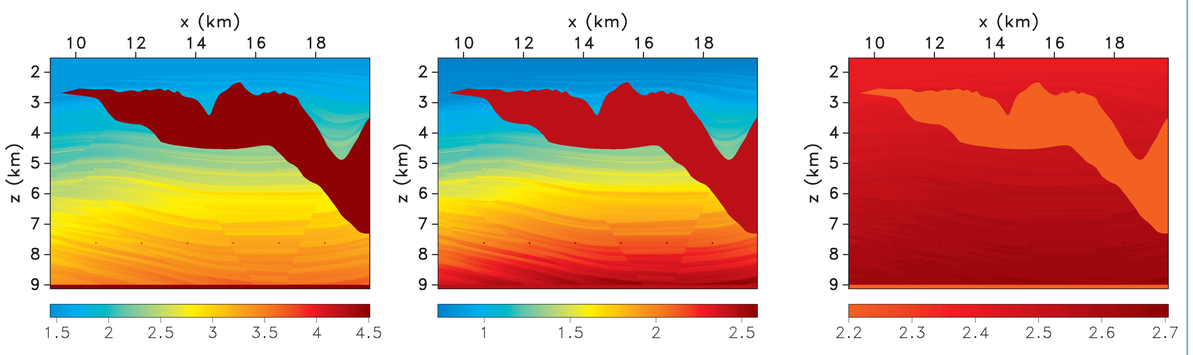
A model with parameters (a) V_{P0} = 3 km/s and (b) V_{S0} = 1.5 km/s except for a low velocity Gaussian anomaly at x = 0.65 km and z = 0.65 km, (c) ρ = 1 g/cm^3 in the top layer and 2 g/cm^3 in the bottom layer, (d) ε smoothly varying from 0 to 0.25 from top to bottom, (e) δ smoothly varying from 0 to 0.29 from left to right. The dots in panel (a) correspond to the locations of the anisotropic operators shown below.

Anisotropic stencils : nonstationary

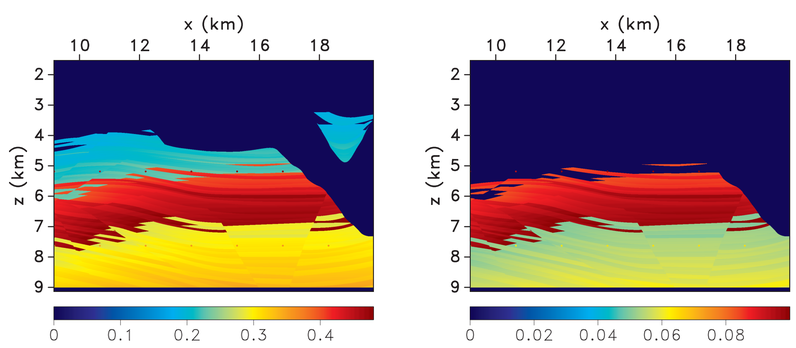


The 8th order anisotropic pseudo derivative operators in the z and x directions at the intersections of x=0.3, 0.6, 0.9 km and z=0.3, 0.6, 0.9 km for the model shown in the above figure.

Sigsbee



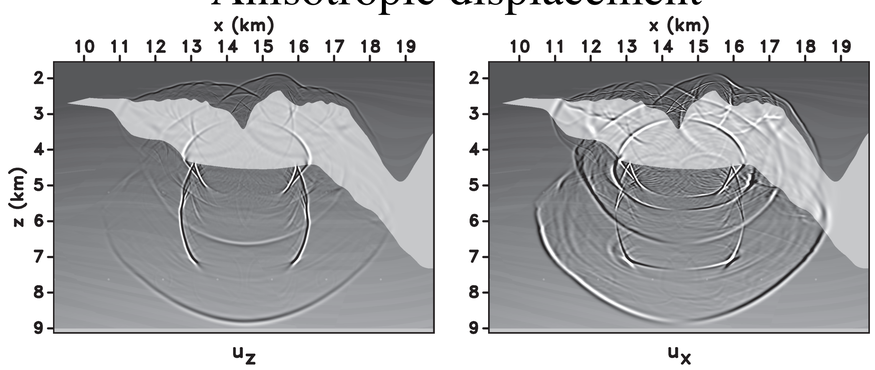
V_p V_s ρ



ε δ

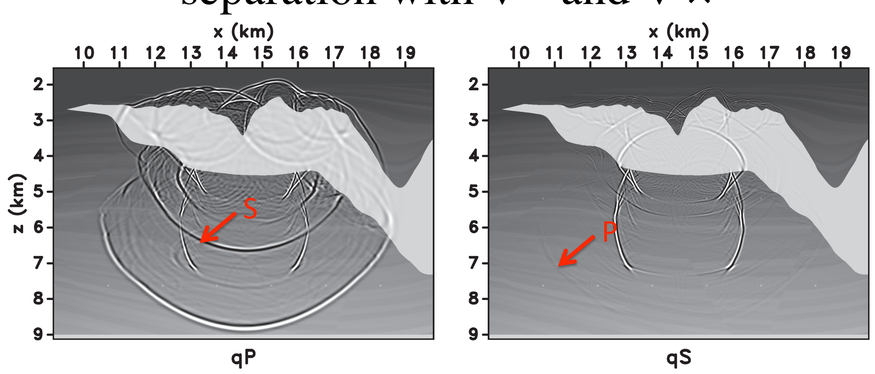
A Sigsbee 2A model with $V_{P0}V_{S0}$ ratio ranging from 1.5 to 2, density ranging from 1 g/cm^3 to 2.2 g/cm^3, ε ranging from 0.2 to 0.48, and δ ranging from 0 to 0.10.

Anisotropic displacement



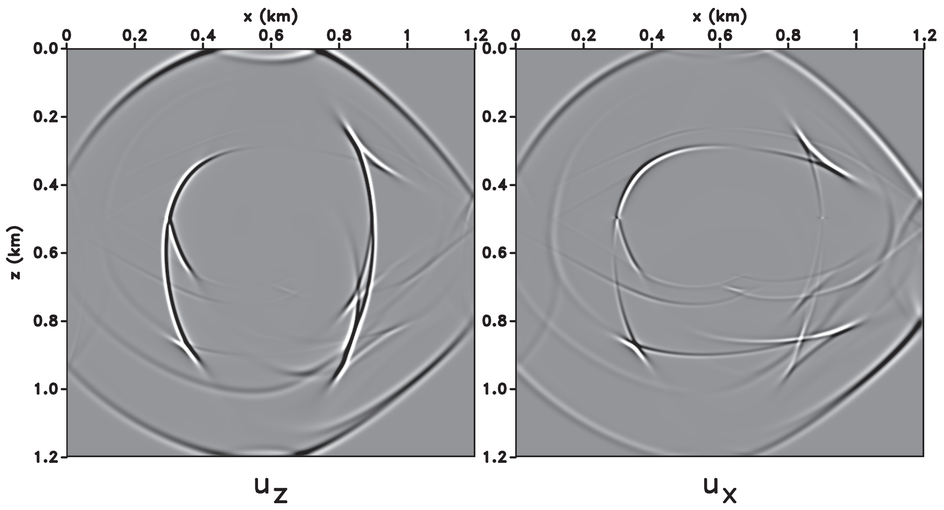
u_z u_x

separation with $\nabla\cdot$ and $\nabla\times$

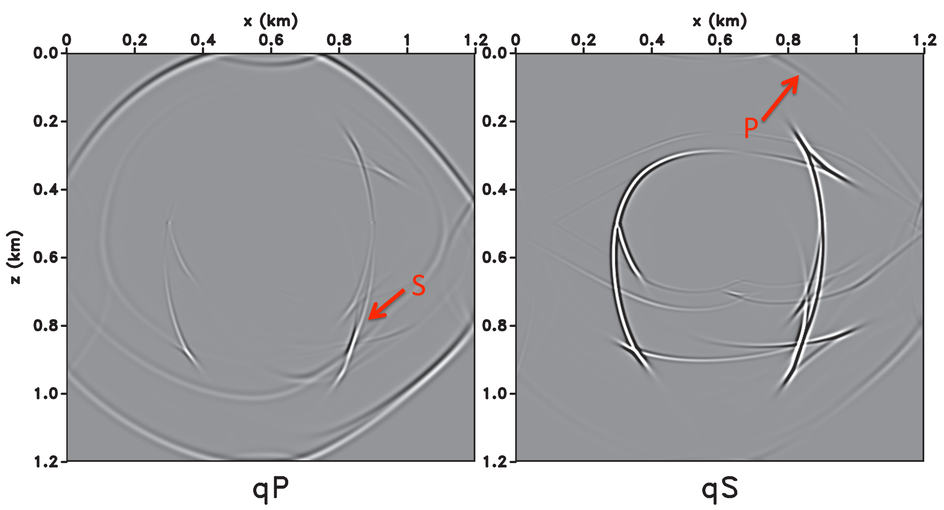


qP qS

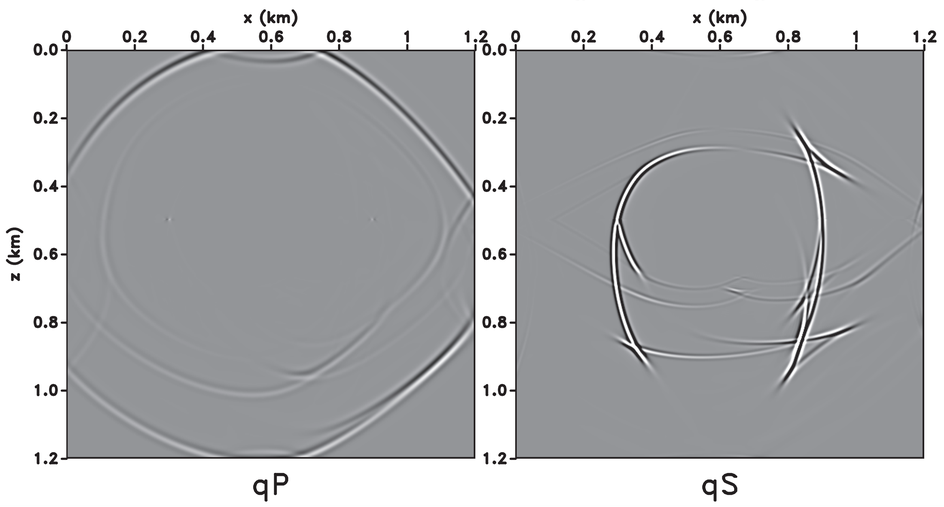
Anisotropic displacement



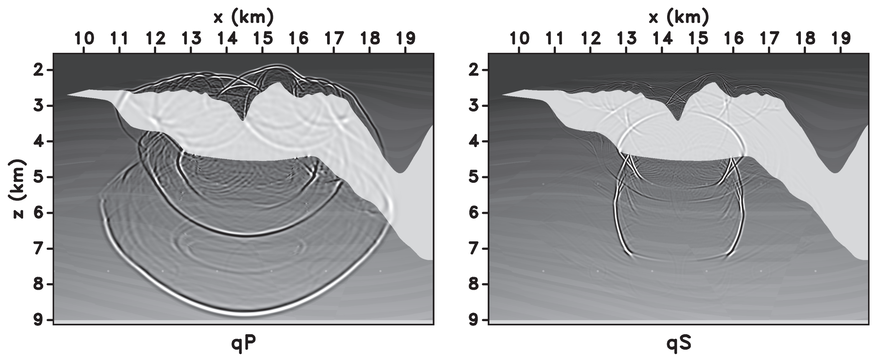
separation with $\nabla\cdot$ and $\nabla\times$



separation with $\nabla_a\cdot$ and $\nabla_a\times$



separation with $\nabla_a\cdot$ and $\nabla_a\times$



CONCLUSIONS

We present a method of obtaining spatially-varying pseudo derivative operators with application to wave mode separation in anisotropic media. The main idea is to utilize polarization vectors constructed in the wavenumber domain using the local medium parameters and then transform these vectors back to the space domain. The main advantage of applying the pseudo derivative operators in the space domain constructed in this way is that they are suitable for heterogeneous media. The wave mode separators obtained using this method are spatially-variable filtering operators and can be used to separate wavefields in VTI media with arbitrary degree of anisotropy. This methodology is applicable for elastic RTM in heterogeneous anisotropic media.

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REFERENCES

Aki, K., and P. Richards, 2002, Quantitative seismology (second edition): University Science Books.

Alkhalifah, T., 2000, An acoustic wave equation for anisotropic media: Geophysics, 65, 1239–1250.

Dellinger, J., and J. Etgen, 1990,Wave-field separation in two- dimensional anisotropic media (short note): Geophysics, 55, 914–919.

Fletcher, R., X. Du, and P. J. Fowler, 2008, A new pseudo-acoustic wave equation for TI media: 78th Annual International Meeting, SEG, Soc. of Expl. Geophys., 2082–2086.

Hokstad, K., 2000, Multicomponent Kirchhoff migration: Geophysics, 65, 861–873.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, 51, 1954–1966.

Tsvankin, I., 2005, Seismic signatures and analysis of reflection data in anisotropic media: 2nd edition: Elsevier Science Publ. Co., Inc.

Yan, J., and P. Sava, 2008, Isotropic angle domain elastic reverse time migration: Geophysics, 73, P.S229–S239.