Moveout analysis for wave-equation extended images

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Abstract

A potentially more robust velocity estimation method can be designed to simultaneously exploit the moveout and focusing information provided by migrated images. We derive the analytic moveout function defined for space- and time-lag common-image gathers and analyze some of its characteristics. Such an analysis may provide deeper insight into the entire extended imaging condition information and thus lead to the possibility of combing semblance and depth focusing analysis for velocity estimation.

Wave-equation imaging condition

•Wave-equation migration consists of two steps: wavefield reconstruction and imaging condition.

•Source wavefield: $u_s = u_s(\boldsymbol{x}, t)$

Receiver wavefield: $u_r = u_r(\boldsymbol{x}, t)$

•Conventional imaging condition (C.I.C)

$$r(\boldsymbol{x}) = \int u_s(\boldsymbol{x}, t)\, u_r(\boldsymbol{x}, t)\, dt$$

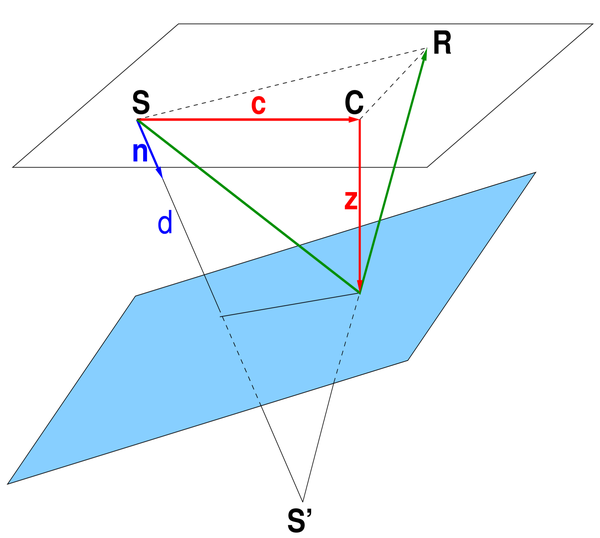
•Extended imaging condition (E.I.C)

$$r(\boldsymbol{x}, \boldsymbol{\lambda}, \tau) = \int u_s(\boldsymbol{x} - \boldsymbol{\lambda}, t - \tau)\, u_r(\boldsymbol{x} + \boldsymbol{\lambda}, t + \tau)\, dt$$

Space-lag is a spatial vector, time-lag is a scalar.

•The extended images can be used for angle decomposition.

•The moveout analysis provides fundamentals for the future velocity estimation techniques.



Geometry of a reflection experiment

Notation

S, **R** and **C** identify the positions of the source, receiver, and image. The reflector is located at distance d from the source position in the direction of the vector **n**. The position of the CIG relative to the source is indicated by the vectors **c** and **z**.

Space- and time-lag moveout analysis

•For C.I.C., the equations represent the source and receiver wavefields. The solution identifies the location where the source and receiver wavefields intersect to form the image.

$$\|\boldsymbol{c} + \boldsymbol{z}\| = vt \qquad \|\boldsymbol{c} + \boldsymbol{z} - 2\mathrm{d}\,\boldsymbol{n}\| = vt$$

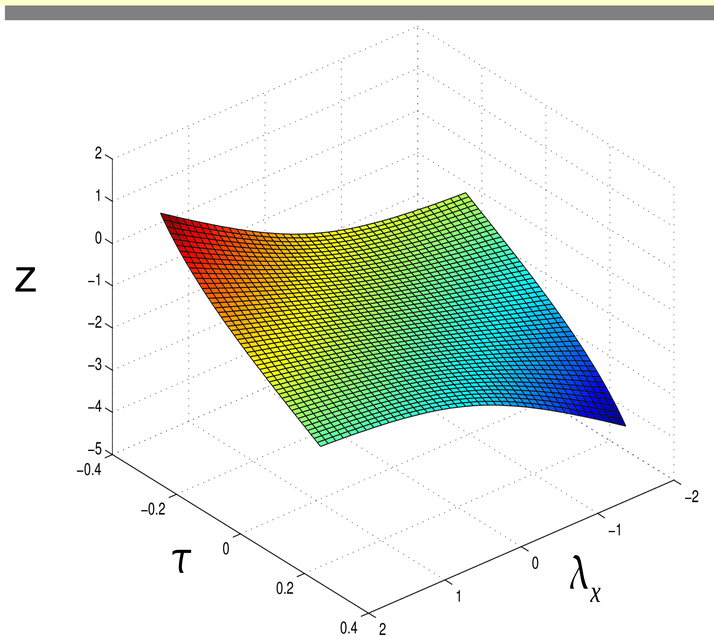
• For E.I.C., the equations represent the shifted source and receiver wavefields. The solution identifies how the image changes with the variation of space-lag and time-lag at a fixed image point location.

$$\|\boldsymbol{c} + \boldsymbol{z} + \boldsymbol{\lambda}\| = v(t + \tau) \qquad \|\boldsymbol{c} + \boldsymbol{z} - 2\mathrm{d}\,\boldsymbol{n} - \boldsymbol{\lambda}\| = v(t - \tau)$$

•A formal solution to the above system leads to the following expression for the reflector depth z whose variables are space-lag $\boldsymbol{\lambda}$ and time-lag τ :

$$z(\boldsymbol{\lambda}, \tau) = (d\, n_z) K + v\tau \sqrt{K^2 + \frac{\|\boldsymbol{c} + \boldsymbol{\lambda}\|^2}{(dn_z)^2 - (v\tau)^2}}$$

$$K = 1 - \frac{(\boldsymbol{c} - d\,\boldsymbol{n}) \cdot \boldsymbol{\lambda} + (\boldsymbol{c} \cdot \boldsymbol{n})\, d - d^2 (n_x^2 + n_y^2)}{(dn_z)^2 - (v\tau)^2}$$



Moveout surface for a horizontal reflector

The analytic surface for the moveout function defined by the above equation in the case of a horizontal reflector. Notice the complex shape and asymmetry of the surface.

Example

We illustrate the moveout function using a synthetic model consisting of a reflector embedded in a constant velocity medium. The extended images are first constructed with the correct velocity model. Then same data are migrated again to build the extended images with an erroneous velocity model.

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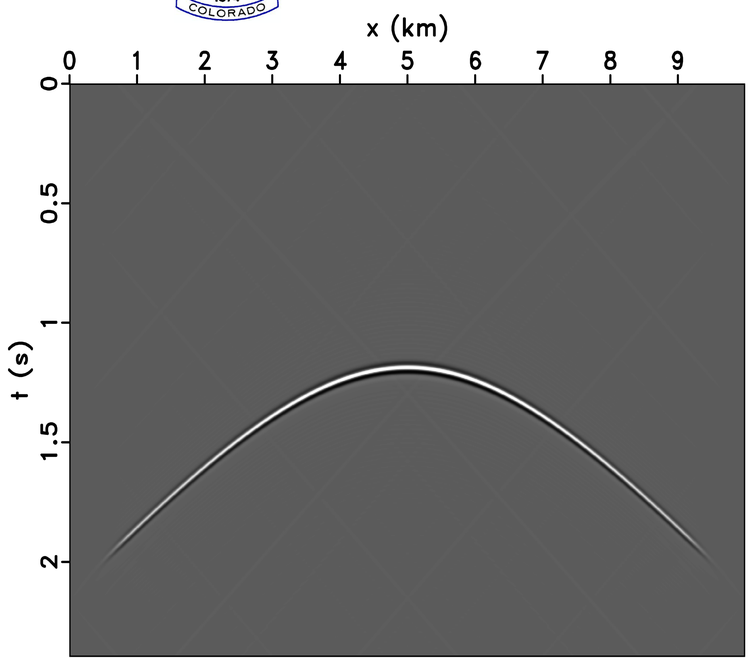
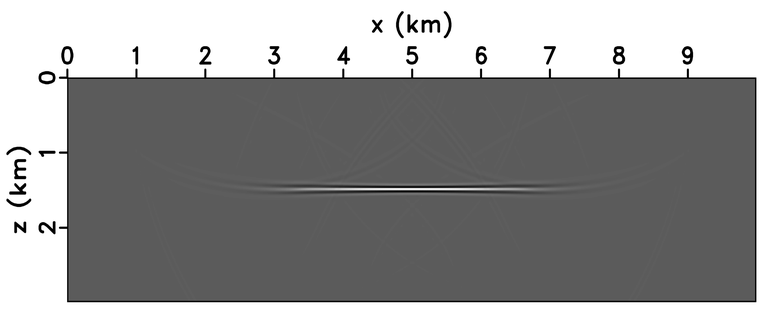
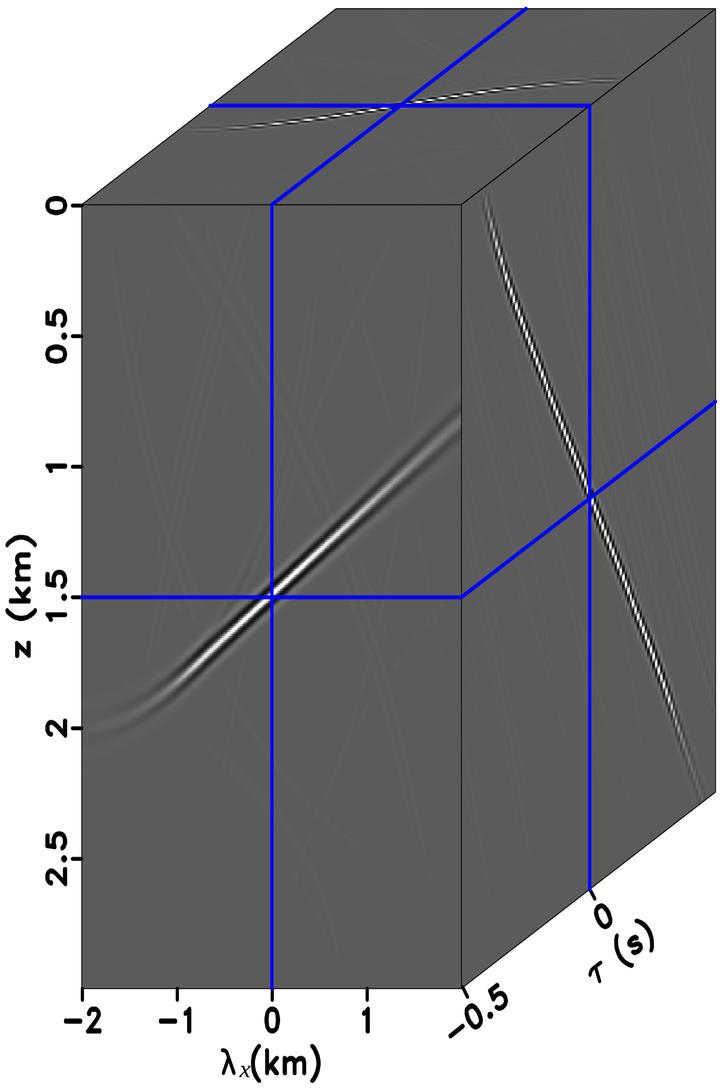


Synthetic data



Migrated image



Extended image gather

- The 3D cube depicts the slices cut from the extended image from different directions.
- The blue lines indicate the cutting location, and the slices are displayed on the corresponding faces of the cube.
- Special case 1: Space-lag moveout, $\tau = 0$
  The moveout function has a linear dependence on horizontal space-lags.
  The linearity justifies the angle decomposition method based on slant-stack.

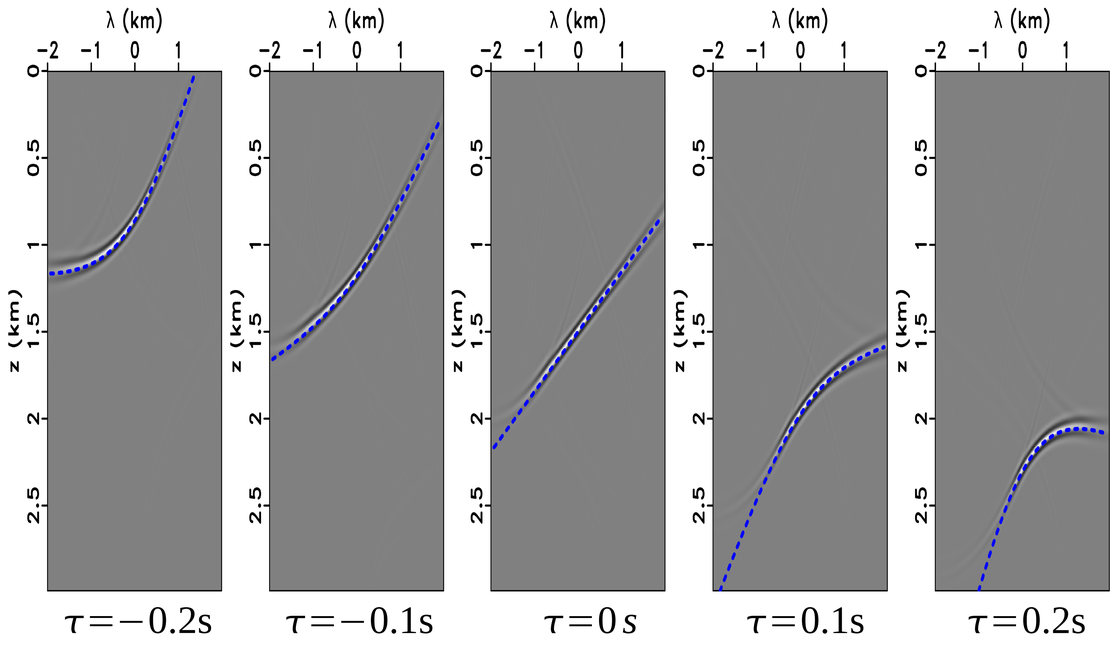
$$z(\lambda) = (dn_z)\, K_\lambda$$

$$K_\lambda = 1 - \frac{(\boldsymbol{c} - d\,\boldsymbol{n})\cdot\lambda + (\boldsymbol{c}\cdot\boldsymbol{n})\, d - d^2(n_x^2 + n_y^2)}{(dn_z)^2}$$

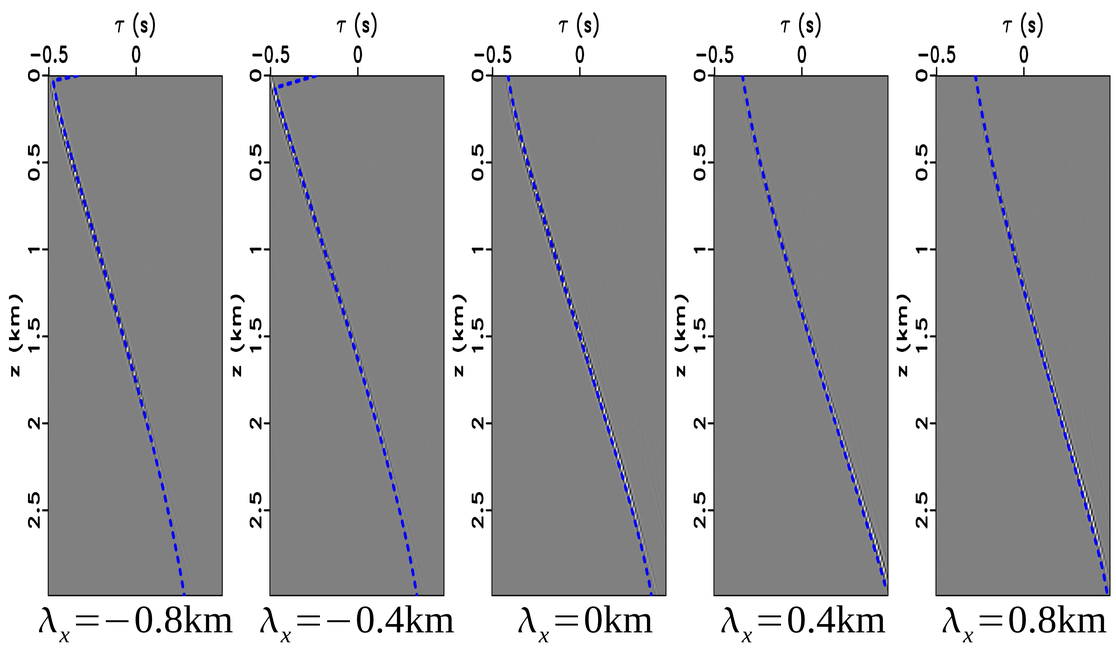
- Special case 2: Time-lag moveout, $\lambda = 0$
  The moveout function has a nonlinear dependence on time-lag.

$$z(\tau) = (dn_z)\, K_\tau + v\tau \sqrt{K_\tau^2 + \frac{\|\boldsymbol{c}\|^2}{(dn_z)^2 - (v\tau)^2}}$$

$$K_\tau = 1 - \frac{(\boldsymbol{c}\cdot\boldsymbol{n})\, d - d^2(n_x^2 + n_y^2)}{(dn_z)^2 - (v\tau)^2}$$



Space-lag CIGs , angle of reflection is 18°



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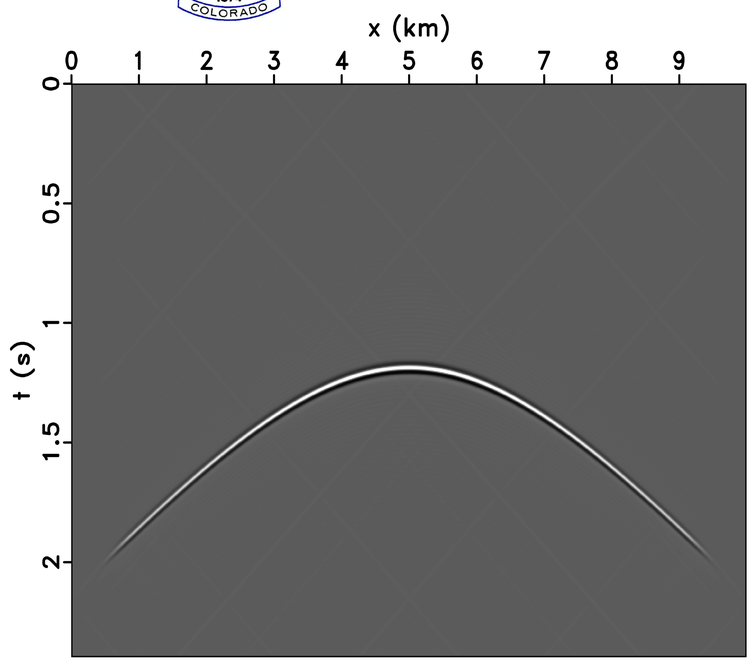
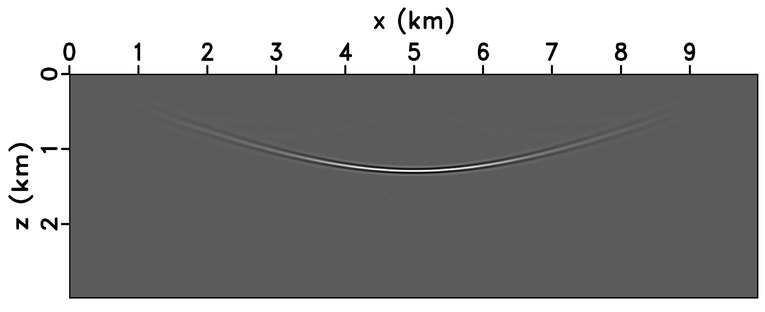
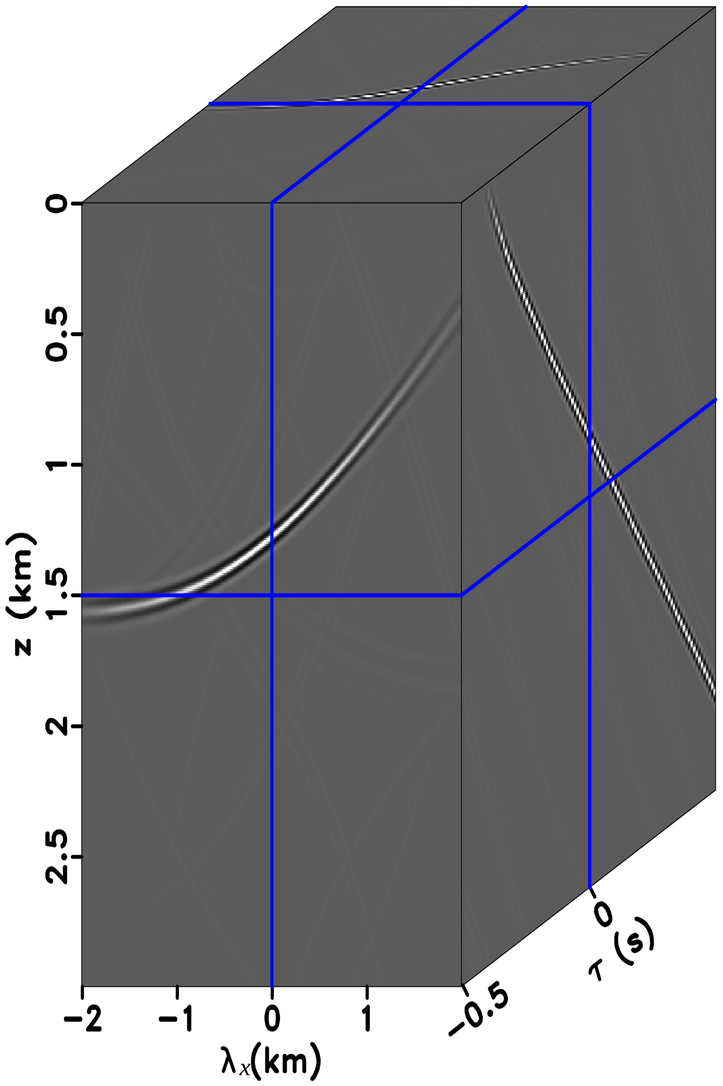


Synthetic data



Migrated image



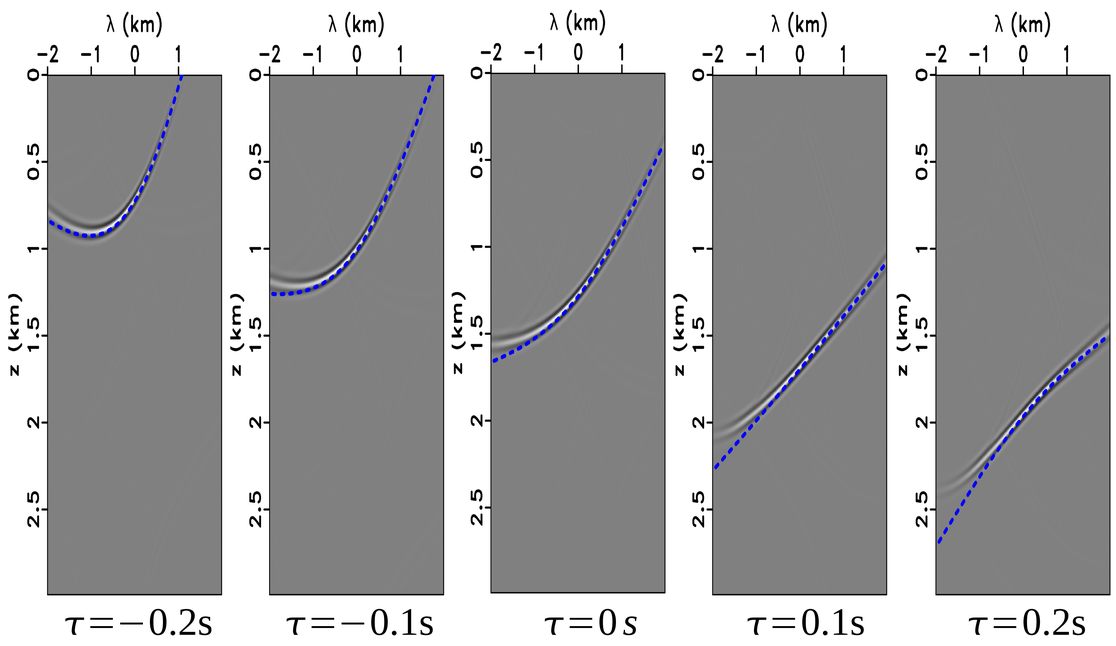
Extended image gather

- The extended images will be influenced by an incorrect velocity model.
- The source and receiver wavefields are propagated with the erroneous velocity model.
- Backward propagation with the erroneous velocity model causes the receiver wavefield to focus at an incorrect depth and time.
- The incorrectly reconstructed wavefields lead to an incorrect image, and thus an incorrect dip and distance of the reflector.
- Below is the moveout function in the erroneous velocity situation. ρ_v is the ratio between erroneous and correct velocity.

$$z(\boldsymbol{\lambda},\tau)=(d_f n'_z)K+v_m(\tau-t_d)\sqrt{K^2+\frac{\|\boldsymbol{c}+\boldsymbol{\lambda}\|^2}{(d_f n_z)^2-v_m^2(\tau-t_d)^2}}$$

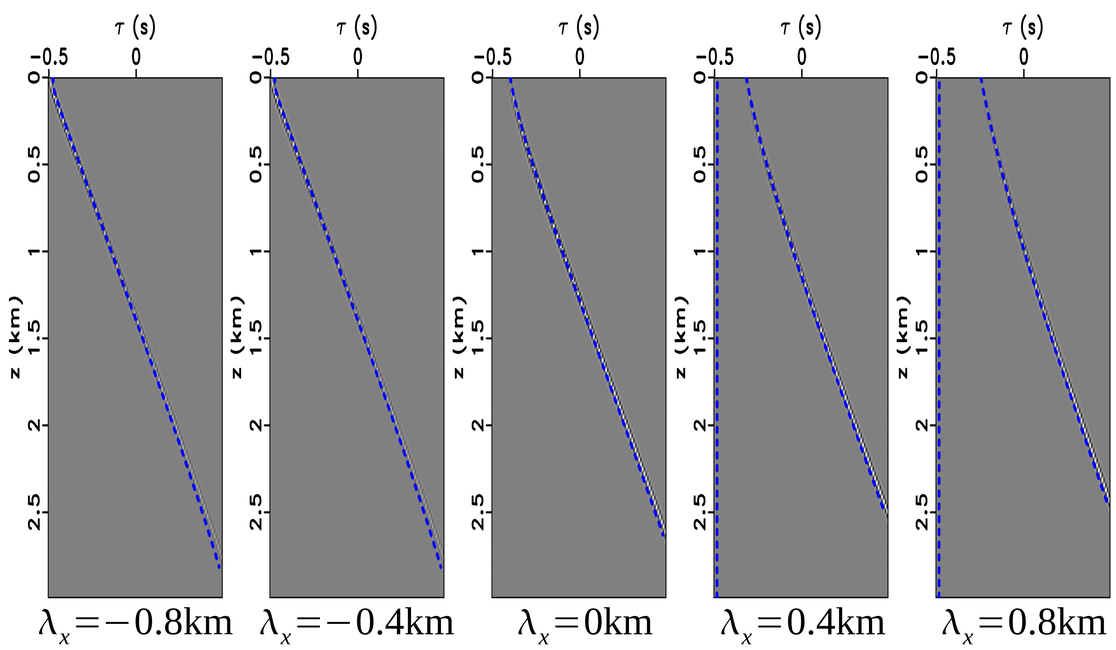
$$K=1-\frac{(\boldsymbol{c}-d_f\boldsymbol{n}')\cdot\boldsymbol{\lambda}+(\boldsymbol{c}\cdot\boldsymbol{n}')d_f-d_f^2(n'^2_x+n'^2_y)}{(d_f n_z)^2-v_m^2(\tau-t_d)^2}$$

$$\rho_v=\frac{v_m}{v_r},\quad d_f=\frac{d}{\rho_v},\quad d_m=d\,\rho_v,\quad t_d=\frac{d_f-d_m}{v_m}=\frac{d(1-\rho_v^2)}{\rho_v v_r}$$



$\tau=-0.2s$ $\tau=-0.1s$ $\tau=0s$ $\tau=0.1s$ $\tau=0.2s$

Space-lag CIGs , angle of reflection is 18°



$\lambda_x=-0.8km$ $\lambda_x=-0.4km$ $\lambda_x=0km$ $\lambda_x=0.4km$ $\lambda_x=0.8km$

Time-lag CIGs, angle of reflection is 18°

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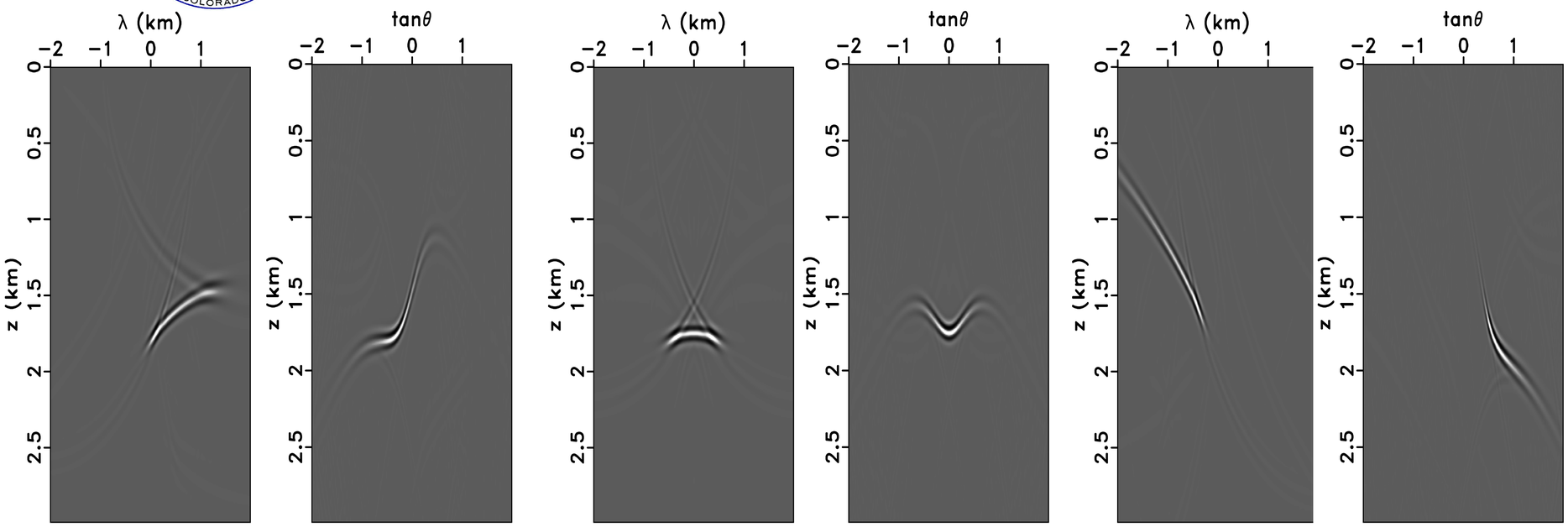
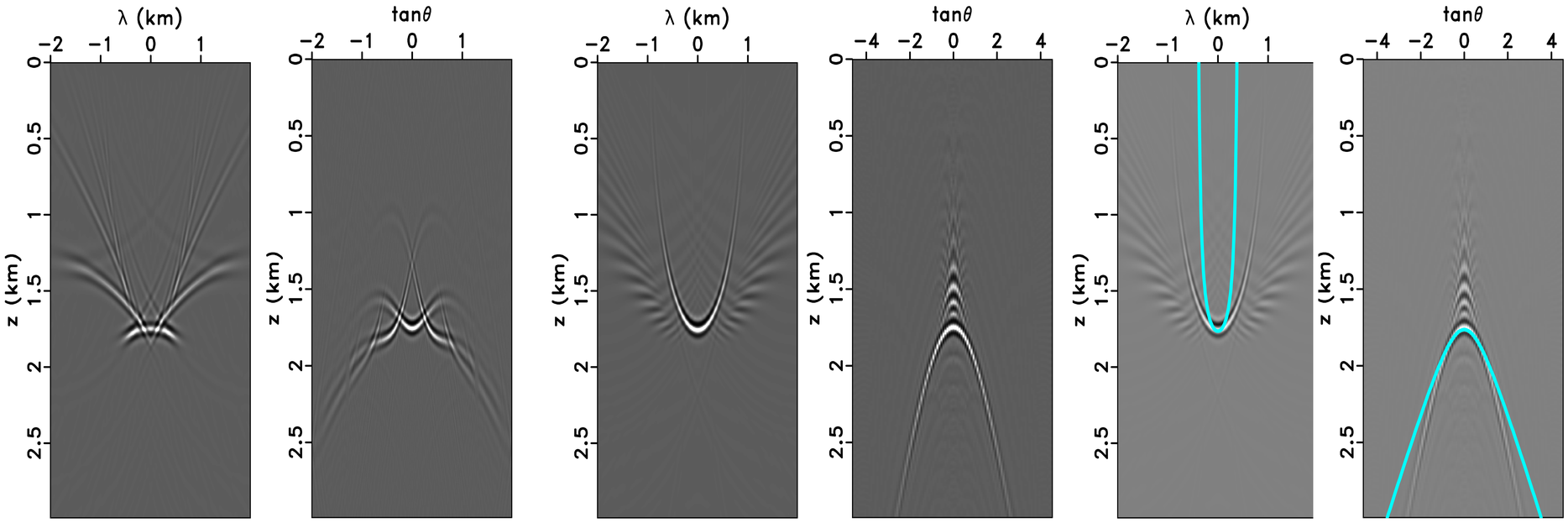


Space-lag CIGs at zero time-lag. Panels correspond to different shot locations



Space-lag CIGs, 15 shots | Space-lag CIGs, 100 shots | Predicted RMO for space-lag CIGs

Application

- The moveout function can be used to predict the RMO.
- For a single shot, we construct the extended image at zero time-lag and map it into angle-domain. The moveout curves from different shot locations have different shapes.
- For many shots, the superimposition of the individual extended images forms the envelope which corresponds to the RMO.
- Taylor expansion is applied to obtain the linearized moveout function, based on which we derive the expressions for the envelope and plot them on the panels.
- Up to the reflection angle of 45°, the envelope of the linearized moveout function can be used as an exact measurement for the RMO.

Conclusions

Extended imaging condition offers the possibility to design robust migration velocity analysis method which exploits conventional semblance analysis and depth focusing analysis simultaneously. The analytic function characterizing the moveout surface is derived under the assumption of homogeneous medium. The synthetic example verifies the correctness and accuracy of the moveout function. The application demonstrates a successful prediction of the RMO in angle-domain, which indicates the future usage in the velocity estimation problems.

Acknowledgments

- Center for Wave Phenomena
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References

Biondi, B., and W. Symes, 2004, Angle-domain common-image gathers for migration velocity analysis by wavefield continuation imaging: Geophysics, **69**, 1283-1298

MacKay, S., and R. Abma, 1992, Imaging and velocity estimation with depth-focusing analysis : Geophysics, **57**, 1608-1622

Faye, J. P,m and J. P. Jeannot, 1986, Prestack migration velocities from focusing depth analysis: 56th Ann. Internat. Mtg., Session:S7.6, Soc. Of Expl. Geophys.

Rickett, J., and P. Sava, 2002, Offset and angle-domain common image-point gathers for shot-profile migration: Geophysics, **67**, 883-889

Sava, P., and B. Biondi, 2004, Wave-equation migration velocity analysis-I: Theory:Geophysical Prospecting, **52**, 593-605

Sava, P., and S. Fomel, 2003, Angle-domain common image gathers by wavefield continuation methods: Geophysics, **68**, 1065-1074